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INTERNATIONAL A LEVEL MATHEMATICS • PURE 2

Pure Mathematics 2

Exam Questions by Topic: Mark Scheme

Pearson Edexcel International Advanced Level (WMA12/01) • Calculator allowed

Every question in the booklet, marked line by line the way an examiner marks it

Chapters (tap a chapter to jump to it)	Qs	Marks	Page	My score
1 Algebraic Methods Algebraic fractions, the factor and remainder theorems, proof	10	66	3	
2 Coordinate Geometry in the (x, y) Plane Equations of circles, tangents, chords and intersections	10	68	9	
3 Exponentials and Logarithms Laws of logarithms, exponential equations, modelling	10	68	15	
4 The Binomial Expansion Expanding $(a + bx)^n$, coefficients and approximations	10	65	21	
5 Sequences and Series Arithmetic and geometric series, sigma notation, recurrence	10	69	26	
6 Trigonometric Identities and Equations $\tan \theta = \frac{\sin \theta}{\cos \theta}$, $\sin^2 \theta + \cos^2 \theta = 1$, solving equations	10	66	32	
7 Differentiation Increasing and decreasing functions, stationary points, optimisation	10	67	37	
8 Integration Definite integrals, areas under curves, the trapezium rule	10	67	42	

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How to use this mark scheme

The same marking rules Pearson examiners use for WMA12.

Mark every line, not just the answer. Method marks are given for a correct method even when the final answer is wrong. If you lost marks, find the *first* line where your working differs from the scheme. That is the thing to fix.

Code	Meaning
M1	Method mark: for knowing a correct method and attempting to apply it. Small slips in the working do not lose this mark.
dM1	Dependent method mark: can only be given if the method mark before it has been given.
A1	Accuracy mark: for a correct answer or line of working. It can only be given if the method mark it depends on has been given.
B1	Independent mark: for a correct result or statement, whatever method was used.
*	The answer is given in the question. Every step must be shown, and the answer must follow from correct working (cso).
cao	Correct answer only.
cso	Correct solution only: no errors anywhere in the working.
ft	Follow through: the mark can be given for correct work using your own earlier (wrong) answer.
oe	Or equivalent: any correct equivalent form is accepted.

“Solutions relying on calculator technology are not acceptable.” When a question says this, a correct final answer with no algebraic working scores **no marks**. The notes under each scheme line say when this applies.



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1

Chapter 1: Algebraic Methods

Mark scheme. Marks are shown on the right of each line.

Question 1

5 marks

Part	Scheme	Marks
(a)	$f(3) = 27 + 9a + 3b + 6 = 0 \Rightarrow 3a + b = -11$	M1
	$f(-2) = -8 + 4a - 2b + 6 = 0 \Rightarrow 2a - b = 1$	M1
	Solves simultaneously: $5a = -10$	M1
	$a = -2, b = -5$	A1
		(4)
(b)	$(x-3)(x+2)(x-1)$	B1
	The third factor comes from the constant term: $-3 \times 2 \times c = 6$, so $c = -1$; or by division.	(1)
Total for Question 1: 5 marks		

Question 2

4 marks

Part	Scheme	Marks
	Attempts to divide: $2x^3 - 3x^2 - 11x + 7 = (x-3)(2x^2 + \dots) + \dots$	M1
	$2x^2 + 3x \dots$	A1
	Quotient $2x^2 + 3x - 2$	A1
	Remainder 1	B1
	The remainder may come from $f(3) = 54 - 27 - 33 + 7 = 1$ (remainder theorem).	(4)
Total for Question 2: 4 marks		



Question 3

5 marks

Part	Scheme	Marks
(a)	$f\left(-\frac{1}{2}\right) = 2\left(-\frac{1}{8}\right) + 3\left(\frac{1}{4}\right) - \frac{a}{2} - 6 = 0$ $\frac{1}{2} - \frac{a}{2} - 6 = 0$, so $a = -11$ * <i>Must set $f\left(-\frac{1}{2}\right)$ equal to 0. Using $f\left(\frac{1}{2}\right)$ scores M0.</i>	M1 A1* (2)
(b)	Divides by $(2x+1)$: $x^2 + x - 6$ $(2x+1)(x^2 + x - 6)$ $(2x+1)(x-2)(x+3)$ <i>The final answer must be the full product on one line.</i>	M1 A1 A1 (3)
Total for Question 3: 5 marks		

Question 4

4 marks

Part	Scheme	Marks
(a)	$f(2) = 16 - 4 + 2k + 4 = 10$ $k = -3$	M1 A1 (2)
(b)	$f\left(-\frac{1}{2}\right) = -\frac{1}{4} - \frac{1}{4} + \frac{3}{2} + 4$ 5	M1 A1 (2)
Total for Question 4: 4 marks		



Question 5

5 marks

Part	Scheme	Marks
(i)	Chooses a value that gives a composite number, e.g. $n = 10$: $100 + 10 + 11 = 121$ $121 = 11 \times 11$, which is not prime, so the statement is not true $n = 11$ ($143 = 11 \times 13$) also works. The conclusion must be stated.	M1 A1 (2)
(ii)	Tries all five values: $n = 1, 2, 3, 4, 5$ give 0, 6, 24, 60, 120 All correct and each shown to be a multiple of 6: $0 = 6 \times 0$, $6 = 6 \times 1$, $24 = 6 \times 4$, $60 = 6 \times 10$, $120 = 6 \times 20$ Conclusion: for all integers $1 \leq n \leq 5$, $n^3 - n$ is a multiple of 6 Checking only some of the cases scores M0.	M1 A1 A1 (3)
Total for Question 5: 5 marks		

Question 6

8 marks

Part	Scheme	Marks
(a)	-9	B1 (1)
(b)	$f\left(\frac{1}{2}\right) = \frac{3}{2}\left(\frac{1}{2} - \frac{1}{2} + k\right) - 9 = 0$ $\frac{3}{2}k = 9$, so $k = 6$ * "= 0" must be seen.	M1 A1* (2)
(c)	(i) Expands: $f(x) = 2x^3 + x^2 + 5x - 3$ Divides by $(2x - 1)$ or compares coefficients $f(x) = (2x - 1)(x^2 + x + 3)$	M1 M1 A1 (3)
	(ii) For $x^2 + x + 3$: $b^2 - 4ac = 1 - 12 = -11$ $-11 < 0$, so $x^2 + x + 3 = 0$ has no real roots and the only real root of $f(x) = 0$ is $x = \frac{1}{2}$ The discriminant must be of the quotient. A calculator giving complex roots with no statement scores M0 A0.	M1 A1 (2)
Total for Question 6: 8 marks		



Question 7

8 marks

Part	Scheme	Marks
(a)	$f(2) = 8 + 8 - 10 - 6$ $= 0$, so $(x - 2)$ is a factor *	M1 A1* (2)
(b)	$(x - 2)(x^2 + 4x + 3)$ $(x - 2)(x + 1)(x + 3)$	M1 A1 (2)
(c)	$(x - 2)(x + 1)(x + 3) = x + 3 \Rightarrow (x + 3)[(x - 2)(x + 1) - 1] = 0$ $(x + 3)(x^2 - x - 3) = 0$ Solves the quadratic: $x = \frac{1 \pm \sqrt{1 + 12}}{2}$ $x = -3, x = \frac{1 \pm \sqrt{13}}{2}$ <i>Dividing both sides by $(x + 3)$ loses the solution $x = -3$ and scores at most M1 A1 M1 A0.</i>	M1 A1 M1 A1 (4)
Total for Question 7: 8 marks		

Question 8

8 marks

Part	Scheme	Marks
(a)	Considers $x^3 + y^3 - xy(x + y) = x^3 - x^2y - xy^2 + y^3$ Factorises: $x^2(x - y) - y^2(x - y) = (x - y)(x^2 - y^2)$ $= (x - y)(x - y)^2$ $(x - y)^2 \geq 0$ and $x + y > 0$ as $x, y > 0$, so $(x + y)(x - y)^2 \geq 0$, hence $x^3 + y^3 \geq xy(x + y)$ <i>The final mark needs both reasons, $(x - y)^2 \geq 0$ and $x + y > 0$ because x and y are positive, and a conclusion.</i>	M1 M1 A1 A1* (4)
(b)	Chooses values with $x + y < 0$ and $x \neq y$, e.g. $x = -2, y = 1$ $x^3 + y^3 = -7$ and $xy(x + y) = 2$; $-7 < 2$, so the inequality is not true for all real x and y	M1 A1 (2)
(c)	Divides both sides of the result in (a) by x^2y^2 , which is positive, so the inequality is unchanged $\frac{x^3}{x^2y^2} + \frac{y^3}{x^2y^2} \geq \frac{xy(x + y)}{x^2y^2}$, giving $\frac{x}{y^2} + \frac{y}{x^2} \geq \frac{1}{x} + \frac{1}{y}$ *	M1 A1* (2)
Total for Question 8: 8 marks		



Question 9

9 marks

Part	Scheme	Marks
(a)	$f(3) = 27 - 27 - 12 + 12$ $= 0$, so $(x - 3)$ is a factor *	M1 A1* (2)
(b)	$(x - 3)(x^2 - 4)$ $(x - 3)(x - 2)(x + 2)$	M1 A1 (2)
(c)	Positive cubic shape Crosses the x -axis at $(-2, 0)$, $(2, 0)$ and $(3, 0)$ Crosses the y -axis at $(0, 12)$	B1 B1ft B1 (3)
(d)	Uses the sketch: $f(x) \geq 0$ between the first two roots and after the third $-2 \leq x \leq 2$ or $x \geq 3$	M1 A1 (2)
Total for Question 9: 9 marks		



Question 10

10 marks

Part	Scheme	Marks
(a)	$f(2) = 8 + 4p + 2q + 12 = 0$ $4p + 2q = -20$, so $2p + q = -10$ *	M1 A1* (2)
(b)	Writes $f(x) = (x-2)^2(x+c)$ and compares constant terms: $4c = 12$, so $c = 3$ $f(x) = (x^2 - 4x + 4)(x+3) = x^3 - x^2 - 8x + 12$ $p = -1$ $q = -8$ <i>Alternative: the curve touches the axis at $x = 2$, so $f'(2) = 12 + 4p + q = 0$; solve with $2p + q = -10$.</i>	M1 M1 A1 A1 (4)
(c)	$(x-2)^2(x+3)$	B1 (1)
(d)	$f(k) = 12: k^3 - k^2 - 8k + 12 = 12 \Rightarrow k(k^2 - k - 8) = 0$ $k \neq 0$, so $k^2 - k - 8 = 0$ and $k = \frac{1 \pm \sqrt{1+32}}{2}$ $k = \frac{1 \pm \sqrt{33}}{2}$	M1 M1 A1 (3)
Total for Question 10: 10 marks		



2

Chapter 2: Coordinate Geometry in the (x, y) Plane

Mark scheme. Marks are shown on the right of each line.

Question 1

4 marks

Part	Scheme	Marks
	Centre is the midpoint of AB : $(2, 0)$	M1
	$(2, 0)$	A1
	Radius: $r^2 = (5 - 2)^2 + (-4 - 0)^2 = 25$ (or $AB = 10$, so $r = 5$)	M1
	$(x - 2)^2 + y^2 = 25$	A1
		(4)
Total for Question 1: 4 marks		

Question 2

5 marks

Part	Scheme	Marks
(a)	Completes the square: $(x - 3)^2 + (y + 5)^2 = 9 + 25 - 9$	M1
	Centre $(3, -5)$	A1
	Radius 5	A1
		(3)
(b)	The centre is 5 units below the x -axis and $r = 5$ (or $y = 0$ gives $(x - 3)^2 = 0$, a repeated root)	M1
	So C touches the x -axis at $(3, 0)$	A1
		(2)
Total for Question 2: 5 marks		



Question 3

6 marks

Part	Scheme	Marks
(a)	(i) $r = \sqrt{(3+1)^2 + (0-3)^2} = 5$	B1 (1)
	(ii) $(x \pm 1)^2 + (y \pm 3)^2 = \dots$ $(x+1)^2 + (y-3)^2 = 25$	M1 A1 (2)
(b)	Gradient of the radius $= \frac{0-3}{3+1} = -\frac{3}{4}$, so the tangent gradient is $\frac{4}{3}$ $y-0 = \frac{4}{3}(x-3)$ $4x-3y-12=0$	M1 M1 A1 (3)
Total for Question 3: 6 marks		

Question 4

5 marks

Part	Scheme	Marks
(a)	Substitutes: $x^2 + (x+1)^2 = 25$ $2x^2 + 2x - 24 = 0 \Rightarrow x^2 + x - 12 = 0$ $(x+4)(x-3) = 0$ and finds y $(3, 4)$ and $(-4, -3)$	M1 A1 M1 A1 (4)
(b)	$\sqrt{7^2 + 7^2} = 7\sqrt{2}$	B1 (1)
Total for Question 4: 5 marks		



Question 5

5 marks

Part	Scheme	Marks
(a)	Midpoint (1, 3) and gradient of $AB = \frac{4}{6} = \frac{2}{3}$	M1
	Perpendicular gradient $-\frac{3}{2}$	M1
	$y - 3 = -\frac{3}{2}(x - 1)$, i.e. $3x + 2y - 9 = 0$	A1
		(3)
(b)	$y = 0$: centre (3, 0); $r^2 = (4 - 3)^2 + 5^2 = 26$	M1
	$(x - 3)^2 + y^2 = 26$	A1
	<i>The centre is on the perpendicular bisector of every chord, so it is where the bisector meets the x-axis.</i>	(2)
Total for Question 5: 5 marks		

Question 6

8 marks

Part	Scheme	Marks
(a)	$(x - 2)^2 + (y + 3)^2 = 12 + 4 + 9$	M1
	Centre (2, -3)	A1
	Radius 5	A1
		(3)
(b)	$(x - 10)^2 + (y - 9)^2 = -145 + 100 + 81 = 36$	M1
	Centre (10, 9), radius 6	A1
		(2)
(c)	Distance between the centres $= \sqrt{8^2 + 12^2} = \sqrt{208} = 14.4 \dots$	M1
	Compares with the sum of the radii, $5 + 6 = 11$	M1
	$\sqrt{208} > 11$, so the circles do not meet (and neither lies inside the other)	A1
	<i>The final mark needs both values and a conclusion. "The centres are in different quadrants" is not a proof.</i>	(3)
Total for Question 6: 8 marks		



Question 7

8 marks

Part	Scheme	Marks
(a)	Substitutes $y = 2x + k$: $(x - 3)^2 + (2x + k - 1)^2 = 20$ Expands: $x^2 - 6x + 9 + 4x^2 + 4(k - 1)x + (k - 1)^2 = 20$ $5x^2 + (4k - 10)x + k^2 - 2k - 10 = 0$ *	M1 M1 A1* (3)
(b)	Tangent: $b^2 - 4ac = 0$: $(4k - 10)^2 - 20(k^2 - 2k - 10) = 0$ $-4k^2 - 40k + 300 = 0 \Rightarrow k^2 + 10k - 75 = 0$ $k = 5$ or $k = -15$	M1 A1 A1 (3)
(c)	$k = 5$: $5x^2 + 10x + 5 = 0$, so $x = -1$ $(-1, 3)$ <i>Alternative for (b) and (c): the radius to the point of contact has gradient $-\frac{1}{2}$, and the distance from the centre to the line equals $\sqrt{20}$.</i>	M1 A1 (2)
Total for Question 7: 8 marks		

Question 8

8 marks

Part	Scheme	Marks
(a)	Gradient of $CA = \frac{2-0}{-1-5} = -\frac{1}{3}$ and gradient of $CB = \frac{6-0}{7-5} = 3$ Both correct $-\frac{1}{3} \times 3 = -1$, so CA is perpendicular to CB and angle $ACB = 90^\circ$ *	M1 A1 A1* (3)
(b)	Angle in a semicircle: AB is a diameter, so the centre is the midpoint $(3, 4)$ $r^2 = (7-3)^2 + (6-4)^2 = 20$ $(x-3)^2 + (y-4)^2 = 20$	M1 M1 A1 (3)
(c)	The centre is the midpoint of CD : $D = (2 \times 3 - 5, 2 \times 4 - 0)$ $D(1, 8)$	M1 A1 (2)
Total for Question 8: 8 marks		



Question 9

9 marks

Part	Scheme	Marks
(a)	$(x-4)^2 + (y-1)^2 = 8 + 16 + 1$ $P(4, 1)$, radius 5	M1 A1 (2)
(b)	Substitutes $y = x + 2$: $(x-4)^2 + (x+1)^2 = 25$ $2x^2 - 6x - 8 = 0 \Rightarrow (x-4)(x+1) = 0$ $A(-1, 1)$ and $B(4, 6)$ (in either order)	M1 A1 A1 (3)
(c)	PA is horizontal and PB is vertical (or $AB^2 = 50 = 5^2 + 5^2$, or $\cos APB = 0$ by the cosine rule) So angle $APB = \frac{\pi}{2}$ *	M1 A1* (2)
(d)	Sector – triangle: $\frac{1}{2} \times 5^2 \times \frac{\pi}{2} - \frac{1}{2} \times 5 \times 5$ $\frac{25\pi}{4} - \frac{25}{2}$	M1 A1 (2)
Total for Question 9: 9 marks		



Question 10

10 marks

Part	Scheme	Marks
(a)	$CT^2 = (14 - 2)^2 + (6 - 1)^2 = 169$ Angle $CPT = 90^\circ$, so $TP^2 = CT^2 - r^2$ $TP^2 = 169 - 25 = 144$, so $TP = 12$ *	M1 M1 A1* (3)
(b)	Two right-angled triangles: $2 \times \frac{1}{2} \times 5 \times 12$ 60	M1 A1 (2)
(c)	$\tan(\angle PCT) = \frac{12}{5}$, so $\angle PCT = 1.176\dots$ $\angle PCQ = 2.35$	M1 A1 (2)
(d)	Sector $PCQ = \frac{1}{2} \times 5^2 \times 2.352\dots = 29.40\dots$ Area of $R = 60 - 29.40\dots$ 30.6 <i>Using degrees in the sector formula scores M0. Premature rounding of the angle to 2.35 gives 30.6 still.</i>	M1 M1 A1 (3)
Total for Question 10: 10 marks		



3

Chapter 3: Exponentials and Logarithms

Mark scheme. Marks are shown on the right of each line.

Question 1

4 marks

Part	Scheme	Marks
(a)	4	B1 (1)
(b)	Power law: $2\log_a 5 = \log_a 25$ Combines: $\log_a \left(\frac{25 \times 4}{10} \right)$ $\log_a 10$	M1 M1 A1 (3)
Total for Question 1: 4 marks		

Question 2

5 marks

Part	Scheme	Marks
(a)	$x = \frac{\log 12}{\log 5}$ (or $\log_5 12$) 1.54	M1 A1 (2)
(b)	$(2x - 1)\log 3 = x\log 7$ $x(2\log 3 - \log 7) = \log 3$, so $x = \frac{\log 3}{\log 9 - \log 7}$ 4.37	M1 M1 A1 (3)
Total for Question 2: 5 marks		



Question 3

6 marks

Part	Scheme	Marks
	Uses the addition law: $\log_3((2x+3)(x-1))$	M1
	Writes 2 as $\log_3 9$ and combines: $(2x+3)(x-1) = 9(x+1)$	M1
	$2x^2 - 8x - 12 = 0 \Rightarrow x^2 - 4x - 6 = 0$	A1
	$x = \frac{4 \pm \sqrt{16+24}}{2} = 2 \pm \sqrt{10}$	M1
	$x = 2 + \sqrt{10}$	A1
	$2 - \sqrt{10} < 0$, so $\log_3(x-1)$ would not be defined: it is rejected	B1
	<i>Removing the logs term by term, e.g. $(2x+3) + (x-1) = \dots$, scores M0 M0.</i>	
		(6)
Total for Question 3: 6 marks		

Question 4

5 marks

Part	Scheme	Marks
(a)	Decreasing curve, approaching $y = 4$ as $x \rightarrow -\infty$	B1
	$(0, 3)$ and $(2, 0)$	B1
	Asymptote $y = 4$	B1
	<i>The y intercept is at $y = 3$, not at the height of the asymptote.</i>	
		(3)
(b)	$4 - 2^x = 1 \Rightarrow 2^x = 3$	M1
	$x = \log_2 3$ (or $\frac{\log 3}{\log 2}$)	A1
		(2)
Total for Question 4: 5 marks		



Question 5

5 marks

Part	Scheme	Marks
(a)	Change of base: $\log_4 x = \frac{\log_2 x}{\log_2 4}$ $\log_2 4 = 2$, so $\log_4 x = \frac{1}{2} \log_2 x$ *	M1 A1* (2)
(b)	$\log_2 x + \frac{1}{2} \log_2 x = 6$ $\log_2 x = 4$ $x = 16$	M1 A1 A1 (3)
Total for Question 5: 5 marks		

Question 6

8 marks

Part	Scheme	Marks
(a)	$2^{2x+1} = 2 \times (2^x)^2 = 2y^2$ $2y^2 - 11y + 12 = 0$ *	M1 A1* (2)
(b)	$(2y - 3)(y - 4) = 0$ $y = \frac{3}{2}$ or $y = 4$ $2^x = 4 \Rightarrow x = 2$ and $2^x = \frac{3}{2} \Rightarrow x = \frac{\log 1.5}{\log 2}$ $x = 2, x = 0.585$	M1 A1 M1 A1 (4)
(c)	$\frac{3}{2} < 2^x < 4$ (the quadratic is negative between its roots) $0.585 < x < 2$ <i>2^x is increasing, so the inequality in y gives the same order of x values.</i>	M1 A1 (2)
Total for Question 6: 8 marks		



Question 7

8 marks

Part	Scheme	Marks
(a)	Two increasing exponential curves, above the x -axis and asymptotic to it for $x < 0$ C_1 through $(0, 1)$ and C_2 through $(0, 2)$ C_1 crosses C_2 for $x > 0$ and is steeper to the right of P	B1 B1 B1 (3)
(b)	$5^x = 2 \times 2^x$ $\left(\frac{5}{2}\right)^x = 2$ (or $x \log 5 = (x+1) \log 2$) $x = \frac{\log 2}{\log 2.5}^*$	M1 M1 A1* (3)
(c)	$x = 0.756 \dots$ and uses the sketch: C_1 is above C_2 to the right of P $x > 0.756$	M1 A1 (2)
Total for Question 7: 8 marks		

Question 8

8 marks

Part	Scheme	Marks
(a)	$\log_a x^2 + \log_a \sqrt{y} - \log_a a$ Uses the power law on both terms and $\log_a a = 1$ $2p + \frac{1}{2}q - 1$	M1 M1 A1 (3)
(b)	$x + y = 2^3 = 8$ $\log_2(xy) = \log_2 15$, so $xy = 15$ Substitutes: $x(8 - x) = 15 \Rightarrow x^2 - 8x + 15 = 0$ $(x - 3)(x - 5) = 0$ $x = 3, y = 5$ and $x = 5, y = 3$ <i>Both pairs are valid because x and y are positive in each.</i>	B1 B1 M1 A1 A1 (5)
Total for Question 8: 8 marks		



Question 9

9 marks

Part	Scheme	Marks
(a)	Change of base: $\log_y a = \frac{1}{\log_a y}$	M1
	Let $u = \log_a y$: $u + 2 = \frac{3}{u}$	M1
	Multiplies by u ($u \neq 0$ as $y \neq 1$): $(\log_a y)^2 + 2\log_a y - 3 = 0$ *	A1*
		(3)
(b)	$(u + 3)(u - 1) = 0$, so $\log_a y = 1$ or $\log_a y = -3$	M1
	Converts from log form	M1
	$y = a$ or $y = a^{-3}$	A1
		(3)
(c)	$a \times a^{-3} = \frac{1}{16}$	M1
	$a^{-2} = \frac{1}{16}$, so $a^2 = 16$	M1
	$a = 4$ (as $a > 1$)	A1
		(3)
Total for Question 9: 9 marks		



Question 10

10 marks

Part	Scheme	Marks
(a)	Uses a subtraction law: $\log_2 \left(\frac{x^2}{x-1} \right) = 3$ Removes the logs: $\frac{x^2}{x-1} = 2^3 = 8$ $x^2 = 8x - 8 \Rightarrow x^2 - 8x + 8 = 0$ *	M1 M1 A1* (3)
(b)	$x = \frac{8 \pm \sqrt{64 - 32}}{2}$ $= \frac{8 \pm 4\sqrt{2}}{2}$ $x = 4 \pm 2\sqrt{2}$ Both valid: $4 - 2\sqrt{2} \approx 1.17 > 1$, so $x - 1 > 0$ and $x^2 > 0$ for both values	M1 A1 A1 B1 (4)
(c)	Recognises $x = 2y$: $\log_2 ((2y)^2) - \log_2 (2y - 1) = 3$ is the same equation $2y = 4 \pm 2\sqrt{2}$ $y = 2 \pm \sqrt{2}$ Both satisfy $y > \frac{1}{2}$ since $2 - \sqrt{2} \approx 0.59$. Solving from scratch ($4y^2 = 8(2y - 1)$) is equally valid.	M1 M1 A1 (3)
Total for Question 10: 10 marks		



4

Chapter 4: The Binomial Expansion

Mark scheme. Marks are shown on the right of each line.

Question 1

4 marks

Part	Scheme	Marks
	Correct binomial coefficients with powers of 2 and $3x$, e.g. $\binom{5}{1}2^4(3x)$	M1
	$32 + 240x$	B1
	One of $720x^2$ or $1080x^3$	A1
	$32 + 240x + 720x^2 + 1080x^3$	A1
		(4)
Total for Question 1: 4 marks		

Question 2

4 marks

Part	Scheme	Marks
(a)	$\binom{7}{3}3^4(-2x)^3 = 35 \times 81 \times (-8)x^3$	M1
	$-22\,680$	A1
		(2)
(b)	$\binom{7}{5}3^2(-2)^5 = 21 \times 9 \times (-32)$	M1
	-6048	A1
	<i>The sign comes from $(-2)^3$ and $(-2)^5$. Leaving x^3 or x^5 in a "coefficient" is condoned once.</i>	
		(2)
Total for Question 2: 4 marks		

Question 3

5 marks

Part	Scheme	Marks
(a)	Coefficient of x^2 : $\binom{8}{2}k^2 = 28k^2$	M1
	$28k^2 = 112 \Rightarrow k^2 = 4$	M1
	$k = 2$ (as $k > 0$)	A1
		(3)
(b)	$\binom{8}{3}k^3 = 56 \times 8$	M1
	448	A1ft
		(2)
Total for Question 3: 5 marks		



Question 4

5 marks

Part	Scheme	Marks
(a)	$1 + \binom{10}{1} \left(-\frac{x}{4}\right) + \binom{10}{2} \left(-\frac{x}{4}\right)^2$	M1
	$1 - \frac{5}{2}x$	A1
	$1 - \frac{5}{2}x + \frac{45}{16}x^2$	A1
		(3)
(b)	$1 - \frac{x}{4} = 0.99$, so $x = 0.04$, substituted into their expansion	M1
	0.9045	A1
	$1 - 0.1 + 0.0045 = 0.9045$. The true value is 0.9044 to 4 d.p., so the estimate is very close because x is small.	(2)
Total for Question 4: 5 marks		

Question 5

4 marks

Part	Scheme	Marks
	$(1 - 2x)^5 = 1 - 10x + 40x^2 + \dots$	M1
	$1 - 10x + 40x^2$	A1
	Combines both products: $3 \times 40 + 1 \times (-10)$	M1
	110	A1
	Using only 3×40 (one of the two products) is the most common error: M1 A1 M0 A0.	
		(4)
Total for Question 5: 4 marks		



Question 6

8 marks

Part	Scheme	Marks
(a)	$A = 64$	B1 (1)
(b)	$B = {}^6_1 2^5 p = 192p$ and $D = {}^6_3 2^3 p^3 = 160p^3$ Both correct $320p^3 = 2880p \Rightarrow p^2 = 9$ (as $p \neq 0$) $p = -3$	M1 A1 M1 A1 (4)
(c)	$C = {}^6_2 2^4 p^2 = 240 \times 9$ 2160	M1 A1 (2)
(d)	$p^6 = 729$	B1 (1)
Total for Question 6: 8 marks		

Question 7

8 marks

Part	Scheme	Marks
(a)	$na = 20$ $\frac{n(n-1)}{2} a^2 = 180$ Eliminates a : $\frac{n(n-1)}{2} \times \frac{400}{n^2} = 180$ $400(n-1) = 360n \Rightarrow 40n = 400$, so $n = 10$ * $a = 2$	B1 B1 M1 A1* A1 (5)
(b)	${}^{10}_3 2^3 = 960$	B1 (1)
(c)	Compares coefficients ${}^{10}_r 2^r$: ..., 8064, 13440, 15360, 11520, ... 15360 , the coefficient of x^7 The coefficients increase while $\frac{2(10-r)}{r+1} > 1$, i.e. up to $r = 7$.	M1 A1 (2)
Total for Question 7: 8 marks		



Question 8

8 marks

Part	Scheme	Marks
(a)	General term $\binom{6}{r} (x^2)^{6-r} \left(-\frac{2}{x}\right)^r$ $= \binom{6}{r} (-2)^r x^{12-2r-r} = \binom{6}{r} (-2)^r x^{12-3r}$ *	M1 A1* (2)
(b)	$12 - 3r = 0 \Rightarrow r = 4: \binom{6}{4} (-2)^4$ 240	M1 A1 (2)
(c)	$12 - 3r = 3 \Rightarrow r = 3: \binom{6}{3} (-2)^3$ -160	M1 A1 (2)
(d)	$12 - 3r = 2$ gives $r = \frac{10}{3}$ r must be a whole number, so there is no term in x^2	M1 A1 (2)
Total for Question 8: 8 marks		

Question 9

9 marks

Part	Scheme	Marks
(a)	Expands both: $(1+x)^5 = 1 + 5x + 10x^2 + 10x^3 + 5x^4 + x^5$ The odd powers cancel $2 + 20x^2 + 10x^4$	M1 M1 A1 (3)
(b)	Substitutes $x = \sqrt{2}$: $2 + 20 \times 2 + 10 \times 4$ $= 2 + 40 + 40 = 82$ *	M1 A1* (2)
(c)	$2 + 20y^2 + 10y^4 = 242 \Rightarrow y^4 + 2y^2 - 24 = 0$ $(y^2 + 6)(y^2 - 4) = 0$ $y^2 = 4$ ($y^2 = -6$ has no real solutions) $y = \pm 2$	M1 M1 A1 A1 (4)
Total for Question 9: 9 marks		



Question 10

10 marks

Part	Scheme	Marks
(a)	$2^5 + \binom{5}{1}2^4(ax) + \binom{5}{2}2^3(ax)^2$ $32 + 80ax$ $32 + 80ax + 80a^2x^2$	M1 A1 A1 (3)
(b)	Coefficient of x : $80a - 32b = 16$ Coefficient of x^2 : $80a^2 - 80ab = 0$ $80a(a - b) = 0$ and $a \neq 0$, so $b = a$ * $48a = 16$ $a = \frac{1}{3}$	M1 M1 A1* M1 A1 (5)
(c)	x^3 term of $(2 + ax)^5$ is $\binom{5}{3}2^2a^3 = \frac{40}{27}$; coefficient = $\frac{40}{27} - b \times 80a^2$ $\frac{40}{27} - \frac{80}{27} = -\frac{40}{27}$	M1 A1 (2)
Total for Question 10: 10 marks		



5

Chapter 5: Sequences and Series

Mark scheme. Marks are shown on the right of each line.

Question 1

5 marks

Part	Scheme	Marks
(a)	$u_2 = 12 - k$ $u_3 = 3(12 - k) - k = 36 - 4k$	B1 B1 (2)
(b)	$36 - 4k = 20$ $k = 4$	M1 A1 (2)
(c)	$4 + 8 + 20 + 56 = 88$	B1 (1)
Total for Question 1: 5 marks		

Question 2

5 marks

Part	Scheme	Marks
(a)	$a + 3d = 23$ and $a + 9d = 53$ Solves: $6d = 30$ $a = 8, d = 5$	M1 M1 A1 (3)
(b)	$S_{20} = \frac{20}{2}(2 \times 8 + 19 \times 5)$ 1110	M1 A1 (2)
Total for Question 2: 5 marks		



Question 3

5 marks

Part	Scheme	Marks
(a)	$81 \times \left(\frac{2}{3}\right)^4 = 16$	B1 (1)
(b)	$\frac{81 \left(1 - \left(\frac{2}{3}\right)^{10}\right)}{1 - \frac{2}{3}}$ 239	M1 A1 (2)
(c)	$\frac{81}{1 - \frac{2}{3}} = 243$	B1 (1)
(d)	4.21 <i>Using the rounded value 239 gives 4 and scores B0: $243 - 238.786 \dots = 4.21$</i>	B1 (1)
Total for Question 3: 5 marks		

Question 4

5 marks

Part	Scheme	Marks
(a)	$S_n = a + (a+d) + \dots + (a + (n-1)d)$ and $S_n = (a + (n-1)d) + \dots + (a+d) + a$ Adds: $2S_n = n(2a + (n-1)d)$ $S_n = \frac{n}{2}(2a + (n-1)d)$ * <i>Both lines must be written with at least the first two and last terms. Each pair adds to $2a + (n-1)d$ and there are n pairs.</i>	M1 M1 A1* (3)
(b)	$a = 5, d = 3, n = 40: \frac{40}{2}(10 + 39 \times 3)$ 2540	M1 A1 (2)
Total for Question 4: 5 marks		



Question 5

5 marks

Part	Scheme	Marks
(a)	$u_2 = -1, u_3 = \frac{1}{2}$	M1
	$u_2 = -1, u_3 = \frac{1}{2}, u_4 = 2$	A1
		(2)
(b)	It is periodic, with period 3 (the terms repeat $2, -1, \frac{1}{2}, \dots$)	B1
		(1)
(c)	Each group of 3 sums to $\frac{3}{2}$; 33 groups plus $u_{100} = u_1 = 2$	M1
	$33 \times \frac{3}{2} + 2 = 51.5$	A1
		(2)
Total for Question 5: 5 marks		

Question 6

9 marks

Part	Scheme	Marks
(a)	Uses a common ratio: $\frac{2k}{k+3} = \frac{3k-2}{2k}$, so $4k^2 = (k+3)(3k-2)$	M1
	$4k^2 = 3k^2 + 7k - 6 \Rightarrow k^2 - 7k + 6 = 0$ *	A1*
		(2)
(b)	$k = 1$ or $k = 6$	B1
		(1)
(c)	$k = 1$: terms 4, 2, 1, so $r = \frac{1}{2}$ and $ r < 1$	M1
	$S_\infty = \frac{4}{1 - \frac{1}{2}}$	M1
	8	A1
		(3)
(d)	$k = 6$: $a = 9, r = \frac{4}{3}$: $\frac{9((\frac{4}{3})^n - 1)}{\frac{4}{3} - 1} > 1000$	M1
	$(\frac{4}{3})^n > \frac{1027}{27}$, so $n > \frac{\log \frac{1027}{27}}{\log \frac{4}{3}} = 12.6 \dots$	M1
	$n = 13$	A1
		(3)
Total for Question 6: 9 marks		



Question 7

8 marks

Part	Scheme	Marks
(a)	$a + 5d = 340$	B1
	$\frac{10}{2}(2a + 9d) = 3230$, so $2a + 9d = 646$	B1
	Solves simultaneously	M1
	$d = 34$ and $a = 170$ *	A1*
		(4)
(b)	$\frac{N}{2}(340 + 34(N - 1)) > 10000 \Rightarrow 17N^2 + 153N - 10000 > 0$	M1
	Solves the quadratic: $N = \frac{-153 + \sqrt{153^2 + 680000}}{34} = 20.1 \dots$	M1
	Rounds up and checks: $S_{20} = 9860 < 10000$, $S_{21} = 10710 > 10000$	M1
	the 21st month	A1
	The negative root is rejected. $N = 20$ is the most common wrong answer.	
		(4)
Total for Question 7: 8 marks		

Question 8

8 marks

Part	Scheme	Marks
(a)	$u_2 = 2a + 2a - 1 = 4a - 1$	B1
	$u_3 = a(4a - 1) + 2a - 1$	M1
	$u_3 = 4a^2 + a - 1$	A1
		(3)
(b)	$2 + (4a - 1) + (4a^2 + a - 1) = 21$	M1
	$4a^2 + 5a - 21 = 0$ *	A1*
		(2)
(c)	$(4a - 7)(a + 3) = 0$, so $a = \frac{7}{4}$ or $a = -3$	M1
	Increasing: $a = -3$ gives 2, -13, 32, ... which is not increasing, so $a = \frac{7}{4}$	A1
	$u_4 = \frac{7}{4} \times 13 + \frac{5}{2} = 25.25$	A1
		(3)
Total for Question 8: 8 marks		



Question 9

9 marks

Part	Scheme	Marks
(a)	$S_n = a + ar + ar^2 + \dots + ar^{n-1}$ and $rS_n = ar + ar^2 + \dots + ar^n$ Subtracts: $S_n - rS_n = a - ar^n$ $S_n(1 - r) = a(1 - r^n)$, so $S_n = \frac{a(1 - r^n)}{1 - r}$ (as $r \neq 1$) *	M1 M1 A1* (3)
(b)	$\frac{a}{1 - r} = 4a$, so $1 - r = \frac{1}{4}$ $r = \frac{3}{4}$ $a\left(1 + \frac{3}{4} + \frac{9}{16}\right) = 74$, so $a = 32$	M1 A1 A1 (3)
(c)	$S_\infty - S_n = \frac{ar^n}{1 - r} = 128\left(\frac{3}{4}\right)^n < 0.01$ $n > \frac{\log\left(\frac{0.01}{128}\right)}{\log 0.75} = 32.87\dots$ (the sign reverses because $\log 0.75 < 0$) $n = 33$	M1 M1 A1 (3)
Total for Question 9: 9 marks		



Question 10

10 marks

Part	Scheme	Marks
(a)	$A: 30\,000 + 4 \times 1500 = \text{£}36\,000$ $B: 28\,000 \times 1.05^4 = \text{£}34\,034$ <i>Year 5 is the 5th term, so the power is 4, not 5.</i>	B1 B1 (2)
(b)	$\frac{10}{2}(2 \times 30\,000 + 9 \times 1500)$ $= 5 \times 73\,500 = 367\,500$ *	M1 A1* (2)
(c)	$\frac{28\,000(1.05^{10} - 1)}{1.05 - 1}$ $\text{£}352\,181$	M1 A1 (2)
(d)	$\frac{28\,000(1.05^n - 1)}{0.05} > 1\,000\,000$ $1.05^n - 1 > \frac{25}{14}, \text{ so } 1.05^n > \frac{39}{14}$ * $n > \frac{\log \frac{39}{14}}{\log 1.05} = 20.998\dots$ $n = 21$	M1 A1* M1 A1 (4)
Total for Question 10: 10 marks		



6

Chapter 6: Trigonometric Identities and Equations

Mark scheme. Marks are shown on the right of each line.

Question 1

4 marks

Part	Scheme	Marks
(a)	$\cos^2 \theta = 1 - \frac{5}{9} = \frac{4}{9}$ Obtuse, so $\cos \theta = -\frac{2}{3}$	M1 A1 (2)
(b)	$\tan \theta = \frac{\sin \theta}{\cos \theta} = \frac{\sqrt{5}}{3} \div \left(-\frac{2}{3}\right)$ $-\frac{\sqrt{5}}{2}$	M1 A1 (2)
Total for Question 1: 4 marks		

Question 2

5 marks

Part	Scheme	Marks
(a)	$x = 48.6^\circ$ $48.6^\circ, 131.4^\circ$	B1 B1 (2)
(b)	$\tan x = -\frac{5}{3}$ Principal value -59.0° , adds 180° $121.0^\circ, 301.0^\circ$	M1 M1 A1 (3)
Total for Question 2: 5 marks		



Question 3

4 marks

Part	Scheme	Marks
	$\sin\left(x - \frac{\pi}{6}\right) = \frac{\sqrt{3}}{2}$ $x - \frac{\pi}{6} = \frac{\pi}{3} \text{ or } \frac{2\pi}{3}$ Adds $\frac{\pi}{6}$ $x = \frac{\pi}{2}, \frac{5\pi}{6}$	M1 A1 M1 A1 (4)
Total for Question 3: 4 marks		

Question 4

6 marks

Part	Scheme	Marks
(a)	$\cos^4 x - \sin^4 x = (\cos^2 x - \sin^2 x)(\cos^2 x + \sin^2 x)$ $= \cos^2 x - \sin^2 x$ using $\cos^2 x + \sin^2 x = 1$ $= 1 - \sin^2 x - \sin^2 x = 1 - 2\sin^2 x$ *	M1 M1 A1* (3)
(b)	$1 - 2\sin^2 x = \frac{1}{2} \Rightarrow \sin x = \pm \frac{1}{2}$ Two correct values $30^\circ, 150^\circ, 210^\circ, 330^\circ$ <i>The negative root $\sin x = -\frac{1}{2}$ gives the two solutions in the third and fourth quadrants.</i>	M1 A1 A1 (3)
Total for Question 4: 6 marks		

Question 5

4 marks

Part	Scheme	Marks
	$2x - 30^\circ = 60^\circ$ Uses the period: $2x - 30^\circ = 60^\circ$ or 240° (in $-30^\circ \leq 2x - 30^\circ < 330^\circ$) Solves for x $x = 45^\circ, 135^\circ$	M1 M1 M1 A1 (4)
Total for Question 5: 4 marks		



Question 6

8 marks

Part	Scheme	Marks
(a)	$3 \frac{\sin x}{\cos x} \sin x = 8 \Rightarrow 3 \sin^2 x = 8 \cos x$ $3(1 - \cos^2 x) = 8 \cos x$ $3 \cos^2 x + 8 \cos x - 3 = 0$ *	M1 M1 A1* (3)
(b)	$(3 \cos x - 1)(\cos x + 3) = 0$; $\cos x = -3$ is impossible $\cos x = \frac{1}{3}$, so $x = 70.5^\circ$ 70.5°, 289.5°	M1 A1 A1 (3)
(c)	$2\theta = 70.5^\circ$ or 289.5° (in $0 \leq 2\theta < 360^\circ$) 35.3°, 144.7°	M1 A1 (2)
Total for Question 6: 8 marks		

Question 7

8 marks

Part	Scheme	Marks
(a)	The student divided by $\sin x$, which may be zero so the solutions of $\sin x = 0$ have been lost	B1 B1 (2)
(b)	$\sin x(5 \cos x - 2) = 0$ $\sin x = 0$: $x = 0^\circ, 180^\circ, 360^\circ$ $\cos x = 0.4$: $x = 66.4^\circ, 293.6^\circ$ 0°, 66.4°, 180°, 293.6°, 360°	M1 A1 A1 A1 (4)
(c)	$0 \leq 3x \leq 1080^\circ$: $\sin 3x = 0$ gives 7 values and $\cos 3x = 0.4$ gives 6 values 13	M1 A1 (2)
Total for Question 7: 8 marks		



Question 8

8 marks

Part	Scheme	Marks
(a)	$4(1 - \sin^2 x) - 4 \sin x = 1$ $4 \sin^2 x + 4 \sin x - 3 = 0$ *	M1 A1* (2)
(b)	$(2 \sin x - 1)(2 \sin x + 3) = 0$ $\sin x = -\frac{3}{2}$ is rejected because $-1 \leq \sin x \leq 1$ $\sin x = \frac{1}{2}$ $x = \frac{\pi}{6}, \frac{5\pi}{6}$	M1 B1 A1 A1 (4)
(c)	Two solutions in each interval of length 2π , and there are five such intervals 10	M1 A1 (2)
Total for Question 8: 8 marks		

Question 9

9 marks

Part	Scheme	Marks
(a)	If $\cos x = 0$ then $\sin^2 x = 1$ and the equation becomes $2 = 0$, so $\cos x \neq 0$ Divides by $\cos^2 x$ and uses $\tan x = \frac{\sin x}{\cos x}$ $2 \tan^2 x + 5 \tan x - 3 = 0$ *	B1 M1 A1* (3)
(b)	$(2 \tan x - 1)(\tan x + 3) = 0$ $\tan x = \frac{1}{2}$: $26.6^\circ, 206.6^\circ$ $\tan x = -3$: $108.4^\circ, 288.4^\circ$ $26.6^\circ, 108.4^\circ, 206.6^\circ, 288.4^\circ$	M1 A1 A1 A1 (4)
(c)	Acute solution has $\tan x = \frac{1}{2}$: a right-angled triangle with sides 1, 2 and hypotenuse $\sqrt{5}$ (or $\sin^2 x + \cos^2 x = 1$ with $\sin x = \frac{1}{2} \cos x$) $\sin x = \frac{\sqrt{5}}{5}$	M1 A1 (2)
Total for Question 9: 9 marks		



Question 10

10 marks

Part	Scheme	Marks
(a)	$\frac{1}{2} \times 5 \times 8 \sin \theta = 12$, so $\sin \theta = \frac{3}{5}$ *	B1* (1)
(b)	$\cos^2 \theta = 1 - \frac{9}{25} = \frac{16}{25}$ $\cos \theta = \frac{4}{5}$ or $\cos \theta = -\frac{4}{5}$	M1 A1 (2)
(c)	$BC^2 = 5^2 + 8^2 - 2 \times 5 \times 8 \cos \theta$ $\cos \theta = \frac{4}{5}$: $BC^2 = 25$, so $BC = 5$ $\cos \theta = -\frac{4}{5}$: $BC^2 = 153$, so $BC = 3\sqrt{17}$	M1 A1 A1 (3)
(d)	$-\frac{3}{4}$	B1 (1)
(e)	$\tan(x - 20^\circ) = \frac{3}{5}$ $x - 20^\circ = 30.96^\circ$ or 210.96° $51.0^\circ, 231.0^\circ$	M1 M1 A1 (3)
Total for Question 10: 10 marks		



7

Chapter 7: Differentiation

Mark scheme. Marks are shown on the right of each line.

Question 1

6 marks

Part	Scheme	Marks
(a)	$\frac{dy}{dx} = 6x^2 - 18x + 12$	M1
	$6(x-1)(x-2) = 0$, so $x = 1, 2$	A1
	Substitutes into y	M1
	$(1, 6)$ and $(2, 5)$	A1
		(4)
(b)	$\frac{d^2y}{dx^2} = 12x - 18$: at $x = 1$ it is -6 and at $x = 2$ it is 6	M1
	$(1, 6)$ is a maximum and $(2, 5)$ is a minimum	A1
		(2)
Total for Question 1: 6 marks		

Question 2

4 marks

Part	Scheme	Marks
	$f'(x) = 3x^2 - 12x - 15$	M1
	$3(x-5)(x+1)$: critical values -1 and 5	A1
	Decreasing where $f'(x) < 0$: chooses the inside region	M1
	$-1 < x < 5$	A1
	Accept $-1 \leq x \leq 5$.	
		(4)
Total for Question 2: 4 marks		



Question 3

5 marks

Part	Scheme	Marks
(a)	$\frac{dy}{dx} = 3x^{\frac{1}{2}} - 12$	M1
	$x^{\frac{1}{2}} = 4$, so $x = 16$	A1
	$(16, -57)$	A1
		(3)
(b)	$\frac{d^2y}{dx^2} = \frac{3}{2}x^{-\frac{1}{2}} = \frac{3}{8}$ at $x = 16$	M1
	$\frac{3}{8} > 0$, so minimum	A1
		(2)
Total for Question 3: 5 marks		

Question 4

4 marks

Part	Scheme	Marks
	$f'(x) = 3x^2 + 6x + 5$	B1
	Completes the square: $3(x+1)^2 + 2$	M1
	$3(x+1)^2 \geq 0$, so $f'(x) \geq 2 > 0$ for all x	A1
	Hence $f(x)$ is increasing for all x	A1
	Alternative: the discriminant of $3x^2 + 6x + 5$ is $36 - 60 < 0$ and the coefficient of x^2 is positive, so $f'(x) > 0$.	(4)
Total for Question 4: 4 marks		

Question 5

5 marks

Part	Scheme	Marks
(a)	$f'(x) = 3x^2 - 6x - 9 = 3(x-3)(x+1) = 0$	M1
	$x = -1, x = 3$	A1
	$(-1, 5)$ and $(3, -27)$	A1
		(3)
(b)	Positive cubic with a maximum at $(-1, 5)$ and a minimum at $(3, -27)$	B1
	Passes through the origin, $(0, 0)$	B1
		(2)
Total for Question 5: 5 marks		



Question 6

8 marks

Part	Scheme	Marks
(a)	Base $(30 - 2x)$ by $(30 - 2x)$ and height x : $V = x(30 - 2x)^2$ $= 4x^3 - 120x^2 + 900x$ *	M1 A1* (2)
(b)	$\frac{dV}{dx} = 12x^2 - 240x + 900$ $12(x - 5)(x - 15) = 0$ $x = 5$ $x = 15$ is rejected: the base would have side $30 - 30 = 0$ cm (no box)	M1 M1 A1 B1 (4)
(c)	$V = 5 \times 20^2 = 2000 \text{ cm}^3$ $\frac{d^2V}{dx^2} = 24x - 240 = -120 < 0$ at $x = 5$, so a maximum	B1 B1 (2)
Total for Question 6: 8 marks		

Question 7

8 marks

Part	Scheme	Marks
(a)	$2x \times x \times h = 72$, so $h = \frac{36}{x^2}$ $S = 2(2x^2) + 2(2xh) + 2(xh) = 4x^2 + 6xh$ $S = 4x^2 + 6x \times \frac{36}{x^2} = 4x^2 + \frac{216}{x}$ *	B1 M1 A1* (3)
(b)	$\frac{dS}{dx} = 8x - 216x^{-2}$ $8x^3 = 216$, so $x = 3$ $S = 36 + 72 = 108 \text{ cm}^2$	M1 A1 A1 (3)
(c)	$\frac{d^2S}{dx^2} = 8 + 432x^{-3}$ $= 24 > 0$ at $x = 3$, so a minimum	M1 A1 (2)
Total for Question 7: 8 marks		



Question 8

8 marks

Part	Scheme	Marks
(a)	Perimeter: $2r + 2h + \pi r = 10$, so $h = \frac{10 - 2r - \pi r}{2}$ $A = 2rh + \frac{1}{2}\pi r^2$ $= r(10 - 2r - \pi r) + \frac{1}{2}\pi r^2 = 10r - \left(2 + \frac{\pi}{2}\right)r^2$ *	B1 M1 A1* (3)
(b)	$\frac{dA}{dr} = 10 - (4 + \pi)r$ $= 0$ gives $r = \frac{10}{4 + \pi}$ Substitutes: $A = \frac{100}{4 + \pi} - \frac{4 + \pi}{2} \times \frac{100}{(4 + \pi)^2}$ $= \frac{100}{4 + \pi} - \frac{50}{4 + \pi} = \frac{50}{4 + \pi}$ *	M1 A1 M1 A1* (4)
(c)	$\frac{d^2A}{dr^2} = -(4 + \pi) < 0$, so a maximum	B1 (1)
Total for Question 8: 8 marks		

Question 9

9 marks

Part	Scheme	Marks
(a)	$f(1) = -2$: $a + b - 12 + 5 = -2$, so $a + b = 5$ $f'(x) = 3ax^2 + 2bx - 12$ and $f'(1) = 0$: $3a + 2b = 12$ Solves simultaneously $a = 2, b = 3$	M1 M1 M1 A1 (4)
(b)	$f'(x) = 6x^2 + 6x - 12 = 6(x + 2)(x - 1)$, so the other point is at $x = -2$: $(-2, 25)$ $f''(x) = 12x + 6$: $f''(1) = 18$ and $f''(-2) = -18$ $(1, -2)$ minimum, $(-2, 25)$ maximum	B1 M1 A1 (3)
(c)	$f'(x) > 0$: outside the roots $x < -2$ or $x > 1$	M1 A1 (2)
Total for Question 9: 9 marks		



Question 10

10 marks

Part	Scheme	Marks
(a)	$\frac{dy}{dx} = 3x^2 - 12x + 9 = 3(x-1)(x-3) = 0$ $x = 1, x = 3$ $(1, 5)$ and $(3, 1)$ $\frac{d^2y}{dx^2} = 6x - 12$: -6 at $x = 1$ and 6 at $x = 3$ $(1, 5)$ maximum, $(3, 1)$ minimum	M1 A1 A1 M1 A1 (5)
(b)	Tangent at the maximum is $y = 5$: $x^3 - 6x^2 + 9x - 4 = 0$ $(x-1)$ is a repeated factor: $(x-1)^2(x-4) = 0$ $P(4, 5)$ <i>Using the factor theorem with $x = 1$ twice, or dividing by $(x-1)^2$, is expected. A calculator solution scores M1 M0 A0.</i>	M1 M1 A1 (3)
(c)	The line $y = k$ must lie strictly between the minimum and the maximum $1 < k < 5$	M1 A1 (2)
Total for Question 10: 10 marks		



8

Chapter 8: Integration

Mark scheme. Marks are shown on the right of each line.

Question 1

4 marks

Part	Scheme	Marks
	Integrates: x^3 or $-8x^{\frac{1}{2}}$	M1
	$\left[x^3 - 8x^{\frac{1}{2}}\right]_1^4$	A1
	Substitutes the limits: $(64 - 16) - (1 - 8)$	M1
	55	A1
		(4)
Total for Question 1: 4 marks		

Question 2

5 marks

Part	Scheme	Marks
	Writes the integrand as $kx^{\frac{1}{2}} - 3x^{-\frac{1}{2}}$	M1
	Integrates: $\frac{2k}{3}x^{\frac{3}{2}} - 6x^{\frac{1}{2}}$	M1
	Both terms correct	A1
	Substitutes the limits and sets equal to 40: $(18k - 18) - \left(\frac{2k}{3} - 6\right) = 40$	M1
	$\frac{52k}{3} = 52$, so $k = 3$	A1
		(5)
Total for Question 2: 5 marks		

Question 3

5 marks

Part	Scheme	Marks
(a)	$x(6 - x) = 0$: $x = 0$ and $x = 6$	B1
		(1)
(b)	Integrates: $3x^2 - \frac{x^3}{3}$	M1
	Correct integral	A1
	Uses the limits 0 and 6: $108 - 72$	M1
	36	A1
		(4)
Total for Question 3: 5 marks		



Question 4

5 marks

Part	Scheme	Marks
(a)	1.1180 and 1.8028	B1 (1)
(b)	$h = 0.5$ $\frac{0.5}{2} (1 + 2.2361 + 2(1.1180 + 1.4142 + 1.8028))$ 2.977	B1 M1 A1 (3)
(c)	Overestimate: the curve is convex (bends upwards), so the tops of the trapezia lie above the curve	B1 (1)
Total for Question 4: 5 marks		

Question 5

5 marks

Part	Scheme	Marks
(a)	$\left[\frac{x^4}{4} - 2x^3 + 4x^2 \right]_0^4$ $= 64 - 128 + 64 - 0 = 0$ *	M1 A1* (2)
(b)	The region for $2 < x < 4$ is below the x-axis, so its integral is negative and cancels the positive part $\int_0^2 \dots = 4$ and $\int_2^4 \dots = -4$ Area = $4 + 4 = 8$	B1 M1 A1 (3)
Total for Question 5: 5 marks		



Question 6

8 marks

Part	Scheme	Marks
(a)	$-x^2 + 8x - 7 = x + 3 \Rightarrow x^2 - 7x + 10 = 0$ $(x - 2)(x - 5) = 0$ $A(2, 5)$ and $B(5, 8)$	M1 A1 A1 (3)
(b)	$\text{Area} = \int_2^5 ((-x^2 + 8x - 7) - (x + 3)) \, dx = \int_2^5 (-x^2 + 7x - 10) \, dx$ Integrates: $-\frac{x^3}{3} + \frac{7x^2}{2} - 10x$ Correct integral Substitutes the limits: $\left(-\frac{125}{3} + \frac{175}{2} - 50\right) - \left(-\frac{8}{3} + 14 - 20\right)$ $\frac{9}{2}$ Alternative: area under the line (a trapezium, $\frac{1}{2}(5+8) \times 3 = \frac{39}{2}$) subtracted from the area under C (24).	M1 M1 A1 M1 A1 (5)
Total for Question 6: 8 marks		

Question 7

8 marks

Part	Scheme	Marks
(a)	Integrates: $[x^3 - x^2]_1^k$ $= k^3 - k^2 - (1 - 1)$ $k^3 - k^2 = 48$, so $k^3 - k^2 - 48 = 0$ *	M1 M1 A1* (3)
(b)	$64 - 16 - 48 = 0$, so $(k - 4)$ is a factor $(k - 4)(k^2 + 3k + 12) = 0$ $k^2 + 3k + 12$ has discriminant $9 - 48 < 0$, so no other real roots: $k = 4$ only	B1 M1 A1 (3)
(c)	$y = 3x^2 + 2x$ is the reflection of $y = 3x^2 - 2x$ in the y-axis, so the integral from -4 to -1 equals the integral from 1 to 4 48	M1 A1 (2)
Total for Question 7: 8 marks		



Question 8

8 marks

Part	Scheme	Marks
(a)	$h = 0.5$ and y values 1, 1.4142, 2, 2.8284, 4 $\frac{0.5}{2}(1 + 4 + 2(1.4142 + 2 + 2.8284))$ Correct structure with their values 4.371	B1 M1 A1ft A1 (4)
(b)	The curve $y = 2^x$ is convex (bends upwards), so each trapezium lies above the curve	B1 (1)
(c)	$2^{x+1} = 2 \times 2^x$, so the integral is $2 \int_0^2 2^x dx + \int_0^2 3 dx$ $2 \times 4.371 + 6$ 14.74	M1 M1 A1 (3)
Total for Question 8: 8 marks		



Question 9

9 marks

Part	Scheme	Marks
(a)	$\frac{dy}{dx} = 3x^2 - 12x + 9 = 3(x-1)(x-3) = 0$ $x = 1$ gives the maximum ($\frac{d^2y}{dx^2} = -6 < 0$) $M(1, 4)$	M1 A1 A1 (3)
(b)	$\int_0^1 (x^3 - 6x^2 + 9x) dx = \left[\frac{x^4}{4} - 2x^3 + \frac{9}{2}x^2 \right]_0^1$ Correct integral with the limits 0 and 1 $\frac{11}{4}$	M1 A1 A1 (3)
(c)	Same integral with the limits 0 and 3: $\frac{81}{4} - 54 + \frac{81}{2}$ $\frac{27}{4}$	M1 A1 (2)
(d)	$\frac{27}{4} - \frac{11}{4} = 4$	B1ft (1)
Total for Question 9: 9 marks		



Question 10

10 marks

Part	Scheme	Marks
(a)	$\frac{dy}{dx} = 4x^{-\frac{1}{2}} - 2$ $= 0$ gives $x = 4$ $(4, 8)$	M1 A1 A1 (3)
(b)	Integrates: $\frac{16}{3}x^{\frac{3}{2}} - x^2$ Correct integral Uses the limits 0 and 16: $\frac{16}{3} \times 64 - 256$ $\frac{256}{3}$	M1 A1 M1 A1 (4)
(c)	$h = 4$, y values 0, 8, 6.6274, 3.7128, 0: $\frac{4}{2}(0 + 0 + 2(8 + 6.6274 + 3.7128))$ 73.36	M1 A1 (2)
(d)	$\frac{85.33... - 73.36...}{85.33...} \times 100 = 14\%$	B1 (1)
Total for Question 10: 10 marks		

