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A-LEVEL MATHEMATICS • DECISION 1

Linear Programming

Worksheet

Pearson Edexcel International A Level (WDM11) • Calculator allowed: show every step

Free Linear Programming worksheet for A-Level and International A-Level Maths, with full worked solutions

A **Examples** & Your turn page 2

B **Fluency** core skills page 4

C **Reasoning** explain it page 6

D **Problem** solving page 7

E **Exam** practice page 11

Tap a section to jump straight to it

By the end of this worksheet you can: formulate constraints, including fractions, percentages and ratios • graph the feasible region and use the objective line or the vertex method • *Stretch:* integer solutions and objectives with an unknown coefficient.

Which questions are for me?

CORE The methods every A-level student must secure. Start here.

STRETCH Turning a problem into inequalities, drawing the feasible region, and finding the best solution exactly and in whole numbers.

Every section has both, clearly labelled. Questions get harder as you go, and every skill in Sections A to D is needed in Section E.

Track your progress

Section	B	C	D	E
Core marks	23	10	13	22
Stretch marks	16	14	17	14
My score				

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A Worked examples & Your turn

Read each example carefully. Cover it up, then answer the question beside it using the same method.

CORE *Secure the method*

Worked example A1: a ratio

Write "x is at least twice y" as an inequality.

$$x \geqslant 2y$$

Your turn A1

Write "y is at most three times x" as an inequality.

Worked example A2: a percentage

At least 30% of the total must be x. Write and simplify the constraint.

$x \geqslant 0.3(x+y)$, *so* $0.7x \geqslant 0.3y$

$$7x \geqslant 3y$$

Your turn A2

At most 40% of the total may be y. Write and simplify the constraint.

Worked example A3: fractions of a resource

In a day a machine can make 200 of A or 300 of B. Write the constraint for x of A and y of B.

$\frac{x}{200} + \frac{y}{300} \leqslant 1$; *multiply by 600*

$$3x + 2y \leqslant 600$$

Your turn A3

In a day a machine can make 150 of A or 250 of B. Write the constraint for x of A and y of B.

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Worked example A4: a vertex

Find where $2x + y = 10$ meets $x + 3y = 15$

$x = 15 - 3y$; $30 - 6y + y = 10$

$(3,\ 4)$

Your turn A4

Find where $3x + 2y = 18$ meets $x + 4y = 16$

Worked example A5: the vertex method

R has vertices $(0,0)$, $(5,0)$, $(3,4)$, $(0,5)$. Maximise $P = 2x + 3y$

P: 0, 10, 18, 15

18 at $(3,4)$

Your turn A5

R has vertices $(0,0)$, $(6,0)$, $(4,3)$, $(0,4)$. Maximise $P = 2x + 3y$

Worked example A6: an objective line

State the gradient of the objective line for $P = 3x + 2y$

$-\frac{3}{2}$

Your turn A6

State the gradient of the objective line for $P = 4x + 5y$

Worked example A7: an integer point

The optimal vertex is $(2.6,\ 4.5)$. Which integer points should be tested?

$(2,4)$, $(3,4)$, $(2,5)$, $(3,5)$, if in R

Your turn A7

The optimal vertex is $(5.2,\ 1.7)$. Which integer points should be tested?

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B Fluency

Each question is a designed sequence. From one part to the next, ask yourself: what changed, and what did it do to the answer?

CORE *Secure the method*

B1 A café makes x litres of soup and y litres of stew each day. Soup uses 2 kg of vegetables per litre and stew uses 3 kg; 60 kg are available. At least 5 litres of stew must be made. Profit is £4 per litre of soup and £5 per litre of stew.

(a) Define the decision variables.

(b) Write down the objective.

(c) Write the vegetable constraint.

(d) Write the stew constraint.

[4 marks]

B2 The constraints are $x+y \leqslant 10$, $3x+y \leqslant 21$, $x+3y \leqslant 24$, $x,\ y \geqslant 0$

(a) Find where $x+y=10$ meets $3x+y=21$

(b) Find where $x+y=10$ meets $x+3y=24$

(c) List all the vertices of R.

(d) Decide whether $(5,5)$ lies in R.

[4 marks]

B3 For the region in B2,

(a) evaluate $P=4x+5y$ at each vertex

(b) state the maximum of P

(c) find the vertex that maximises $Q=5x+2y$

(d) state the maximum of Q

[4 marks]

B4 The constraints are $x+y \geqslant 8$, $2x+y \geqslant 11$, $x+4y \geqslant 12$, $x,\ y \geqslant 0$

(a) Find the vertices of R.

(b) Minimise $C=3x+5y$

(c) Minimise $D=x+y$

(d) Explain your answer to (c).

[4 marks]

B5 For the region in B2, $P=2x+5y$ is to be maximised.

(a) State the gradient of the objective line.

(b) State the direction in which to slide it.

(c) Find the optimal vertex and the maximum value. **[3 marks]**

B6 The constraints are $5x+3y \leqslant 31$, $2x+5y \leqslant 26$, $x,\ y \geqslant 0$, and $P=4x+5y$ is to be maximised.

(a) Find the optimal vertex exactly.

(b) Find the maximum value.

(c) Test the integer points near it.

(d) State the integer optimum.

[4 marks]

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STRETCH

B7 Write each condition as a simplified inequality in x and y

(a) At least 40% of the items are x

(b) y is at most 3 times x

(c) There are at least 10 more y than x

(d) There are at least 50 items in total.

[4 marks]

B8 R is the region above the line $y = 1$, below the line through $(0, 6)$ and $(9, 0)$, and on the same side as $(5, 1.5)$ of the line through $(0, 0)$ and $(3, 6)$.

(a) Find the first line's equation.

(b) Find the second line's equation.

(c) Write the three inequalities.

(d) Find the vertices of R.

[4 marks]

B9 For the region in B2, the objective is to maximise $P = x + ky$

(a) State the gradients of the two lines through $(3, 7)$

(b) State the gradient of the objective line.

(c) Find the range of k for which $(3, 7)$ is optimal.

(d) State the optimal vertex when $k = 4$

[4 marks]

B10 In a day machine I can make 60 of A or 90 of B, and machine II can make 120 of A or 40 of B

(a) Write machine I's constraint.

(b) Write machine II's constraint.

(c) Find where the two lines meet.

(d) Explain what this point means.

[4 marks]

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C Reasoning

Show your working and explain your thinking. A correct answer without a reason does not score full marks.

CORE *Secure the method*

C1 Amir writes the objective for B1 as "Maximum $P = 4x + 5y$". Explain his error. **[2 marks]**

C2 Bea defines the variables in B1 as "x = soup, y = stew". Explain why this is not good enough. **[2 marks]**

C3 For "at least 30% of the trays are herbs (x)", Cal writes $x \geqslant 0.3y$. Explain his error and give the correct simplified constraint. **[3 marks]**

C4 In B6, Dev rounds the optimal vertex to $(4, 4)$ and gives the integer optimum as 36. Explain his error. **[3 marks]**

STRETCH

C5 To minimise $C = 3x + 5y$ in B4, Eve slides the objective line away from the origin until it last touches R. Explain her error. **[3 marks]**

C6 In B2, Fay reads the vertex where $x + y = 10$ meets $3x + y = 21$ from her graph as (5.4, 4.6). Explain why this is not acceptable, and give the exact vertex. **[3 marks]**

C7 For the constraint $y \geqslant 2$, Gus shades the region above the line $y = 2$ and labels the unshaded part R. Explain his error. **[3 marks]**

C8 **(a)** Explain why, if a linear objective has a maximum on a bounded feasible region, the maximum occurs at a vertex.

(3)

(b) Explain how to decide which side of the line $ax + by = c$ satisfies $ax + by \leqslant c$, when the line does not pass through the origin.

(2)

[5 marks]

D Problem solving

You decide the method. Read the whole question first, and show every step.

CORE *Secure the method*

D1 A workshop makes x kites and y banners each week. Sam cuts the material and could cut 20 kites or 50 banners in a week. Ana paints them and could paint 30 kites or 40 banners in a week. There must be at least twice as many banners as kites. Write down these three constraints, simplifying each so that it has integer coefficients. **[3 marks]**

D2 Figure 1 shows four lines and the feasible region R, which is shaded.

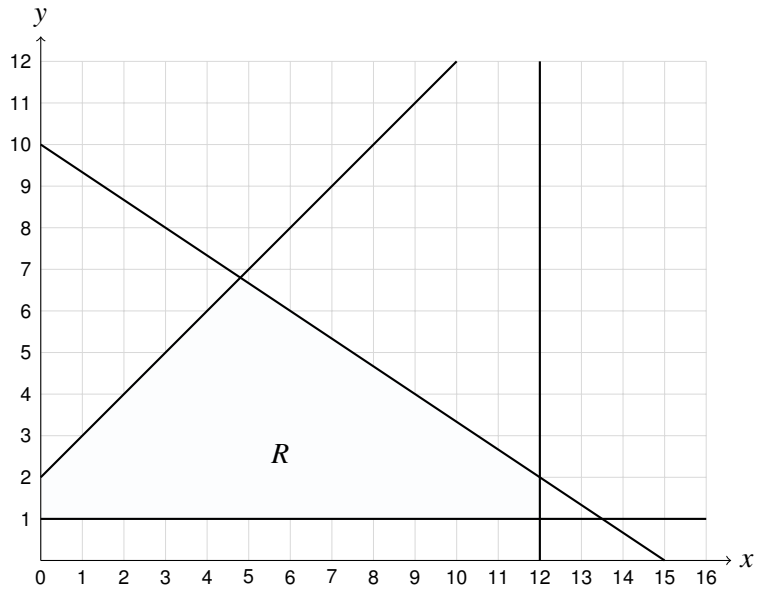


Figure 1

Determine the four inequalities, other than $x \geqslant 0$, that define R, giving each with integer coefficients. **[4 marks]**

D3 Figure 2 shows the feasible region R for a linear programming problem.

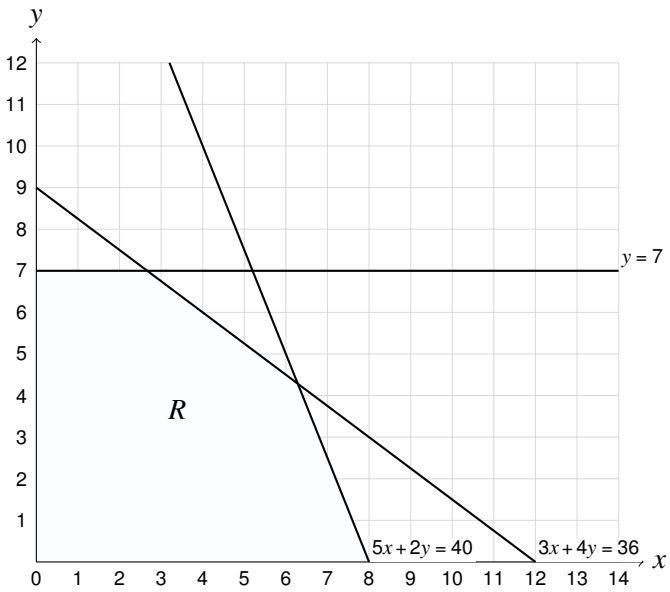
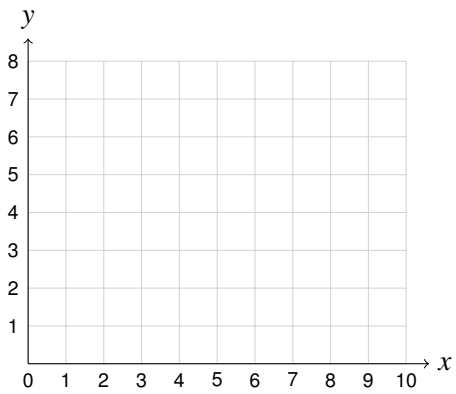


Figure 2

Use the vertex method to find the maximum value of $P = 3x + 5y$ on R, finding any vertices you use exactly.

[3 marks]

D4 Represent the constraints $x + y \leqslant 9$, $y \geqslant 2$ and $y \leqslant 2x$ on the grid, label the feasible region R, and write down the coordinates of its vertices.



Grid for D4

[3 marks]

STRETCH

D5 Figure 3 shows the feasible region R for a linear programming problem.

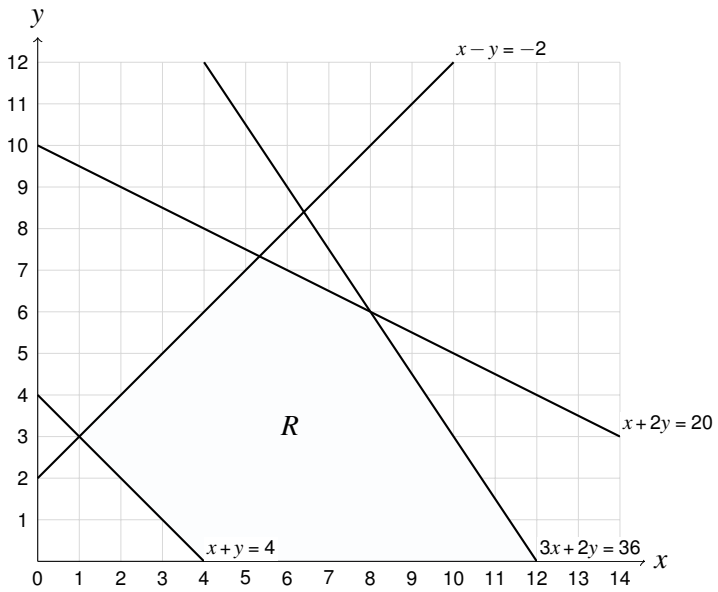


Figure 3

(a) Use an objective line to find the vertex of R at which $P = x + 3y$ is greatest, and find this greatest value exactly.

(2)

(b) Find the least value of P on R, explaining how you used the objective line.

(2)

[4 marks]

D6 A garden centre sells x trays of herbs and y trays of flowers each week. It can sell at most 200 trays, and at least 30 trays of flowers. At most 60% of the trays sold may be herbs. Each tray of herbs makes a profit of £3 and each tray of flowers £4. The centre wishes to maximise its profit.

(a) Define the decision variables and write down the objective.

(2)

(b) Write down the constraints, simplifying the percentage constraint so that it has integer coefficients.

(2)

[4 marks]

D7 Figure 4 shows the feasible region R for a problem in which $P = 5x + 4y$ is to be maximised, where x and y must be integers.

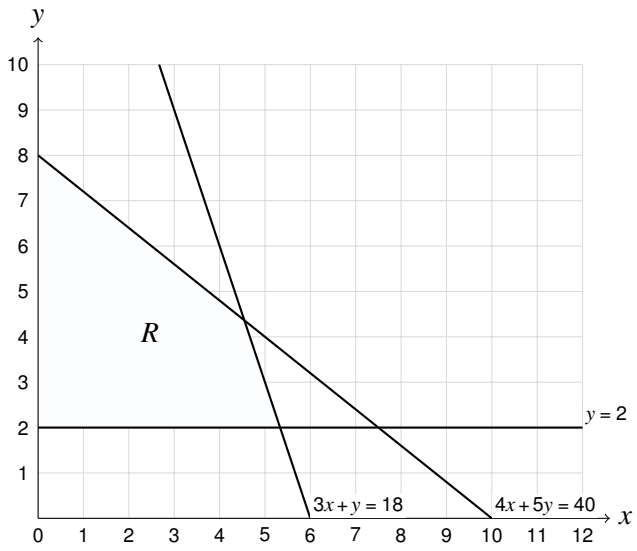


Figure 4

(a) Find the exact coordinates of the optimal vertex of R and the value of P there.

(2)

(b) Find the optimal integer solution, showing which points you tested.

(2)

[4 marks]

D8 The constraints of a problem are $2x + y \leqslant 16$, $x + 3y \leqslant 18$, $x \leqslant 7$ and $x,\ y \geqslant 0$. The objective is to maximise $P = kx + 3y$, where k is a positive constant.

(a) Find the coordinates of all the vertices of the feasible region.

(2)

(b) Determine the range of values of k for which the vertex $(6, 4)$ is optimal.

(3)

[5 marks]

E Exam practice

Exam-style questions in the style of WDM11. Write your answers in the spaces, and show every step.

CORE *Exam questions for everyone*

E1 A florist makes x small bouquets and y large bouquets each day. Each small bouquet uses 3 stems and each large bouquet uses 5 stems, and 60 stems are available each day. Each bouquet takes 15 minutes to make, and the florist works for 4 hours each day. The number of small bouquets must be no more than three times the number of large bouquets. The profit on a small bouquet is £5 and on a large bouquet is £6, and the florist wishes to maximise the daily profit.

(a) Write down three constraints, other than $x,\ y \geqslant 0$, simplifying each so that it has integer coefficients.

(3)

(b) Represent your constraints on the grid on the answer-book page. Make your shading clear and label the feasible region R.

(4)

(c) Use an objective line to find the number of each type of bouquet the florist should make, and the maximum daily profit. You must make your objective line clear.

(3)

(Total for Question E1 is 10 marks)

Answer book: Question E1

Work in the space on this page, as you would in the WDM11 answer book.

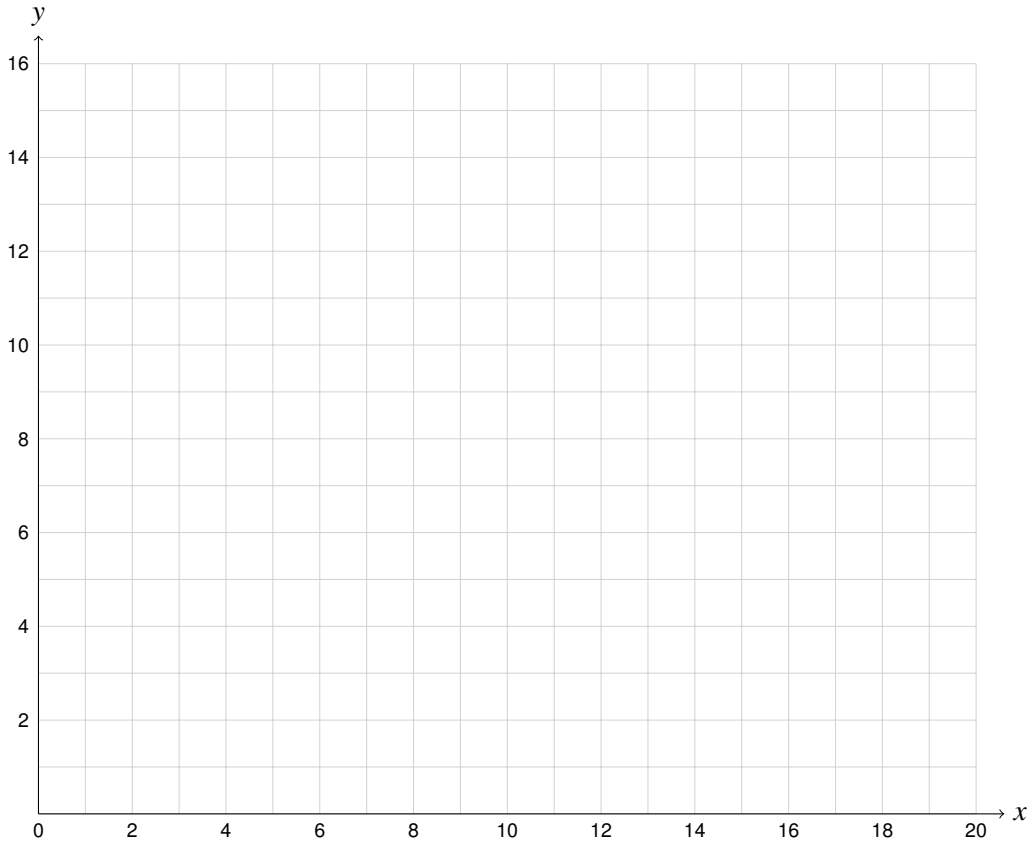


Diagram 1

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E2 A factory makes two products, A and B, using two machines. In one day, machine I could make 300 of A or 200 of B, or a combination of the two in proportion. In one day, machine II could make 250 of A or 400 of B, or a combination in proportion. At least 20% of the items made each day must be B. The profit is £5 on each A and £6 on each B, and the factory wishes to maximise its daily profit. Let x and y be the numbers of A and B made each day.

(a) Show that the machine constraints simplify to $2x + 3y \leqslant 600$ and $8x + 5y \leqslant 2000$, and write down the percentage constraint with integer coefficients.

(3)

(b) Represent the constraints on the grid on the answer-book page and label the feasible region R.

(3)

(c) Use the vertex method to find the exact optimal vertex and the exact maximum profit.

(3)

(d) Given that x and y must be integers, find the optimal numbers of A and B and the profit. You must show the points you test.

(3)

(Total for Question E2 is 12 marks)

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Answer book: Question E2

Work in the space on this page, as you would in the WDM11 answer book.

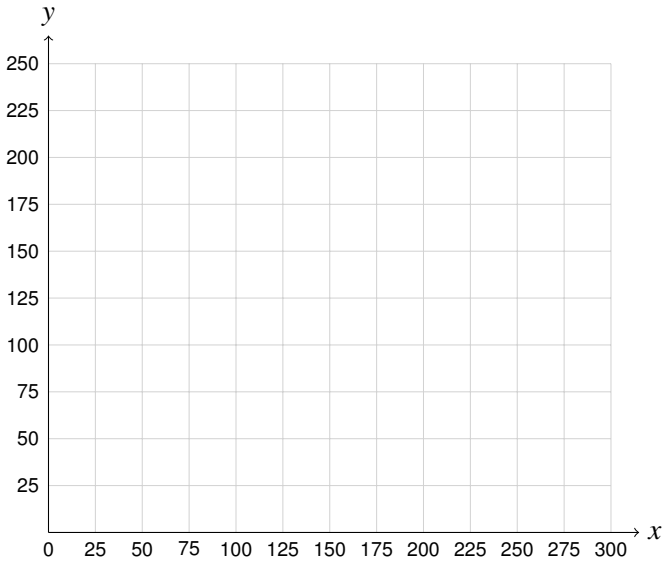


Diagram 2

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STRETCH

E3 A farmer mixes x bags of feed A and y bags of feed B. Each bag of A provides 3 units of protein and 1 unit of fibre; each bag of B provides 2 units of protein and 4 units of fibre. The mix must provide at least 30 units of protein and at least 24 units of fibre. The number of bags of A must be no more than twice the number of bags of B. A bag of A costs £5 and a bag of B costs £4, and the farmer wishes to minimise the cost.

(a) Formulate this as a linear programming problem, stating the objective and the constraints.

(3)

(b) Represent the constraints on the grid on the answer-book page and label the feasible region R.

(3)

(c) Find the exact optimal vertex of R and the exact minimum cost, explaining your method.

(3)

(d) The farmer can only buy whole bags. Find the optimal integer solution and its cost.

(2)

(e) The cost of a bag of A changes to £c, while the cost of B stays at £4. Determine the range of values of c for which the vertex in part (c) remains optimal.

(3)

(Total for Question E3 is 14 marks)

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Answer book: Question E3

Work in the space on this page, as you would in the WDM11 answer book.

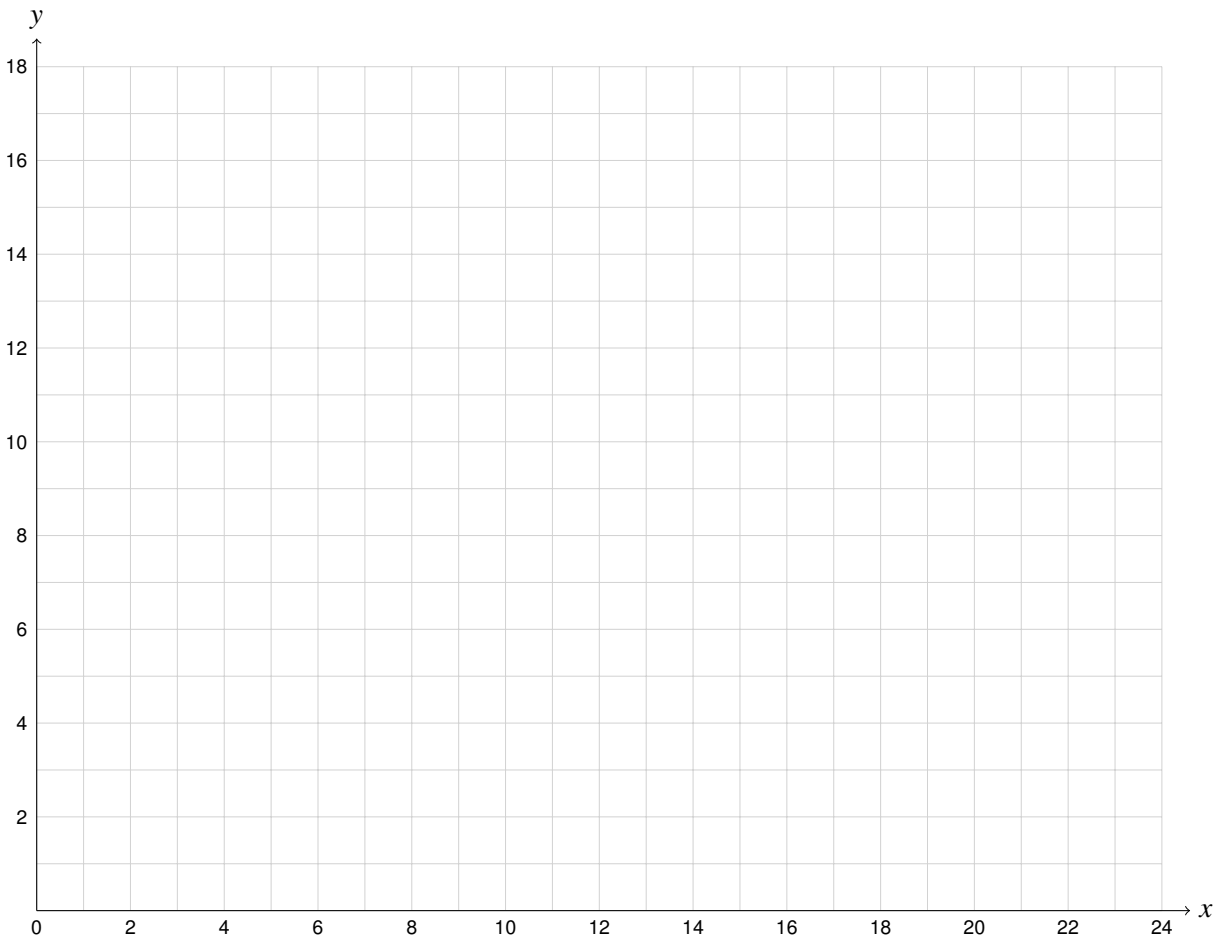


Diagram 3

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Open the booklet

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