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INTERNATIONAL A LEVEL MATHEMATICS • MECHANICS 2

Mechanics 2

Chapter 1: Projectiles

Exam Questions by Topic

Pearson Edexcel International Advanced Level (WME02/01) • Calculator allowed • Take $g = 9.8\,\text{m}\,\text{s}^{-2}$

Multi-step exam questions at the difficulty of the real WME02 paper, with a full mark scheme

Chapters (tap a chapter to jump to it)		Qs	Marks	Page	My score
1	**Projectiles** Horizontal projection, projection at any angle, the equation of the path	10	84	2	
2	**Variable Acceleration** Calculus for motion in a line and in $\mathbf{i}, \mathbf{j}$ form	10	85	separate file	
3	**Centres of Mass** Particles, laminas, frameworks, suspended and tilting bodies	10	81	separate file	
4	**Work and Energy** Work done, kinetic and potential energy, the work–energy principle, power	10	82	separate file	
5	**Impulses and Collisions** Impulse as a vector, Newton's law of restitution, loss of kinetic energy	10	82	separate file	
6	**Statics of Rigid Bodies** Rods and ladders in equilibrium, friction, hinge forces	10	81	separate file	

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Name ..... Class ..... Date .....

CHAPTER 1 • KEY SKILLS

1 Projectiles

Specification M2: Projectiles • 10 questions • 84 marks • about 101 minutes

1 Components

Speed U at angle α above the horizontal: $U\cos\alpha$ across and $U\sin\alpha$ up. $\tan\alpha = \frac{3}{4}$ gives $\cos\alpha = \frac{4}{5}$, $\sin\alpha = \frac{3}{5}$

2 Horizontal motion

No horizontal force: the horizontal velocity stays $U\cos\alpha$, so $x = (U\cos\alpha)t$

3 Vertical motion

Acceleration $9.8\,\text{m s}^{-2}$ downwards: use suvat with up positive and $a = -9.8$. A point below the start has $s < 0$

4 Time of flight

Set the vertical displacement equal to its final value and solve for t in one equation. Reject the negative root.

5 Greatest height

The vertical component of velocity is zero at the top: $0 = (U\sin\alpha)^2 - 2gh$. The speed there is $U\cos\alpha$, not 0.

6 Velocity at an instant

$v_x = U\cos\alpha$, $v_y = U\sin\alpha - gt$. Speed $= \sqrt{v_x^2 + v_y^2}$; direction $\tan\theta = \frac{|v_y|}{v_x}$, above or below the horizontal.

7 Speed conditions

Speed $\leqslant k$ means $v_y^2 \leqslant k^2 - v_x^2$, a condition on the vertical component only.

8 Equation of the path

$y = x\tan\alpha - \frac{gx^2}{2U^2\cos^2\alpha}$ and $\frac{1}{\cos^2\alpha} = \sec^2\alpha = 1 + \tan^2\alpha$ give a quadratic in $\tan\alpha$

9 Vectors

Velocity $(p\mathbf{i} + q\mathbf{j})$: horizontal part p, vertical part q. Perpendicular directions: gradients multiply to -1

Skills map every skill above is tested at least once; interleaved skills come from GCSE or other chapters

Skill	Q1	Q2	Q3	Q4	Q5	Q6	Q7	Q8	Q9	Q10
Marks	7	6	6	6	7	11	10	10	10	11
1 Components		•		•		•	•	•	•	
2 Horizontal motion	•	•	•		•		•	•	•	•
3 Vertical motion	•	•	•	•		•	•		•	•
4 Time of flight	•	•				•			•	•
5 Greatest height		•		•		•			•	
6 Velocity at an instant	•		•	•		•	•			•
7 Speed conditions				•						
8 Equation of the path					•			•		
9 Vectors			•			•				•
Interleaved: constant acceleration and vectors (M1)	•		•			•	•			•
Interleaved: quadratics and inequalities (P1)				•	•	•		•		•
Interleaved: trigonometric identities, exact values (P2, P3)		•			•			•	•	

Exam difficulty. Questions 1 to 5 check each skill on its own. Questions 6 to 10 are multi-step questions built like the last questions on a real WME02 paper, where one part leads into the next and skills from different topics are combined.

Must learn. Write R(→) and R(↑) before each equation and keep up as positive. Take $g = 9.8\,\text{m s}^{-2}$ and give answers to 2 or 3 significant figures, not as fractions. A speed is a magnitude; a direction needs an angle and a reference line.

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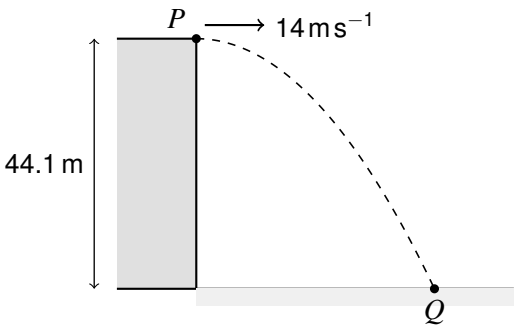


Figure 1

A stone is thrown horizontally with speed $14\,\text{m}\,\text{s}^{-1}$ from a point P at the top of a vertical cliff. The point P is 44.1 m above the sea. The stone hits the sea at the point Q, as shown in Figure 1. The stone is modelled as a particle moving freely under gravity.

(a) Find the time taken by the stone to travel from P to Q.

(2)

(b) Find the horizontal distance from the foot of the cliff to Q.

(1)

(c) Find the speed of the stone as it hits the sea, and the angle its direction of motion makes with the horizontal at this instant.

(4)

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Question 1 continued

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(Total for Question 1 is 7 marks)

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2. A ball is projected from a point O on horizontal ground with speed $35\,\text{m}\,\text{s}^{-1}$ at an angle of elevation α, where $\tan\alpha = \frac{3}{4}$. The ball moves freely under gravity until it hits the ground at the point A.

(a) Find the greatest height above the ground reached by the ball.

(3)

(b) Find the distance OA.

(3)

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Question 2 continued

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(Total for Question 2 is 6 marks)

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3. *[In this question,* $\mathbf{i}$ *and* $\mathbf{j}$ *are horizontal and vertically upward unit vectors respectively.]*

A particle P is projected from a fixed point O with velocity $(9.8\mathbf{i} + 14.7\mathbf{j})\,\text{m s}^{-1}$ and moves freely under gravity.

(a) Find the time after projection at which P is moving in a direction 45° below the horizontal.

(3)

(b) Find the position vector of P relative to O at this time.

(3)

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Question 3 continued

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4. A particle is projected from a point on horizontal ground with speed $30\,\text{m}\,\text{s}^{-1}$ at an angle of 60° above the horizontal, and moves freely under gravity until it hits the ground.

(a) Find the least speed of the particle during its flight.

(2)

(b) Find the length of time for which the speed of the particle is less than or equal to $25\,\text{m}\,\text{s}^{-1}$

(4)

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Question 4 continued

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5. A particle is projected from a point O with speed U at an angle α above the horizontal and moves freely under gravity. When the particle has moved a horizontal distance x from O, its height above O is y.

(a) Show that $y = x\tan\alpha - \frac{gx^2}{2U^2}\left(1+\tan^2\alpha\right)$

(4)

A particle is projected from O with speed $28\,\text{m}\,\text{s}^{-1}$ and passes through the point which is 40 m horizontally from O and 10 m above O.

(b) Find the two possible values of α, giving your answers to the nearest 0.1°

(3)

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Question 5 continued

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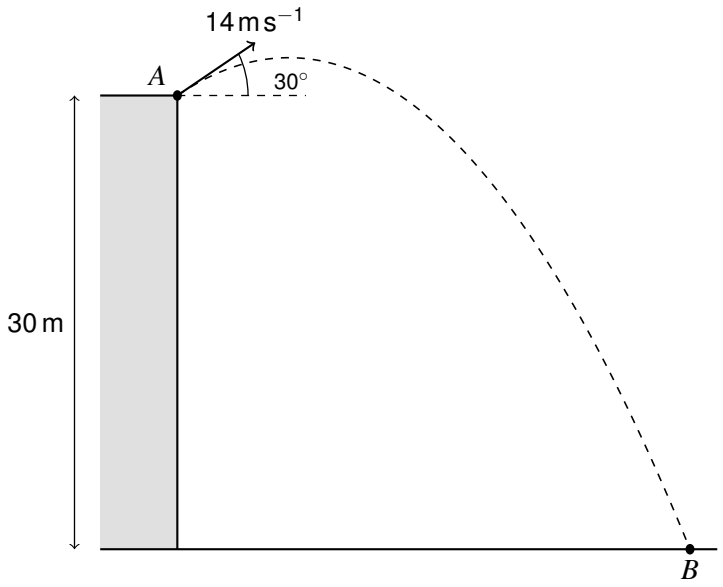


Figure 2

A ball is projected from a point A which is 30 m above horizontal ground, with speed 14 m s^{-1} at an angle of 30° above the horizontal. The ball moves freely under gravity and hits the ground at the point B, as shown in Figure 2.

(a) Find the greatest height above the ground reached by the ball.

(2)

(b) Find the time taken by the ball to travel from A to B.

(3)

(c) Show that the ball hits the ground with speed 28 m s^{-1}

(2)

At the point C on its path, the ball is moving perpendicular to its initial direction of motion.

(d) Find the height of C above the ground.

(4)

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Question 6 continued

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Question 6 continued

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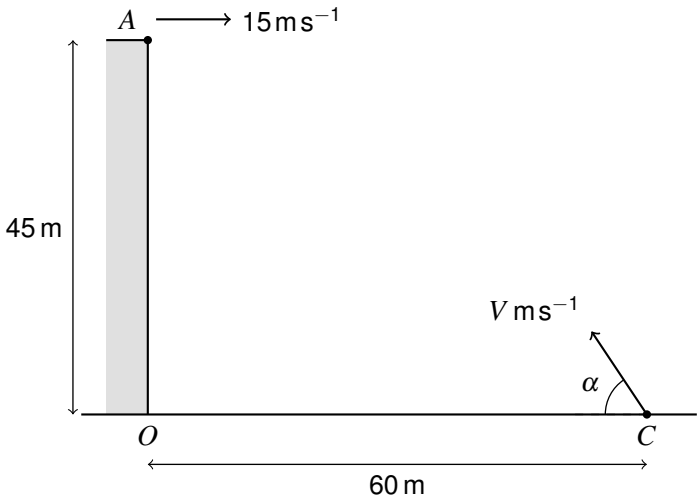


Figure 3

A ball A is projected horizontally with speed $15\,\text{m s}^{-1}$ from the top of a vertical tower OA of height 45 m. At the same instant, a second ball B is projected from the point C on horizontal ground, where $OC = 60$ m, with speed $V\,\text{m s}^{-1}$ at an angle α above the horizontal, towards the tower, as shown in Figure 3. The balls move freely under gravity in the same vertical plane and collide 2 s after they are projected.

(a) Find the height above the ground at which the balls collide.

(2)

(b) By considering the horizontal motion of the balls, show that $V\cos\alpha = 15$

(2)

(c) Find the value of V and the value of α.

(4)

(d) Determine whether B is moving upwards or downwards at the instant of the collision.

(2)

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Question 7 continued

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Question 7 continued

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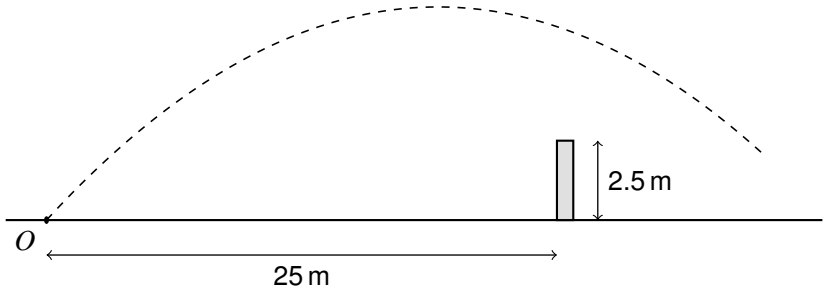


Figure 4

A ball is kicked from a point O on horizontal ground with speed $20\,\text{m s}^{-1}$ at an angle α above the horizontal. A vertical wall of height 2.5 m stands 25 m from O, as shown in Figure 4. The ball moves freely under gravity in a vertical plane perpendicular to the wall, and the wall is modelled as having no thickness.

(a) Show that, when the ball is a horizontal distance x m from O, its height y m above the ground is given by

$$y = x\tan\alpha - \frac{49x^2}{4000}\left(1+\tan^2\alpha\right)$$

(3)

(b) Show that the ball passes over the wall only if $49\tan^2\alpha - 160\tan\alpha + 65 < 0$

(4)

(c) Hence find the range of values of α for which the ball passes over the wall, giving the limits to the nearest 0.1°

(3)

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Question 8 continued

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Question 8 continued

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(Total for Question 8 is 10 marks)

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9. A particle is projected from a point O on horizontal ground with speed U at an angle α above the horizontal, where $0 < \alpha < \frac{\pi}{2}$. It moves freely under gravity until it hits the ground.

(a) Show that the time of flight is $\dfrac{2U\sin\alpha}{g}$

(2)

(b) Show that the horizontal distance from O to the point where the particle lands is $\dfrac{U^2\sin 2\alpha}{g}$

(2)

A stone is thrown from O with speed $21\,\text{m}\,\text{s}^{-1}$ and lands 22.5 m from O.

(c) Find the two possible angles of projection.

(3)

(d) Show that the ratio of the greatest heights reached for these two angles is $\left(7-4\sqrt{3}\right) : 1$

(3)

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Question 9 continued

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Question 9 continued

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10. *[In this question,* $\mathbf{i}$ *and* $\mathbf{j}$ *are horizontal and vertically upward unit vectors respectively.]*

Two particles P and Q are projected at the same instant from a point O on horizontal ground. The initial velocity of P is $(3\mathbf{i}+12\mathbf{j})\,\mathrm{m\,s^{-1}}$ and the initial velocity of Q is $(2\mathbf{i}+4\mathbf{j})\,\mathrm{m\,s^{-1}}$. The particles move freely under gravity, and each comes to rest when it hits the ground.

(a) Find the distance between P and Q at the instant Q hits the ground.

(4)

(b) Show that the directions of motion of P and Q are perpendicular when $(9.8t)^2 - 16(9.8t) + 54 = 0$, where t seconds is the time after projection.

(4)

(c) Hence find the time at which the directions of motion of P and Q are perpendicular, explaining why only one root of the equation in part (b) is valid.

(3)

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Question 10 continued

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TOTAL FOR CHAPTER 1 IS 84 MARKS

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