



eClassroom

INTERNATIONAL A LEVEL MATHEMATICS • PURE 1

Pure Mathematics 1

Exam Questions by Topic

Pearson Edexcel International Advanced Level (WMA11/01) • Calculator allowed

Multi-step exam questions at the difficulty of the real WMA11 paper, with a full mark scheme

Chapters (tap a chapter to jump to it)	Qs	Marks	Page	My score
1 Algebraic Expressions Indices, surds, expanding and factorising	10	57	2	
2 Quadratics Completing the square, the discriminant, quadratic graphs	10	65	22	
3 Equations and Inequalities Simultaneous equations, inequalities, regions	10	69	44	
4 Graphs and Transformations Cubic and reciprocal graphs, intersections, transformations	10	68	66	
5 Straight-line Graphs Gradients, equations of lines, parallel and perpendicular	10	68	89	
6 Trigonometric Ratios Sine and cosine rules, area of a triangle, trig graphs	10	66	110	
7 Radians Radian measure, arc length, sector and segment areas	10	71	132	
8 Differentiation Gradients, second derivatives, tangents and normals	10	71	156	
9 Integration Integrating x^n , finding $y = f(x)$ from $f'(x)$	10	67	180	

[Full mark scheme](#)

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CHAPTER 1 • KEY SKILLS

1

Algebraic Expressions

Specification P1: 1.1, 1.2, 1.10 • 10 questions • 57 marks • about 70 minutes

1 Laws of indices

$$a^m \times a^n = a^{m+n}, \quad a^m \div a^n = a^{m-n},$$

$$(a^m)^n = a^{mn}$$

e.g. $\frac{6x^5}{2x^2} = 3x^3$

2 Negative, zero, fractional indices

$$a^{-n} = \frac{1}{a^n}, \quad a^0 = 1, \quad a^{\frac{m}{n}} = (\sqrt[n]{a})^m$$

e.g. $8^{-\frac{2}{3}} = \frac{1}{4}$

3 Expanding brackets

Multiply two brackets, then the third, and collect like terms.

e.g. $(x+2)(x-1)(x+3)$
 $= x^3 + 4x^2 + x - 6$

4 Factorising completely

Common factor first, then the quadratic or the difference of two squares.

e.g. $x^3 - 9x = x(x-3)(x+3)$

5 Writing as $Ax^p + Bx^q$

Write $\sqrt{x}$ as $x^{\frac{1}{2}}$ and divide every term by the denominator.

e.g. $\frac{x^2+3}{\sqrt{x}} = x^{\frac{3}{2}} + 3x^{-\frac{1}{2}}$

6 Simplifying surds

$\sqrt{ab} = \sqrt{a}\sqrt{b}$: take out the largest square factor.

e.g. $\sqrt{12} + \sqrt{27} = 5\sqrt{3}$

7 Rationalising the denominator

Multiply top and bottom by the conjugate.

e.g. $\frac{1}{3+\sqrt{2}} = \frac{3-\sqrt{2}}{9-2} = \frac{3-\sqrt{2}}{7}$

8 Hidden quadratics

$$4^x = (2^x)^2, \text{ so let } p = 2^x.$$

e.g. $4^x - 5 \times 2^x + 4 = 0$ gives

$$p^2 - 5p + 4 = 0, \text{ so } p = 1, 4 \text{ and } x = 0, 2$$

9 Equating powers

Write both sides as powers of the same prime, then equate the indices.

e.g. $4^x = 8\sqrt{2}$ gives $2^{2x} = 2^{\frac{7}{2}}$, so $x = \frac{7}{4}$

Skills map every skill above is tested at least once; interleaved skills come from GCSE or other chapters

Skill	Q1	Q2	Q3	Q4	Q5	Q6	Q7	Q8	Q9	Q10
Marks	4	6	5	5	6	6	6	7	6	6
1 Laws of indices	•			•		•	•			
2 Negative, zero and fractional indices	•			•			•			
3 Expanding brackets		•		•	•				•	•
4 Factorising completely		•				•				
5 Writing as $Ax^p + Bx^q$				•						
6 Simplifying surds			•		•				•	
7 Rationalising the denominator			•		•			•	•	•
8 Hidden quadratics						•				
9 Equating powers						•	•			
Interleaved: Pythagoras' theorem (GCSE)					•					
Interleaved: simultaneous linear equations (GCSE)							•			
Interleaved: gradients, perpendicular lines (Chapter 5)								•		

Exam difficulty. Questions 1 to 5 check each skill on its own. Questions 6 to 10 are multi-step questions built like the last questions on a real WMA11 paper, where one part leads into the next and skills from different topics are combined.

Must learn. Nothing in this chapter is in the formula booklet. “Hence” means use the result you have just found. “Show that” means every step must be written down. “Exact” means leave surds and fractions in your answer.



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1. In this question you must show all stages of your working.

Solutions relying on calculator technology are not acceptable.

(a) Find the value of $27^{-\frac{2}{3}}$

(2)

(b) Simplify fully $\frac{(27x^6)^{\frac{2}{3}}}{(3x)^2}$

(2)

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(Total for Question 1 is 4 marks)



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2. (a) Factorise completely $2x^3 - 7x^2 - 15x$

(3)

- (b) Expand $(x+3)(2x-1)(4-x)$
giving your answer as a polynomial in x in simplest form.

(3)

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Question 2 continued

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(Total for Question 2 is 6 marks)



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3. In this question you must show all stages of your working.

Solutions relying on calculator technology are not acceptable.

(a) Simplify $\sqrt{50} - \sqrt{18}$

giving your answer in the form $a\sqrt{2}$, where a is an integer.

(2)

(b) Hence, or otherwise, express $\frac{\sqrt{50} - \sqrt{18}}{3\sqrt{2} - 4}$ in the form $p + q\sqrt{2}$, where p and q are integers.

(3)

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Question 3 continued

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(Total for Question 3 is 5 marks)**Join students already acing their exams.**Book your FREE lesson at www.eclassroom.org.uk

4. In this question you must show all stages of your working.

Given that $x > 0$,

- (a) expand and simplify $(3 - 2\sqrt{x})^2$ (2)

- (b) Hence write $\frac{(3 - 2\sqrt{x})^2}{\sqrt{x}}$ in the form $Ax^p + B + Cx^q$
where A , B , C , p and q are constants to be found. (3)

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Question 4 continued

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(Total for Question 4 is 5 marks)**Join students already acing their exams.**Book your FREE lesson at www.eclassroom.org.uk

5. In this question you must show all stages of your working.
Solutions relying on calculator technology are not acceptable.

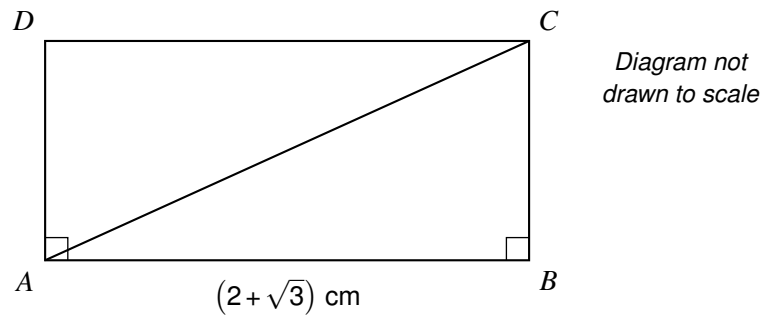
**Figure 1**

Figure 1 shows a rectangle $ABCD$.

The length of AB is $(2 + \sqrt{3})$ cm.

The area of the rectangle is $(13 + 7\sqrt{3})$ cm².

- (a) Find the exact length of BC , giving your answer in the form $a + b\sqrt{3}$ cm, where a and b are integers.

(3)

- (b) Show that $AC^2 = 7(5 + 2\sqrt{3})$

(3)

Question 5 continued

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(Total for Question 5 is 6 marks)



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6. In this question you must show all stages of your working.

Solutions relying on calculator technology are not acceptable.

(a) By substituting $p = 2^x$, show that the equation

$$2^{2x+1} + 8 = 2^{x+4} + 2^x$$

can be written in the form

$$2p^2 - 17p + 8 = 0$$

(3)

(b) Hence solve $2^{2x+1} + 8 = 2^{x+4} + 2^x$

(3)

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Question 6 continued

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(Total for Question 6 is 6 marks)



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7. In this question you must show all stages of your working.

Solutions relying on calculator technology are not acceptable.

Solve the simultaneous equations

$$2^x \times 4^{y+1} = 32\sqrt{2}$$

$$\frac{27^x}{9^y} = 729\sqrt{3}$$

(6)

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Question 7 continued

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(Total for Question 7 is 6 marks)



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Solutions relying on calculator technology are not acceptable.

(a) Show that the gradient of AB can be written in the form $p+q\sqrt{3}$, where p and q are integers to be found.

(3)

The line l crosses the x -axis at the point C .

(b) Find the exact x coordinate of C , giving your answer in the form $a+b\sqrt{3}$, where a and b are integers.

(4)



Question 8 continued

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(Total for Question 8 is 7 marks)



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9. In this question you must show all stages of your working.
Solutions relying on calculator technology are not acceptable.

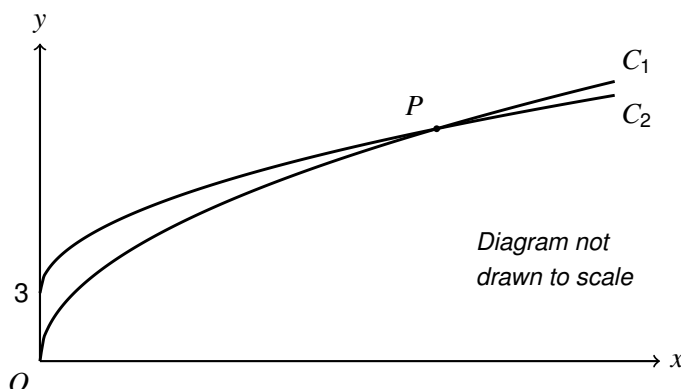
**Figure 2**

Figure 2 shows a sketch of part of the curve C_1 with equation

$$y = \sqrt{2x} \quad x \geq 0$$

and part of the curve C_2 with equation

$$y = \sqrt{x} + 3 \quad x \geq 0$$

The curves meet at the point P .

- (a) Show that the x coordinate of P satisfies $\sqrt{x} = 3(\sqrt{2} + 1)$ (3)
- (b) Hence find the exact coordinates of P , giving each coordinate in the form $a + b\sqrt{2}$, where a and b are integers. (3)



Question 9 continued

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(Total for Question 9 is 6 marks)



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10. In this question you must show all stages of your working.

Solutions relying on calculator technology are not acceptable.

Given that k is a positive constant,

(a) show that $(\sqrt{k+2} - \sqrt{k})(\sqrt{k+2} + \sqrt{k}) = 2$ (1)

(b) Hence show that $\frac{1}{\sqrt{k} + \sqrt{k+2}} = \frac{\sqrt{k+2} - \sqrt{k}}{2}$ (2)

(c) Use the result of part (b) to show that

$$\frac{1}{\sqrt{1} + \sqrt{3}} + \frac{1}{\sqrt{3} + \sqrt{5}} + \frac{1}{\sqrt{5} + \sqrt{7}} + \cdots + \frac{1}{\sqrt{47} + \sqrt{49}} = 3$$

(3)

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Question 10 continued

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(Total for Question 10 is 6 marks)

TOTAL FOR CHAPTER 1 IS 57 MARKS

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2

CHAPTER 2 • KEY SKILLS

Quadratics

Specification P1: 1.3, 1.4, 1.5 • 10 questions • 65 marks • about 80 minutes

1 Solving quadratic equations

Factorise, or use the formula for exact answers.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

e.g. $x^2 - 4x + 1 = 0$ gives $x = 2 \pm \sqrt{3}$ **2 Completing the square**Take out the x^2 coefficient first, including a negative one.

e.g. $3x^2 + 12x + 5 = 3(x+2)^2 - 7$

3 Turning points

$y = a(x+p)^2 + q$ has its turning point at $(-p, q)$: a minimum if $a > 0$, a maximum if $a < 0$.

e.g. $y = 4 - (x-1)^2$: maximum (1, 4)**4 Sketching quadratic graphs**Shape from the sign of a ; mark the y intercept, the roots and the turning point with coordinates.**5 The discriminant** $b^2 - 4ac > 0$: two distinct real roots $b^2 - 4ac = 0$: equal roots $b^2 - 4ac < 0$: no real roots**6 Discriminant with a constant k** Form $b^2 - 4ac$ in terms of k , then solve the equation or inequality in k . Check the x^2 coefficient is not zero.**7 A quadratic from its graph**

Roots p, q : $y = a(x-p)(x-q)$. Turning point (h, k) : $y = a(x-h)^2 + k$. Find a from one more point.

8 Hidden quadraticsSpot a quadratic in x^2 or $\sqrt{x}$.

e.g. $x - 5\sqrt{x} + 4 = 0$:

$(\sqrt{x} - 1)(\sqrt{x} - 4) = 0$, so $x = 1$ or 16

9 Modelling with quadratics

Completed square gives the maximum and when it happens. Roots give when the value is zero. Question the model.

Skills map every skill above is tested at least once; interleaved skills come from GCSE or other chapters

Skill	Q1	Q2	Q3	Q4	Q5	Q6	Q7	Q8	Q9	Q10
Marks	5	5	6	4	6	7	8	6	9	9
1 Solving quadratic equations	•				•		•		•	•
2 Completing the square		•	•			•	•			•
3 Turning points		•	•			•	•			
4 Sketching quadratic graphs			•							
5 The discriminant				•		•		•	•	•
6 Discriminant with a constant k				•				•	•	
7 A quadratic from its graph						•				
8 Hidden quadratics					•					•
9 Modelling with quadratics							•			
Interleaved: surds and exact answers (Chapter 1)	•						•			•
Interleaved: quadratic inequalities (Chapter 3)								•	•	•
Interleaved: intersections of graphs (Chapter 4)						•			•	•

Exam difficulty. Questions 1 to 5 check each skill on its own. Questions 6 to 10 are multi-step questions built like the last questions on a real WMA11 paper, where one part leads into the next and skills from different topics are combined.

Must learn. The quadratic formula is *not* in the formula booklet. When a question gives a constant k , always check the value that makes the x^2 coefficient zero, and give *both* ends of any range.



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1. In this question you must show all stages of your working.

Solutions relying on calculator technology are not acceptable.

(a) Solve $3x^2 + 10x - 8 = 0$

(2)

(b) Solve $2x^2 - 6x + 1 = 0$

giving your answers in the form $\frac{a \pm \sqrt{b}}{c}$, where a , b and c are integers.

(3)

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Question 1 continued

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(Total for Question 1 is 5 marks)



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2. (a) Express $2x^2 - 12x + 23$ in the form $a(x+b)^2 + c$
where a , b and c are integers to be found.

(3)

- (b) Hence write down the minimum value of $2x^2 - 12x + 23$

(1)

- (c) Find the maximum value of $\frac{1}{2x^2 - 12x + 23}$

(1)

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Question 2 continued

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(Total for Question 2 is 5 marks)



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3. The curve C has equation $y = 5 + 4x - x^2$

- (a) Write $5 + 4x - x^2$ in the form $p - (x + q)^2$
where p and q are integers to be found.

(2)

- (b) Sketch the curve C . On your sketch show clearly

- the coordinates of the turning point of C
- the coordinates of the points where C meets the coordinate axes

(4)

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Question 3 continued

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(Total for Question 3 is 6 marks)



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4. The equation $3x^2 + kx + 12 = 0$, where k is a constant, has equal roots.

(a) Find the possible values of k .

(2)

(b) For each value of k , find the root of the equation.

(2)

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(Total for Question 4 is 4 marks)



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5. In this question you must show all stages of your working.

Solutions relying on calculator technology are not acceptable.

(a) Solve $x^4 - 10x^2 + 9 = 0$

(3)

(b) Solve $2x - 7\sqrt{x} + 3 = 0$, $x > 0$

(3)

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Question 5 continued

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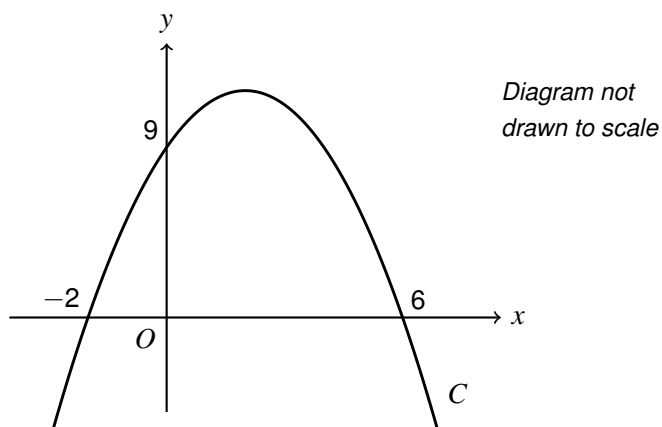
**Figure 1**

Figure 1 shows a sketch of the curve C with equation $y = f(x)$, where $f(x)$ is a quadratic expression. The curve C cuts the x -axis at -2 and 6 and cuts the y -axis at 9

- (a) Find $f(x)$, giving your answer in the form $ax^2 + bx + c$, where a , b and c are constants. (3)
- (b) Write $f(x)$ in the form $p - q(x - r)^2$, where p , q and r are constants. (2)
- (c) Hence write down the coordinates of the maximum point of C . (1)
- (d) Find the set of values of k for which the equation $f(x) = k$ has two distinct real roots. (1)



Question 6 continued

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(Total for Question 6 is 7 marks)



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7. A stone is thrown upwards from the top of a platform.

The height, h metres, of the stone above the ground, t seconds after it is thrown, is modelled by the equation

$$h = 2 + 12t - 5t^2 \quad t \geq 0$$

- (a) Write $2 + 12t - 5t^2$ in the form $A - B(t - C)^2$, where A , B and C are constants to be found. (3)
- (b) Hence state, according to the model,
- (i) the maximum height of the stone above the ground,
 - (ii) the time taken for the stone to reach this maximum height. (2)
- (c) Find, according to the model, the time taken for the stone to hit the ground. Give your answer to 3 significant figures. (2)
- (d) Give one limitation of the model. (1)



Question 7 continued

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(Total for Question 7 is 8 marks)



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8. In this question you must show all stages of your working.

Solutions relying on calculator technology are not acceptable.

The equation

$$(k+3)x^2 + 6x + k = 5$$

where k is a constant, has two distinct real roots.

(a) Show that $k^2 - 2k - 24 < 0$

(3)

(b) Hence find the set of possible values of k .

(3)

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Question 8 continued

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Question 9 continued

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Question 9 continued

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(Total for Question 9 is 9 marks)



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10. In this question you must show all stages of your working.

Solutions relying on calculator technology are not acceptable.

$$f(x) = x^4 - 6x^2 \quad x \in \mathbb{R}$$

- (a) Write $f(x)$ in the form $(x^2 + p)^2 + q$, where p and q are integers to be found. (2)
- (b) Hence solve $f(x) = 16$, giving your answers in simplest surd form. (3)
- (c) Explain why the equation $f(x) = -10$ has no real solutions. (1)
- (d) Find the set of values of A for which the equation $f(x) = A$ has exactly four real solutions. (3)



Question 10 continued

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Question 10 continued

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(Total for Question 10 is 9 marks)

TOTAL FOR CHAPTER 2 IS 65 MARKS

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3

CHAPTER 3 • KEY SKILLS

Equations and Inequalities

Specification P1: 1.6, 1.7, 1.8, 1.9 • 10 questions • 69 marks • about 85 minutes

1 Simultaneous equations

Make one letter the subject of the linear equation and substitute into the other. Pair each x with its y .

e.g. $y = x + 1$, $x^2 + y^2 = 13$ gives $(2, 3)$ and $(-3, -2)$

2 Linear inequalities

Solve like an equation, but reverse the sign when you multiply or divide by a negative number.

e.g. $3 - 2x > 9$ gives $x < -3$

3 Quadratic inequalities

Find the critical values, sketch the curve, then choose the region.

Inside: $a < x < b$. Outside: $x < a$ or $x > b$.

4 Combining inequalities

Draw a number line to find the values that satisfy *both*. Never write

$-1 > x > 2$.

5 x in the denominator

Multiply by x^2 , which is positive, not by x .

e.g. $\frac{4}{x} < 1$ gives $4x < x^2$, so $x < 0$ or $x > 4$

6 Inequalities as graphs

$f(x) > g(x)$ means the graph of $y = f(x)$ is *above* the graph of $y = g(x)$.

7 Shading regions

Solid line for $\leq$ or $\geq$ (boundary included); dashed line for $<$ or $>$.

e.g. $y > x^2$: dashed curve, shade above

8 Defining a region

One inequality for each boundary, including any vertical boundary such as $x > 0$.

9 Forming equations from a context

Write each fact as an equation or inequality, then check your answers make sense in the context.

Skills map every skill above is tested at least once; interleaved skills come from GCSE or other chapters

Skill	Q1	Q2	Q3	Q4	Q5	Q6	Q7	Q8	Q9	Q10
Marks	6	7	4	6	5	7	9	8	7	10
1 Simultaneous equations	•			•		•	•	•	•	
2 Linear inequalities		•								
3 Quadratic inequalities		•			•		•			•
4 Combining inequalities		•					•			
5 x in the denominator			•							
6 Inequalities as graphs					•	•				
7 Shading regions				•						
8 Defining a region						•			•	
9 Forming equations from a context							•		•	•
Interleaved: quadratic graphs, discriminant, completing the square (Chapter 2)				•				•		•
Interleaved: exact surd answers (Chapter 1)								•		
Interleaved: straight lines and distances (Chapter 5, GCSE)									•	•

Exam difficulty. Questions 1 to 5 check each skill on its own. Questions 6 to 10 are multi-step questions built like the last questions on a real WMA11 paper, where one part leads into the next and skills from different topics are combined.

Must learn. Examiners take marks for $-1 > x > 2$ (write $x < -1$ or $x > 2$), for “and” where you mean “or”, and for leaving out a boundary such as $x > 0$ when you define a region.



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1. In this question you must show all stages of your working.

Solutions relying on calculator technology are not acceptable.

Solve the simultaneous equations

$$y - 3x + 2 = 0$$

$$x^2 + y^2 = 20$$

(6)

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Question 1 continued

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(Total for Question 1 is 6 marks)**Join students already acing their exams.**Book your FREE lesson at www.eclassroom.org.uk

2. Find the set of values of x for which

(a) $5x - 3 > 2(x + 6)$

(2)

(b) $2x^2 - 9x - 18 < 0$

(4)

(c) both $5x - 3 > 2(x + 6)$ and $2x^2 - 9x - 18 < 0$

(1)

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3. In this question you must show all stages of your working.

Solutions relying on calculator technology are not acceptable.

Find the set of values of x for which

$$5 - \frac{12}{x} > 2 \quad x \neq 0$$

(4)

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Question 4 continued

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(Total for Question 4 is 6 marks)**Join students already acing their exams.**Book your FREE lesson at www.eclassroom.org.uk

5. In this question you must show all stages of your working.

Solutions relying on calculator technology are not acceptable.

The curve C has equation $y = 2x^2 - x - 4$

The line l has equation $y = x + 8$

(a) Find the set of values of x for which C lies above l .

(4)

(b) Hence write down the set of values of x for which $2x^2 - x - 4 \leq x + 8$

(1)

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Question 5 continued

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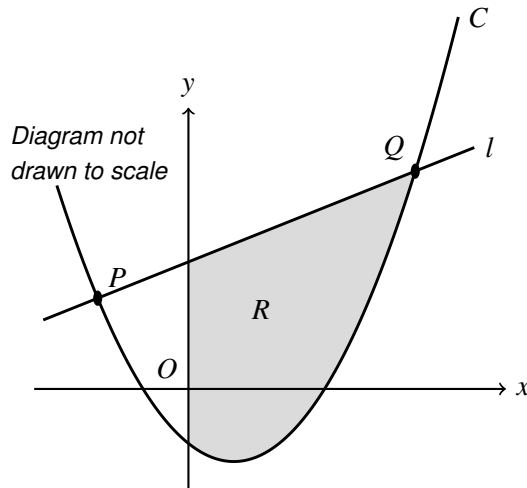
**Figure 1**

Figure 1 shows a sketch of the curve C with equation $y = x^2 - 2x - 3$ and the line l with equation $y = x + 7$

The line l meets C at the points P and Q , as shown in Figure 1.

- (a) Using algebra, find the coordinates of P and the coordinates of Q .
(Solutions relying on calculator technology are not acceptable.)

(4)

The region R , shown shaded in Figure 1, is bounded by C , l and the y -axis.

- (b) Use inequalities to define R .

(3)

Question 6 continued

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(Total for Question 6 is 7 marks)



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7. A rectangular sheet of card has length x cm and width y cm, where $x > y$

The perimeter of the sheet is 46 cm and the length of a diagonal of the sheet is 17 cm.

- (a) Write down two equations linking x and y .

(2)

- (b) Hence find the value of x and the value of y .

(4)

A second rectangular sheet of card also has a perimeter of 46 cm. Its area is at least 120 cm^2 and its length, x cm, is greater than its width.

- (c) Find the set of possible values of x .

(3)

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Question 7 continued

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Solutions relying on calculator technology are not acceptable.

The curve C has equation $y = \frac{1}{x}$, $x \neq 0$

- (a)** Show that the x coordinates of any points where l meets C satisfy the equation

$$kx^2 + 3x - 1 = 0$$

(2)

- (b)** Find the set of values of k for which l meets C at two distinct points.

(2)

(c) find the exact coordinates of the points where l meets C , giving each coordinate in simplest surd form.

(4)

Question 8 continued

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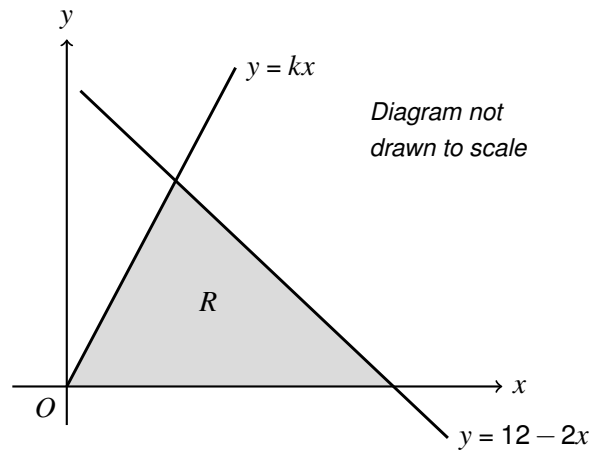


Figure 2

Figure 2 shows the line with equation $y = kx$, where k is a positive constant, and the line with equation $y = 12 - 2x$

The region R , shown shaded in Figure 2, is bounded by the two lines and the x -axis.

- (a) Show that the x coordinate of the point where the two lines meet is $\frac{12}{k+2}$ (2)

Given that the area of R is 24 square units,

- (b) find the value of k . (3)
- (c) Hence use inequalities to define R . (2)



Question 9 continued

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(Total for Question 9 is 7 marks)**Join students already acing their exams.**Book your FREE lesson at www.eclassroom.org.uk

Solutions relying on calculator technology are not acceptable.

The point $P(p, 2p - 1)$ lies on l .

- Given that $AP = \sqrt{50}$,

- (c)** Find the set of values of p for which $AP > 5$ **(3)**

- (d) Find the exact shortest distance from A to l , and the coordinates of the point on l where it occurs. (2)

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Question 10 continued

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Question 10 continued

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(Total for Question 10 is 10 marks)

TOTAL FOR CHAPTER 3 IS 69 MARKS

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4

CHAPTER 4 • KEY SKILLS

Graphs and Transformations

Specification P1: 1.11, 1.12 • 10 questions • 68 marks • about 80 minutes

1 Sketching cubics

Find the roots and the y intercept. A squared factor touches the x -axis. A negative x^3 term flips the shape.
e.g. $y = (x - 1)^2(x + 2)$ touches at $(1, 0)$

2 Factorise, then sketch

Take out a factor of x first, then factorise the quadratic.
e.g. $x^3 - 4x^2 + 4x = x(x - 2)^2$

3 Reciprocal graphs

$y = \frac{k}{x}$, $k > 0$: first and third quadrants.
 $y = \frac{k}{x^2}$, $k > 0$: both branches above the x -axis.

4 Asymptotes

State them as equations. They move with the graph.
e.g. $y = \frac{1}{x} + 3$ has asymptotes $x = 0$ and $y = 3$

5 Counting solutions

Rearrange to $f(x) = g(x)$ for two graphs you can sketch. The number of solutions is the number of intersections: give this reason.

6 Intersections by algebra

Set the equations equal. Look for common factors before you expand.

7 Translations

$y = f(x + a)$: a units left.
 $y = f(x) + a$: a units up.

8 Stretches

$y = af(x)$: y coordinates $\times a$.
 $y = f(ax)$: x coordinates $\div a$.

9 Equations from graphs

Roots give the factors; include a constant k .
e.g. repeated root 3: $y = kx(x - 3)^2$, then find k from one more point.

Skills map every skill above is tested at least once; interleaved skills come from GCSE or other chapters

Skill	Q1	Q2	Q3	Q4	Q5	Q6	Q7	Q8	Q9	Q10
Marks	5	5	6	5	5	9	8	8	8	9
1 Sketching cubics	•			•		•				•
2 Factorise, then sketch	•					•	•			
3 Reciprocal graphs		•		•	•			•		
4 Asymptotes		•			•			•	•	
5 Counting solutions				•						•
6 Intersections by algebra						•	•	•		
7 Translations		•	•		•				•	•
8 Stretches			•						•	
9 Equations from graphs							•		•	
Interleaved: surds and exact answers (Chapter 1)		•				•				
Interleaved: solving quadratics (Chapter 2)							•	•		
Interleaved: inequalities from graphs (Chapter 3)						•				•

Exam difficulty. Questions 1 to 5 check each skill on its own. Questions 6 to 10 are multi-step questions built like the last questions on a real WMA11 paper, where one part leads into the next and skills from different topics are combined.

Must learn. Every sketch needs the coordinates of the points where it meets the axes, and the *equation* of every asymptote. $f(x) - 2$ moves the graph down; $f(x - 2)$ moves it right.



1. (a) Factorise completely $x^3 - 6x^2 + 9x$

(2)

- (b) Sketch the curve with equation $y = x^3 - 6x^2 + 9x$
showing clearly the coordinates of the points where the curve meets the coordinate axes.

(3)

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2. (a) Sketch the curve with equation $y = \frac{3}{x^2}$, $x \neq 0$

(2)

- (b) On a separate diagram, sketch the curve with equation $y = \frac{3}{x^2} - 1$, $x \neq 0$

On your sketch show clearly the equation of the horizontal asymptote and the exact coordinates of the points where the curve crosses the x -axis.

(3)

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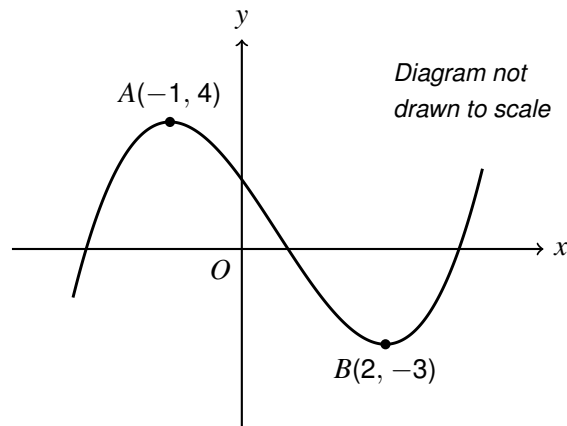


Figure 1

Figure 1 shows a sketch of part of the curve with equation $y = f(x)$.

The curve has a maximum point at $A(-1, 4)$ and a minimum point at $B(2, -3)$.

Write down the coordinates of the maximum point and the coordinates of the minimum point on the curve with equation

(a) $y = f(x + 2)$

(2)

(b) $y = 3f(x)$

(2)

(c) $y = f(2x)$

(2)



Question 3 continued

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4. (a) On the same axes, sketch the curves with equations

$$y = x^2(x - 3) \quad \text{and} \quad y = \frac{4}{x}, \quad x \neq 0$$

On your sketch show clearly the coordinates of the points where the curve $y = x^2(x - 3)$ meets the x -axis.

(3)

- (b) Hence state the number of real solutions of the equation $x^4 - 3x^3 = 4$ giving a reason for your answer.

(2)

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Question 5 continued

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Solutions relying on calculator technology are not acceptable.

$$f(x) = (2x + 1)^2(x - 3)$$

- (3)**

- (1)

- (2)

The line l passes through the point where the curve meets the y -axis and is parallel to the x -axis.

- (3)**

Question 6 continued

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Question 6 continued

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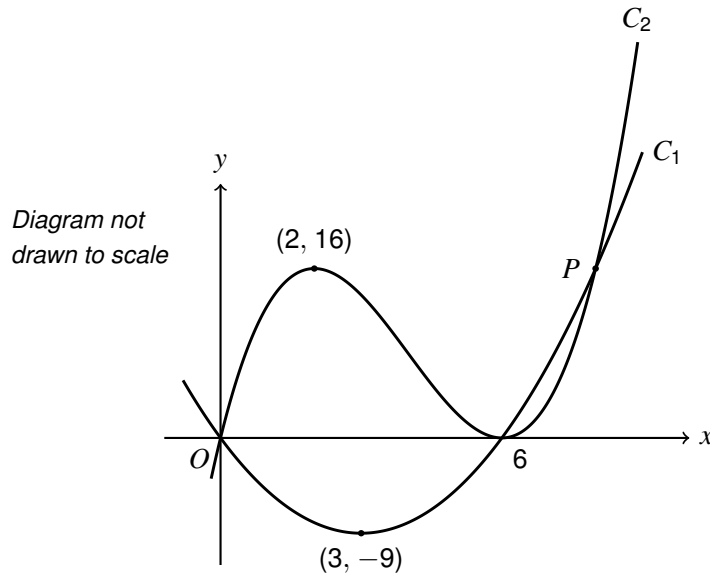


Figure 2

Figure 2 shows a sketch of the curve C_1 with equation $y = f(x)$, where $f(x)$ is a quadratic expression, and the curve C_2 with equation $y = g(x)$, where $g(x)$ is a cubic expression.

- C_1 passes through O and $(6, 0)$ and has a minimum point at $(3, -9)$
- C_2 passes through O , touches the x -axis at $(6, 0)$ and has a maximum point at $(2, 16)$

(a) Find $f(x)$.

(2)

(b) Find $g(x)$.

(3)

The curves also meet at the point P , shown in Figure 2.

(c) Using algebra, find the coordinates of P .

(3)



Question 7 continued

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(a) Sketch C , stating the equations of its asymptotes and the coordinates of the point where C crosses the x -axis.

(3)

The line l meets C at the point P , where P has x coordinate -3

(b) Find the value of k .

(2)

The line l meets C again at the point Q .

(c) Using algebra, find the coordinates of Q .

(Solutions relying entirely on calculator technology are not acceptable.)

(3)

Question 8 continued

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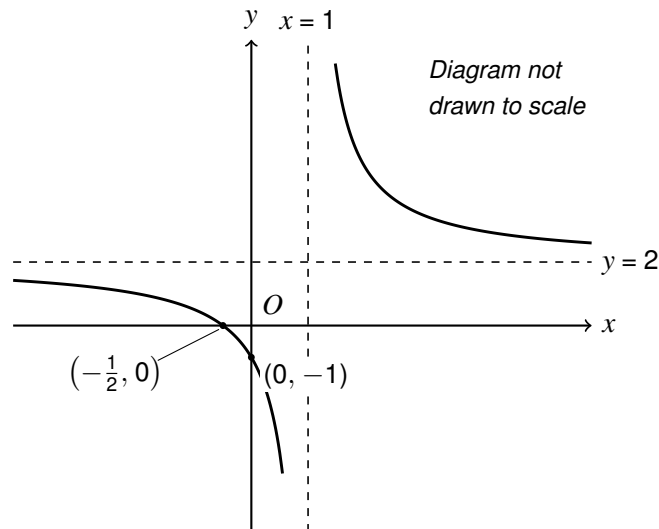


Figure 3

Figure 3 shows a sketch of the curve with equation $y = f(x)$.

The curve has asymptotes $x = 1$ and $y = 2$, crosses the x -axis at $(-\frac{1}{2}, 0)$ and crosses the y -axis at $(0, -1)$.

On separate diagrams, sketch the curve with equation

(a) $y = f(x) - 2$ (3)

(b) $y = f(\frac{1}{2}x)$ (3)

On each sketch, show clearly the equations of the asymptotes and the coordinates of any points where the curve meets the coordinate axes.

Given that $f(x) = a + \frac{b}{x-1}$, where a and b are constants,

(c) find the value of a and the value of b . (2)



Question 9 continued

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10.

$$f(x) = (x + 1)(x - 2)^2$$

The curve with equation $y = f(x)$ has a maximum point at $(0, 4)$.

- (a) Sketch the curve with equation $y = f(x)$, showing clearly the coordinates of the points where the curve meets the coordinate axes. (3)
- (b) Find the set of values of k for which the equation $f(x) = k$ has three distinct real solutions. (2)
- (c) State the number of real solutions of the equation $f(x + 3) = 2$ giving a reason for your answer. (2)
- (d) Find the values of the constant c for which the curve with equation $y = f(x) + c$ touches the x -axis. (2)

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Question 10 continued

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5

CHAPTER 5 • KEY SKILLS

Straight-line Graphs

Specification P1: 2.1, 2.2 • 10 questions • 68 marks • about 80 minutes

1 Gradient

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

e.g. (1, 2) and (4, 11): $m = \frac{9}{3} = 3$

2 Equation of a line

$y - y_1 = m(x - x_1)$, then rearrange as asked.

e.g. $y - 2 = 3(x - 1)$ gives
 $3x - y - 1 = 0$ (integers, = 0)

3 Parallel and perpendicular

Parallel: equal gradients.

Perpendicular: $m_1 m_2 = -1$.

e.g. $3x + 4y = 18$ has $m = -\frac{3}{4}$;
 perpendicular $m = \frac{4}{3}$

4 Midpoints and bisectors

Midpoint $(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2})$. Perpendicular bisector: through the midpoint with the negative reciprocal gradient.

5 Distance between points

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

e.g. (1, 2) to (4, 6) is 5

6 Where lines meet

Solve the equations simultaneously.

e.g. $y = 2x + 1$ and $x + y = 7$ meet at (2, 5)

7 Areas of triangles

$\frac{1}{2} \times \text{base} \times \text{perpendicular height}$.

Look for a right angle or a side on an axis.

8 Proving properties

Show $m_1 m_2 = -1$ for a right angle, or equal gradients for parallel sides, and state the conclusion.

9 Linear models

In $C = 50d + 55$: the gradient 50 is the cost per day; the intercept 55 is the fixed charge.

Skills map every skill above is tested at least once; interleaved skills come from GCSE or other chapters

Skill	Q1	Q2	Q3	Q4	Q5	Q6	Q7	Q8	Q9	Q10
Marks	4	5	4	7	6	8	8	7	9	10
1 Gradient	•			•	•			•		
2 Equation of a line	•	•	•		•	•	•		•	•
3 Parallel and perpendicular		•	•	•		•	•	•	•	•
4 Midpoints and bisectors			•				•			•
5 Distance between points				•			•		•	•
6 Where lines meet						•			•	•
7 Areas of triangles				•		•	•		•	•
8 Proving properties				•		•		•		
9 Linear models					•					
Interleaved: surds and exact answers (Chapter 1)				•				•	•	•
Interleaved: quadratics and inequalities (Chapters 2, 3)					•		•	•		•
Interleaved: curves meeting lines (Chapter 4)								•		

Exam difficulty. Questions 1 to 5 check each skill on its own. Questions 6 to 10 are multi-step questions built like the last questions on a real WMA11 paper, where one part leads into the next and skills from different topics are combined.

Must learn. Perpendicular means *negative* reciprocal. " $ax + by + c = 0$ " means integers and " $= 0$ ". When no diagram is given, draw your own sketch first.



1. The line L passes through the points $A(-2, 7)$ and $B(4, -2)$.

Find an equation for L , giving your answer in the form $ax + by + c = 0$, where a , b and c are integers.

(4)

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(Total for Question 1 is 4 marks)



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2. The line l_1 has equation $2x - 5y + 10 = 0$

(a) Find the gradient of l_1 .

(1)

The line l_2 is perpendicular to l_1 and passes through the point $(4, -1)$.

(b) Find an equation for l_2 , giving your answer in the form $y = mx + c$, where m and c are constants.

(3)

(c) Find the coordinates of the point where l_2 crosses the x -axis.

(1)

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(Total for Question 2 is 5 marks)



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3. The points A and B have coordinates $(-3, 4)$ and $(5, 8)$ respectively.

Find an equation of the perpendicular bisector of AB , giving your answer in the form $ax + by + c = 0$, where a , b and c are integers.

(4)

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(Total for Question 3 is 4 marks)



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4. In this question you must show all stages of your working.**Solutions relying on calculator technology are not acceptable.**

The points P , Q and R have coordinates $(-2, 1)$, $(2, 3)$ and $(5, -3)$ respectively.

(a) Prove that angle $PQR = 90^\circ$

(3)

Given that $PQRS$ is a rectangle,

(b) find the coordinates of S .

(2)

(c) Find the area of the rectangle $PQRS$.

(2)

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Question 4 continued

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(Total for Question 4 is 7 marks)



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5. The total cost, £ C , of hiring a van for d days from company A is modelled by the linear equation

$$C = pd + q$$

where p and q are constants. Hiring the van for 3 days costs £205 and hiring it for 7 days costs £405

- (a) Find the value of p and the value of q . (3)

- (b) Interpret the value of p in the context of the model. (1)

Company B charges £65 per day with no fixed charge.

- (c) Find the least number of whole days for which hiring from company A is cheaper than hiring from company B . (2)



Question 5 continued

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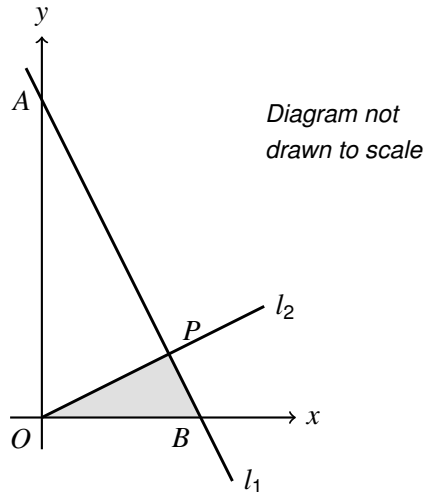
(Total for Question 5 is 6 marks)



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6.

**Figure 1**

In Figure 1, the line l_1 passes through the points $A(0, 10)$ and $B(5, 0)$.

The line l_2 passes through the origin O and is perpendicular to l_1 . The lines meet at the point P .

- (a) Find an equation for l_1 . (2)
- (b) Find an equation for l_2 . (2)
- (c) Find the coordinates of P . (2)
- (d) Show that the area of triangle OPA is four times the area of triangle OPB . (2)



Question 6 continued

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(Total for Question 6 is 8 marks)



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Solutions relying on calculator technology are not acceptable.

The line l is the perpendicular bisector of AB .

- (3)**

- (1)**

(c) Find the possible coordinates of C .

(4)



Question 7 continued

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(Total for Question 7 is 8 marks)



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8. In this question you must show all stages of your working.
Solutions relying on calculator technology are not acceptable.

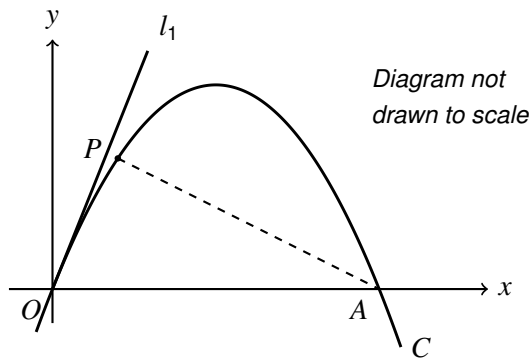


Figure 2

Figure 2 shows a sketch of the curve C with equation $y = x(6 - x)$ and the line l_1 with equation $y = kx$, where k is a constant and $0 < k < 6$

The line l_1 meets C at O and at the point P . The point A has coordinates $(6, 0)$.

- (a) Show that P has coordinates $(6 - k, 6k - k^2)$ (2)

- (b) Show that the gradient of the line PA is $k - 6$ (2)

Given that l_1 is perpendicular to the line PA and that $k > 1$,

- (c) find the exact value of k , giving your answer in the form $a + b\sqrt{2}$, where a and b are integers. (3)



Question 8 continued

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(Total for Question 8 is 7 marks)



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Solutions relying on calculator technology are not acceptable.

The quadrilateral $ABCD$ is a trapezium with AB parallel to DC and angle $DAB = 90^\circ$

- (a) Find an equation of the line through C that is parallel to AB . (2)
- (b) Find an equation of the line through A that is perpendicular to AB . (2)
- (c) Hence find the coordinates of D . (2)
- (d) Find the exact area of the trapezium $ABCD$. (3)

Question 9 continued

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Question 9 continued

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(Total for Question 9 is 9 marks)



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Solutions relying on calculator technology are not acceptable.

(a) Find an equation of the line through P that is perpendicular to l .

(2)

(2)

(c) Find the coordinates of P' .

(2)

(d) Find the exact length of PP' , giving your answer in the form $a\sqrt{5}$

(1)

(e) Find the possible coordinates of Q .

(3)

Question 10 continued

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Question 10 continued

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(Total for Question 10 is 10 marks)**TOTAL FOR CHAPTER 5 IS 68 MARKS****Join students already acing their exams.**Book your FREE lesson at www.eclassroom.org.uk

6

CHAPTER 6 • KEY SKILLS

Trigonometric Ratios

Specification P1: 3.1, 3.3 • 10 questions • 66 marks • about 80 minutes

1 Cosine rule for a side

$$a^2 = b^2 + c^2 - 2bc \cos A$$

Use it when you know two sides and the angle between them.

2 Cosine rule for an angle

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$

A negative value means A is obtuse.

3 Sine rule

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

Use it with a side and its opposite angle.

4 The ambiguous case

$\sin \theta = \sin(180^\circ - \theta)$, so a sine rule angle may be acute *or* obtuse. Check both fit in the triangle.

5 Area of a triangle

Area = $\frac{1}{2}ab \sin C$, where C is the angle between a and b .

Given the area, $\sin C$ may give two angles.

6 Multi-step triangle problems

Split the shape into triangles, find the shared side first, and keep full accuracy until the end.

7 Graphs of sin, cos, tan

sin and cos: period 360° , between -1 and 1 .

tan: period 180° , asymptotes at $x = 90^\circ, 270^\circ$

8 Transforming trig graphs

$y = a \sin x$: height a . $y = \sin(x + 30^\circ)$: 30° left.

$y = \sin bx$: period $\frac{360^\circ}{b}$

9 Counting solutions

Sketch both graphs over the whole interval and count the intersections. Check end points, asymptotes and repeating periods.

Skills map every skill above is tested at least once; interleaved skills come from GCSE or other chapters

Skill	Q1	Q2	Q3	Q4	Q5	Q6	Q7	Q8	Q9	Q10
Marks	4	5	5	5	5	8	9	8	8	9
1 Cosine rule for a side	•					•				•
2 Cosine rule for an angle			•							
3 Sine rule		•				•				•
4 The ambiguous case		•				•				•
5 Area of a triangle	•		•			•				
6 Multi-step triangle problems						•				•
7 Graphs of sin, cos, tan				•	•		•	•	•	
8 Transforming trig graphs				•	•		•	•		
9 Counting solutions				•	•		•	•	•	
Interleaved: surds and exact values (Chapter 1)			•					•		•
Interleaved: quadratics and straight lines (Chapters 2, 5)							•		•	•
Interleaved: graph transformations (Chapter 4)				•	•		•	•		

Exam difficulty. Questions 1 to 5 check each skill on its own. Questions 6 to 10 are multi-step questions built like the last questions on a real WMA11 paper, where one part leads into the next and skills from different topics are combined.

Must learn. The sine rule, the cosine rule and $\frac{1}{2}ab \sin C$ are *not* in the formula booklet. Read whether an angle is acute or obtuse, and keep full calculator accuracy until the final answer.



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1. In the triangle ABC , $AB = 8$ cm, $AC = 11$ cm and angle $BAC = 50^\circ$

(a) Find the length of BC , giving your answer in cm to 3 significant figures.

(2)

(b) Find the area of triangle ABC , giving your answer in cm^2 to 3 significant figures.

(2)

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(Total for Question 1 is 4 marks)



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2. In the triangle PQR , $PQ = 9$ cm, $QR = 7$ cm and angle $QPR = 42^\circ$

- (a) Find the two possible values of angle PRQ , giving each answer in degrees to one decimal place.

(3)

Given that angle PRQ is obtuse,

- (b) find the length of PR , giving your answer in cm to 3 significant figures.

(2)

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Solutions relying on calculator technology are not acceptable.

(a) Show that $\cos \theta = -\frac{1}{10}$

(2)

(b) Find the exact area of the triangle, giving your answer in the form $\frac{a\sqrt{11}}{b}$ cm², where a and b are integers.

(3)

Question 3 continued

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(Total for Question 3 is 5 marks)**Join students already acing their exams.**Book your FREE lesson at www.eclassroom.org.uk

4. (a) Sketch the graph of $y = \tan(x - 45^\circ)$, $0 \leq x \leq 360^\circ$
On your sketch show clearly the equations of any asymptotes and the coordinates of the points where the graph meets the coordinate axes. (3)
- (b) State the number of solutions of the equation $\tan(x - 45^\circ) = 2$ in the interval $0 \leq x \leq 360^\circ$ (1)
- (c) State the number of solutions of the equation $\tan(x - 45^\circ) + 1 = 0$ in the interval $0 \leq x \leq 360^\circ$ (1)

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Question 4 continued

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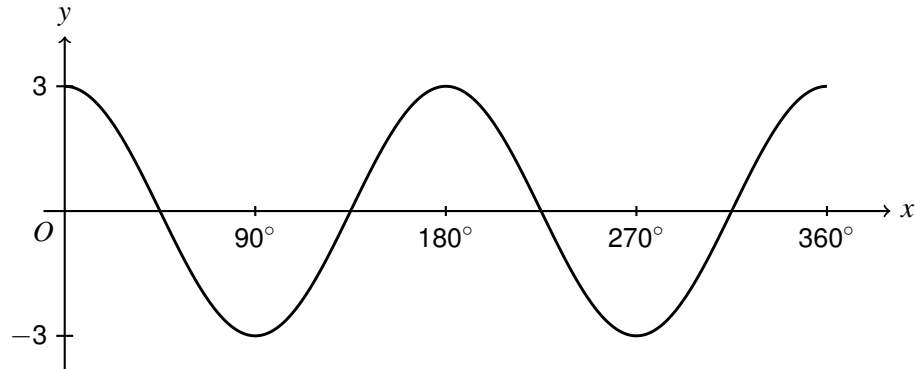


Figure 1

Figure 1 shows the graph of $y = f(x)$, $0 \leq x \leq 360^\circ$, where $f(x) = a \cos bx$ and a and b are positive constants.

- (a) Write down the value of a and the value of b . (2)
- (b) Write down the coordinates of the minimum points of the graph. (2)
- (c) State the number of solutions of the equation $f(x) = 1$ in the interval $0 \leq x \leq 360^\circ$ (1)



Question 5 continued

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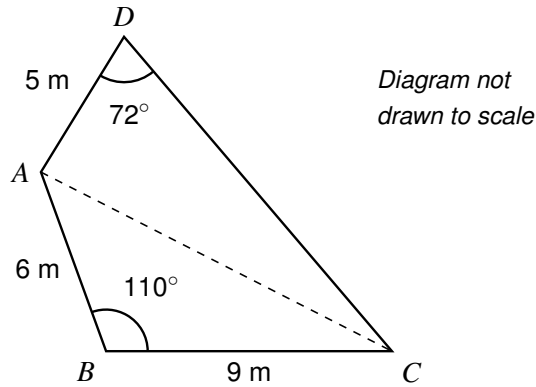


Figure 2

Figure 2 shows the plan of a garden $ABCD$.

$AB = 6$ m, $BC = 9$ m, $AD = 5$ m, angle $ABC = 110^\circ$ and angle $ADC = 72^\circ$

- (a) Find the length of AC , giving your answer in metres to 3 significant figures. (2)
- (b) Find the size of angle ACD , giving your answer in degrees to one decimal place. Explain why there is only one possible value. (3)
- (c) Find the total area of the garden, giving your answer in m^2 to 3 significant figures. (3)



Question 6 continued

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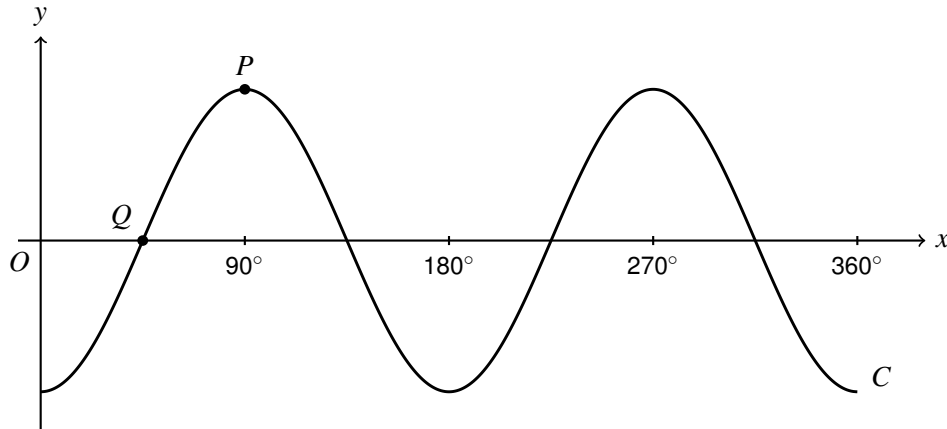


Figure 3

Figure 3 shows part of the curve C with equation

$$y = -4 \cos 2x \quad 0 \leq x \leq 360^\circ$$

The point P is a maximum point of C and the point Q is where C first crosses the positive x -axis.

- (a) Write down the coordinates of P and the coordinates of Q . (3)

The curve C is translated k units in the positive y direction, where k is a constant. The translated curve passes through the point $(30^\circ, 5)$

- (b) Find the value of k . (2)

- (c) Hence write down the coordinates of the minimum points of the translated curve for $0 \leq x \leq 360^\circ$ (2)

- (d) By drawing a suitable straight line on Figure 3, state the number of solutions of the equation

$$4 \cos 2x + 2 = \frac{x}{45} \quad 0 \leq x \leq 360^\circ$$

giving a reason for your answer.

(2)



Question 7 continued

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Question 7 continued

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(Total for Question 7 is 9 marks)



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8. The curve C has equation

$$y = 2 \sin(x + 60^\circ) \quad 0 \leq x \leq 360^\circ$$

- (a) Find the exact coordinates of the points where C meets the coordinate axes. (3)
- (b) Find the coordinates of the maximum point and the minimum point of C . (2)
- (c) Find the set of values of k for which the line with equation $y = k$ meets C at exactly two points. (2)
- (d) State the number of solutions of the equation $2 \sin(2x + 60^\circ) = 1$ in the interval $0 \leq x \leq 360^\circ$ (1)

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Question 8 continued

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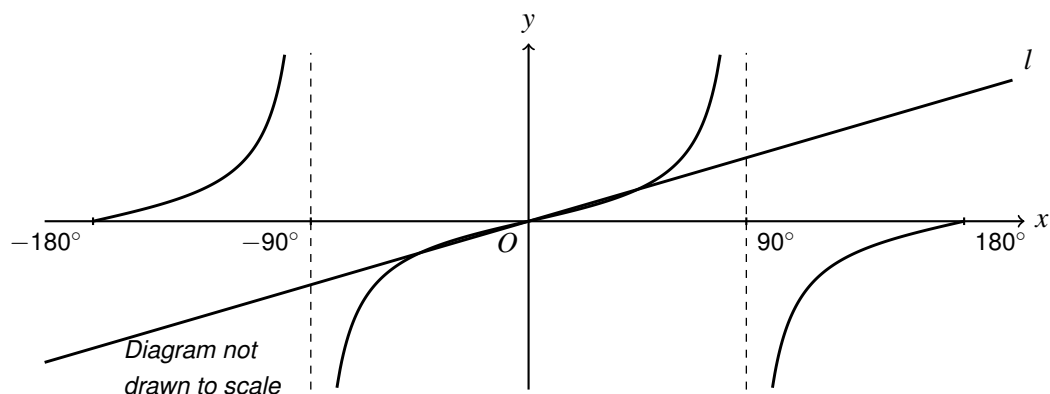


Figure 4

Figure 4 shows part of the curve with equation $y = \tan x$ and the straight line l with equation $y = \frac{x}{45}$, in the interval $-180^\circ < x < 180^\circ$

- (a) State the period of $\tan x$. (1)
- (b) Show that $x = 45^\circ$ is a solution of the equation $\tan x = \frac{x}{45}$. (1)
- (c) Write down the number of solutions of the equation $\tan x = \frac{x}{45}$ in the interval
- (i) $-180^\circ < x < 180^\circ$
 - (ii) $-360^\circ < x < 360^\circ$
 - (iii) $-9000^\circ < x < 9000^\circ$
- (4)
- (d) State the number of solutions of the equation $\tan x = -\frac{x}{45}$ in the interval $-180^\circ < x < 180^\circ$, giving a reason for your answer. (2)



Question 9 continued

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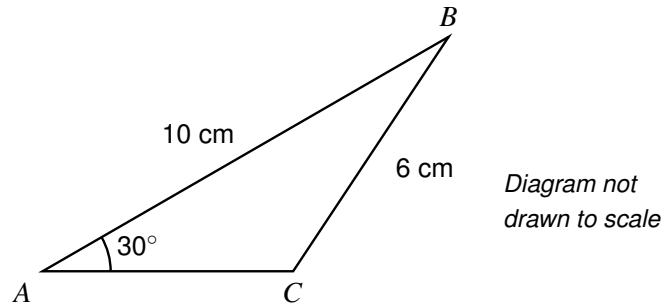


Figure 5

Figure 5 shows the triangle ABC with $AB = 10$ cm, $BC = 6$ cm and angle $BAC = 30^\circ$. The angle ACB is obtuse.

- (a) Find the size of angle ACB , giving your answer in degrees to one decimal place.

(3)

- (b) By applying the cosine rule to triangle ABC , show that $AC = x$ cm satisfies

$$x^2 - 10\sqrt{3}x + 64 = 0$$

and hence show that $AC = 5\sqrt{3} - \sqrt{11}$ cm.

(Solutions relying on calculator technology are not acceptable.)

(4)

The point D lies on AC extended, beyond C , such that $BD = 6$ cm.

- (c) Find the exact length of CD .

(2)



Question 10 continued

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(Total for Question 10 is 9 marks)

TOTAL FOR CHAPTER 6 IS 66 MARKS

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7

CHAPTER 7 • KEY SKILLS

Radians

Specification P1: 3.2, 3.3 • 10 questions • 71 marks • about 85 minutes

1 Radians and exact values π radians = 180° .

e.g. $150^\circ = \frac{5\pi}{6}$, $\sin \frac{\pi}{3} = \frac{\sqrt{3}}{2}$,

$\cos \frac{\pi}{4} = \frac{\sqrt{2}}{2}$

2 Arc length $s = r\theta$, with θ in radians.

e.g. $r = 5$, $\theta = 1.2$: $s = 6$

3 Area of a sector $A = \frac{1}{2}r^2\theta$, with θ in radians.

e.g. $r = 4$, $\theta = \frac{\pi}{4}$: $A = 2\pi$

4 Area of a segment

Sector minus triangle:

$\frac{1}{2}r^2\theta - \frac{1}{2}r^2\sin\theta = \frac{1}{2}r^2(\theta - \sin\theta)$

5 Perimeters

Add every outside edge: arcs, radii and straight sides. Never include an internal line.

6 Triangles in radiansSine rule, cosine rule and $\frac{1}{2}ab\sin C$ work in radians: set your calculator to radian mode.**7 Equations from a sector**Write the perimeter and area in r and θ , then eliminate one of them. Check $0 < \theta < 2\pi$.**8 Trig graphs in radians** $\sin$, $\cos$: period 2π . $\tan$: period π .

$y = \sin\left(x + \frac{\pi}{3}\right)$: $\frac{\pi}{3}$ to the left.

9 Solutions from graphs

Rearrange so one side is a graph you know and the other a line you can draw; count intersections.

Skills map every skill above is tested at least once; interleaved skills come from GCSE or other chapters

Skill	Q1	Q2	Q3	Q4	Q5	Q6	Q7	Q8	Q9	Q10
Marks	4	6	5	5	6	9	9	9	8	10
1 Radians and exact values	•		•	•					•	
2 Arc length	•	•			•	•	•	•		•
3 Area of a sector	•	•			•	•	•	•		•
4 Area of a segment					•					
5 Perimeters		•			•	•	•	•		•
6 Triangles in radians		•			•	•	•	•		
7 Equations from a sector		•				•				•
8 Trig graphs in radians			•	•					•	
9 Solutions from graphs				•						
Interleaved: sine rule, cosine rule, ambiguous case (Chapter 6)					•	•	•	•		
Interleaved: quadratics, completing the square (Chapter 2)										•
Interleaved: transformations, straight lines (Chapters 4, 5)			•	•					•	

Exam difficulty. Questions 1 to 5 check each skill on its own. Questions 6 to 10 are multi-step questions built like the last questions on a real WMA11 paper, where one part leads into the next and skills from different topics are combined.

Must learn. $s = r\theta$ and $A = \frac{1}{2}r^2\theta$ are *not* in the formula booklet. Stay in radians: a straight line is π , not 2π . Keep full accuracy until the final answer.



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1. In this question you must show all stages of your working.

Solutions relying on calculator technology are not acceptable.

(a) Write 150° in radians, giving your answer as an exact multiple of π .

(1)

The sector OAB of a circle has centre O , radius 6 cm and angle $AOB = 150^\circ$

(b) Find, in terms of π ,

- (i) the length of the arc AB ,
- (ii) the area of the sector OAB .

(3)

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(Total for Question 1 is 4 marks)



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2. The sector OAB of a circle has centre O and radius r cm. The angle AOB is 1.25 radians and the area of the sector is 40 cm^2

(a) Show that $r = 8$

(2)

(b) Find the perimeter of the sector.

(2)

(c) Find the length of the chord AB , giving your answer in cm to 3 significant figures.

(2)

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(Total for Question 2 is 6 marks)



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3. (a) Sketch the graph of $y = \cos\left(x - \frac{\pi}{3}\right)$, $0 \leq x \leq 2\pi$

Show clearly the exact coordinates of the points where the graph meets the coordinate axes.

(3)

- (b) Write down the exact coordinates of the maximum point and the minimum point of the graph in this interval.

(2)

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4.

$$f(x) = 3 \sin 2x \quad 0 \leq x \leq 2\pi$$

- (a) State the period of $f(x)$. (1)
- (b) Find the exact value of $f\left(\frac{\pi}{6}\right)$ (1)
- (c) Write down the exact coordinates of the maximum points of the graph of $y = f(x)$. (2)
- (d) State the number of solutions of the equation $3 \sin 2x = 2$ in the interval $0 \leq x \leq 2\pi$ (1)

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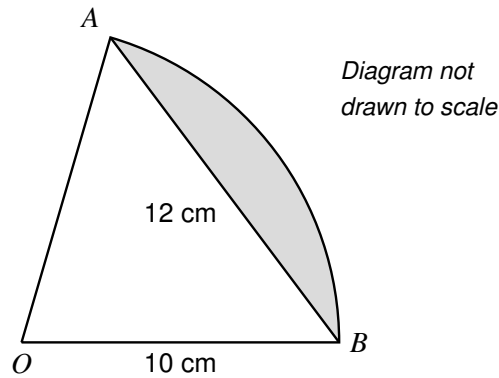


Figure 1

Figure 1 shows the sector OAB of a circle with centre O and radius 10 cm.

The chord AB has length 12 cm. The segment bounded by the arc AB and the chord AB is shown shaded.

- (a) Show that angle $AOB = 1.287$ radians, correct to 4 significant figures. (2)
- (b) Find the area of the shaded segment, giving your answer in cm^2 to 3 significant figures. (2)
- (c) Find the perimeter of the shaded segment, giving your answer in cm to 3 significant figures. (2)



Question 5 continued

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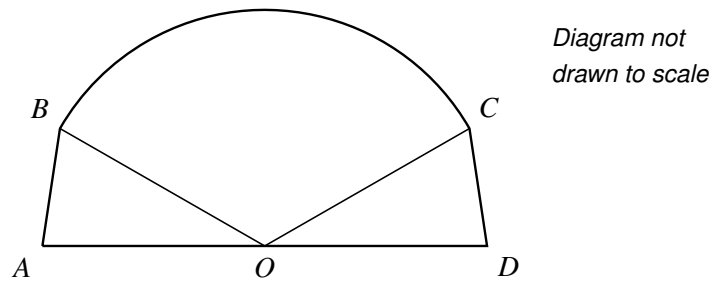
**Figure 2**

Figure 2 shows the plan of a stage. It consists of the sector OBC of a circle with centre O , joined to two congruent triangles OAB and ODC .

Given that

- angle $BOC = 2.1$ radians
- the area of the sector OBC is 58 m^2
- AOD is a straight line of length 14 m

(a) show that the radius of the sector is 7.43 m , correct to 3 significant figures, (2)

(b) find the size of angle AOB , in radians, to 3 significant figures. (1)

Hence find

(c) the total area of the stage, giving your answer in m^2 to 3 significant figures, (3)

(d) the total perimeter of the stage, giving your answer in m to 3 significant figures. (3)



Question 6 continued

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Question 6 continued

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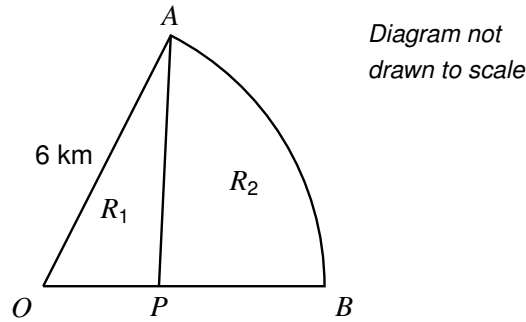
**Figure 3**

Figure 3 shows a plot of land in the shape of a sector OAB of a circle with centre O and radius 6 km. The angle AOB is 1.1 radians.

The point P lies on OB . A straight fence AP divides the plot into two regions: R_1 , the triangle OAP , and R_2 , the rest of the sector.

Given that the area of R_2 is twice the area of R_1 ,

- (a) find the perimeter of the sector OAB , (2)
- (b) show that the area of R_1 is 6.6 km^2 (3)
- (c) find the length OP , giving your answer in km to 3 significant figures, (2)
- (d) find the length of the fence AP , giving your answer to the nearest 100 m. (2)



Question 7 continued

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Question 7 continued

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Diagram not drawn to scale

(d) Find the perimeter of the shape $OABC$, giving your answer in cm to 3 significant figures. **(2)**

Question 8 continued

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Question 8 continued

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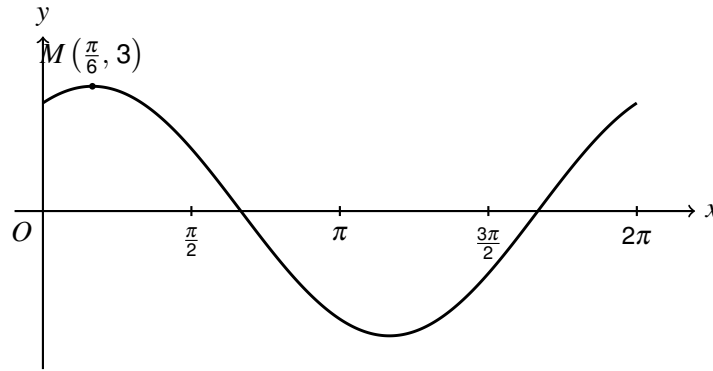


Figure 5

Figure 5 shows part of the curve with equation $y = a \sin(x + b)$, $0 \leq x \leq 2\pi$, where a and b are constants, $a > 0$ and $0 < b < \pi$

The curve has a maximum point at $M\left(\frac{\pi}{6}, 3\right)$.

(a) Write down the value of a and the value of b . (2)

(b) Find the exact coordinates of the point where the curve crosses the y -axis. (1)

(c) Find the exact x coordinates of the points where the curve crosses the x -axis. (2)

The same curve can also be written as $y = a \cos(x + c)$, where $-\pi < c < 0$

(d) Find the value of c . (2)

(e) Write down the exact coordinates of the minimum point of the curve for $0 \leq x \leq 2\pi$ (1)



Question 9 continued

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Solutions relying on calculator technology are not acceptable.

The perimeter of the sector is 20 cm and the area of the sector is 16 cm^2

- (b)** Find the value of r and the value of θ , explaining clearly why one solution of the equation in part (a) must be rejected. **(4)**

(c) Show that its area, $A \text{ cm}^2$, is given by $A = 10r - r^2$, and hence find the maximum possible area of the sector and the value of θ for which it occurs. (3)



Question 10 continued

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Question 10 continued

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TOTAL FOR CHAPTER 7 IS 71 MARKS

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8

CHAPTER 8 • KEY SKILLS

Differentiation

Specification P1: 4.1, 4.2, 4.3 • 10 questions • 71 marks • about 85 minutes

1 Differentiating x^n

$$\frac{d}{dx}(ax^n) = anx^{n-1} \text{ for any rational } n.$$

e.g. $\frac{d}{dx}(4x^{-2}) = -8x^{-3}$

2 Rewrite first

Expand brackets and split fractions into powers of x before differentiating.

e.g. $\frac{x^2+1}{\sqrt{x}} = x^{\frac{3}{2}} + x^{-\frac{1}{2}}$

3 Gradients and rates of change

Substitute into $\frac{dy}{dx}$, showing the

powers exactly. $\frac{dV}{dt} < 0$ means V is decreasing.

4 Second derivative

Differentiate $\frac{dy}{dx}$ again to get $\frac{d^2y}{dx^2}$ or $f''(x)$.

5 Gradient as a limit

Gradient of the chord from x to $x+h$, then let $h \rightarrow 0$.

e.g. chord gradient $5+h \rightarrow 5$

6 Tangents

Gradient $m = f'(a)$ at $(a, f(a))$, then

$$y - y_1 = m(x - x_1).$$

7 Normals

The normal is perpendicular to the

tangent: gradient $-\frac{1}{m}$

8 A given gradient

Solve $f'(x) = m$. It may be a hidden quadratic, e.g. in x^2 .

9 Equations with unknowns

Use given gradients or $f' = f''$ at a point to form an equation for the unknown constant.

Skills map every skill above is tested at least once; interleaved skills come from GCSE or other chapters

Skill	Q1	Q2	Q3	Q4	Q5	Q6	Q7	Q8	Q9	Q10
Marks	5	5	5	6	6	8	9	9	8	10
1 Differentiating x^n	•	•		•	•	•	•	•	•	•
2 Rewrite first		•				•				•
3 Gradients and rates of change				•	•	•	•	•		•
4 Second derivative	•							•		
5 Gradient as a limit			•							
6 Tangents					•	•	•		•	•
7 Normals					•			•		•
8 A given gradient							•		•	
9 Equations with unknowns								•	•	
Interleaved: indices and surds (Chapter 1)		•		•					•	•
Interleaved: quadratics, hidden quadratics (Chapter 2)							•	•	•	
Interleaved: cubics, lines and areas (Chapters 4, 5)						•	•			•

Exam difficulty. Questions 1 to 5 check each skill on its own. Questions 6 to 10 are multi-step questions built like the last questions on a real WMA11 paper, where one part leads into the next and skills from different topics are combined.

Must learn. Differentiation of x^n is *not* in the formula booklet. A tangent uses $f'(a)$, not $f(a)$. Never add “+c” after differentiating.



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1. Given that

$$y = 4x^3 - \frac{6}{x^2} + 5\sqrt{x} \quad x > 0$$

(a) find $\frac{dy}{dx}$, giving each term in simplest form,

(3)

(b) find $\frac{d^2y}{dx^2}$

(2)

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Question 1 continued

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(Total for Question 1 is 5 marks)**Join students already acing their exams.**Book your FREE lesson at www.eclassroom.org.uk

2. In this question you must show all stages of your working.

$$y = \frac{3x^2 - 4}{2\sqrt{x}} \quad x > 0$$

(a) Write y in the form $Ax^p + Bx^q$, where A , B , p and q are constants to be found.

(3)

(b) Hence find $\frac{dy}{dx}$, giving each coefficient in simplest form.

(2)

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(Total for Question 2 is 5 marks)



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3. The curve C has equation $y = x^2 - 3x$

The point $P(4, 4)$ lies on C . The point Q also lies on C and has x coordinate $4 + h$, where $h \neq 0$

- (a) Show that the gradient of the chord PQ is $5 + h$

(3)

- (b) Explain how this result can be used to find the gradient of the tangent to C at P , and state its value.

(2)

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Question 3 continued

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Question 4 continued

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5. The curve C has equation $y = x^3 - 4x^2 + 7$

(a) Verify that the point $P(2, -1)$ lies on C .

(1)

(b) Find an equation of the tangent to C at P , giving your answer in the form $y = mx + c$

(3)

(c) Find an equation of the normal to C at P , giving your answer in the form $ax + by + c = 0$, where a , b and c are integers.

(2)

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Question 5 continued

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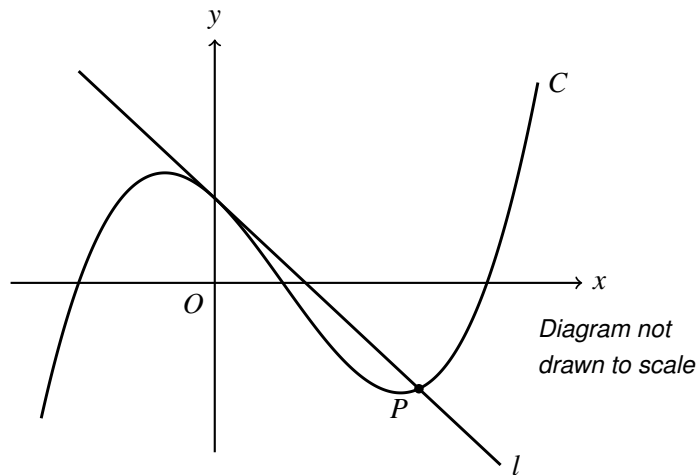


Figure 1

Figure 1 shows a sketch of the curve C with equation $y = f(x)$, where

$$f(x) = (x+2)(x-1)(x-4)$$

(a) Expand $f(x)$, writing your answer as a polynomial in simplest form.

(2)

The straight line l is the tangent to C at the point where C crosses the y -axis.

(b) Find an equation of l .

(3)

The line l meets C again at the point P , as shown in Figure 1.

(c) Using algebra, find the coordinates of P .

(Solutions relying on calculator technology are not acceptable.)

(3)



Question 6 continued

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- (a) Find $\frac{dy}{dx}$

(2)

- (2)

(c) Using algebra, find the coordinates of P .

(3)

- (d) Show that the midpoint of RP also lies on C .

(2)

Question 7 continued

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Question 7 continued

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8.

$$f(x) = 2x^3 + kx^2 - 12x + 1$$

where k is a constant. Given that $f'(2) = f''(2)$

(a) show that $k = 6$

(3)

(b) Find the x coordinate of the other point on the curve $y = f(x)$ at which $f'(x) = f''(x)$, and the y coordinate of this point.

(3)

(c) Find an equation of the normal to the curve $y = f(x)$ at the point where $x = 1$, giving your answer in the form $ax + by + c = 0$, where a , b and c are integers.

(3)

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Question 8 continued

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Question 8 continued

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Solutions relying on calculator technology are not acceptable.

(a) Find $\frac{dy}{dx}$

(2)

(b) Show that the x coordinate of the point where the line touches C satisfies $3x^4 - 5x^2 - 2 = 0$

(2)

(c) Hence find the exact value of c .

(4)

Question 9 continued

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10. In this question you must show all stages of your working.

Solutions relying on calculator technology are not acceptable.

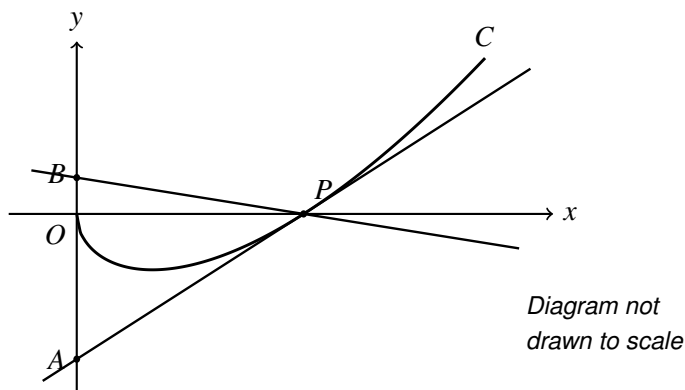


Figure 2

Figure 2 shows a sketch of part of the curve C with equation

$$y = (x - 4)\sqrt{x} \quad x \geq 0$$

The point P lies on C and has x coordinate 4

- (a) Find $\frac{dy}{dx}$ (3)

- (b) Show that the tangent to C at P has equation $y = 2x - 8$ (2)

The tangent to C at P meets the y -axis at A . The normal to C at P meets the y -axis at B .

- (c) Find an equation of the normal to C at P . (2)

- (d) Find the area of the triangle APB . (3)



Question 10 continued

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9

CHAPTER 9 • KEY SKILLS

Integration

Specification P1: 5.1, 5.2 • 10 questions • 67 marks • about 80 minutes

1 Integrating x^n

$$\int ax^n dx = \frac{ax^{n+1}}{n+1} + c, \quad n \neq -1$$

Raise the power by one, then divide by the new power.

2 Rewrite first

Split fractions and expand brackets. Never integrate the top and bottom separately.

e.g. $\frac{x^3 + 2}{x^2} = x + 2x^{-2}$

3 Negative and fractional powers

Divide by a fraction by multiplying by its reciprocal.

e.g. $\int x^{-\frac{1}{2}} dx = 2x^{\frac{1}{2}} + c$

4 The constant c

Every indefinite integral needs $+c$. Simplify each coefficient.

5 $f(x)$ from $f'(x)$

Integrate, then substitute the given point to find c . Write $f(x)$ out in full.

6 Integrating twice

From $f''(x)$: integrate, find the first constant using a gradient, then integrate again and use a point.

7 Using tangents and normals

A tangent gives $f(a)$ and $f'(a)$. A normal with gradient m gives $f'(a) = -\frac{1}{m}$.

8 Unknown constants

Each condition gives one equation. Two unknowns need two equations: solve them simultaneously.

9 Check by differentiating

Differentiate your answer: you should get back to the expression you integrated.

Skills map every skill above is tested at least once; interleaved skills come from GCSE or other chapters

Skill	Q1	Q2	Q3	Q4	Q5	Q6	Q7	Q8	Q9	Q10
Marks	4	5	5	5	5	9	8	8	8	10
1 Integrating x^n	•	•	•	•	•	•	•	•	•	•
2 Rewrite first		•	•							•
3 Negative and fractional powers	•	•	•	•	•		•	•	•	•
4 The constant c	•	•	•		•					
5 $f(x)$ from $f'(x)$				•		•	•	•		•
6 Integrating twice									•	
7 Using tangents and normals							•	•	•	•
8 Unknown constants							•	•		
9 Check by differentiating					•	•				•
Interleaved: tangents and normals (Chapter 8)					•		•	•	•	•
Interleaved: simultaneous equations, discriminant (Chapters 2, 3)								•		•
Interleaved: factorising and sketching cubics (Chapters 1, 4)						•				

Exam difficulty. Questions 1 to 5 check each skill on its own. Questions 6 to 10 are multi-step questions built like the last questions on a real WMA11 paper, where one part leads into the next and skills from different topics are combined.

Must learn. Integration of x^n is *not* in the formula booklet. A gradient condition finds the constant in $f'(x)$; a point condition finds the constant in $f(x)$. Don't mix them up.



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1. Find

$$\int \left(6x^5 + \frac{4}{x^3} - \frac{3}{\sqrt{x}} \right) dx \quad x > 0$$

giving each term in simplest form.

(4)

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2. In this question you must show all stages of your working.

Find

$$\int \frac{3x^4 - 2}{x^2\sqrt{x}} dx \quad x > 0$$

giving your answer in simplest form.

(5)

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Question 2 continued

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3. Given that $x > 0$,

- (a) show that $\frac{(2\sqrt{x}-3)^2}{\sqrt{x}}$ can be written in the form $4x^{\frac{1}{2}} + A + Bx^{-\frac{1}{2}}$, where A and B are constants to be found.

(2)

- (b) Hence find $\int \frac{(2\sqrt{x}-3)^2}{\sqrt{x}} dx$, giving each term in simplest form.

(3)

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Question 3 continued

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(Total for Question 3 is 5 marks)



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4. The curve C has equation $y = f(x)$, $x > 0$, where

$$f'(x) = 3\sqrt{x} - \frac{4}{x^2}$$

Given that the point $(4, 10)$ lies on C , find $f(x)$.

(5)

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Question 4 continued

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(Total for Question 4 is 5 marks)



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5.

$$y = 2x^3 - \frac{6}{\sqrt{x}} \quad x > 0$$

(a) Find $\frac{dy}{dx}$

(2)

(b) Find $\int y dx$

(3)

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Question 5 continued

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6. The curve C has equation $y = f(x)$, where

$$f'(x) = 3x^2 - 8x + 5$$

The point $(3, 4)$ lies on C .

- (a) Find $f(x)$.

(4)

- (b) Show that $f(x) = (x - 1)^2(x - 2)$

(2)

- (c) Sketch C , showing clearly the coordinates of the points where C meets the coordinate axes.

(3)

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Question 6 continued

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Question 6 continued

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(Total for Question 6 is 9 marks)**Join students already acing their exams.**Book your FREE lesson at www.eclassroom.org.uk

7. The curve C has equation $y = f(x)$, $x > 0$, where

$$f'(x) = kx - 3\sqrt{x}$$

and k is a constant. The point $P(4, 20)$ lies on C .

The normal to C at P has equation $x + 10y = 204$

- (a) Show that $k = 4$

(3)

- (b) Find $f(x)$, giving each term in simplest form.

(5)

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Question 7 continued

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8. The curve C has equation $y = f(x)$, $x > 0$, where

$$f'(x) = ax + b\sqrt{x}$$

and a and b are constants. Given that

- the tangent to C at the point where $x = 1$ has equation $y = 5x - 2$
- the normal to C at the point where $x = 4$ has gradient $-\frac{1}{16}$

- (a) show that $a + b = 5$ and $2a + b = 8$

(3)

- (b) Hence find $f(x)$.

(5)

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Question 8 continued

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(Total for Question 8 is 8 marks)



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9. A curve C has equation $y = f(x)$, $x > 0$. Given that

- $f''(x) = 12x - \frac{4}{x^3}$

- the point P has x coordinate 1 and lies on C

- the tangent to C at P has equation $y = 2x + 3$

(a) find an equation of the normal to C at P , giving your answer in the form $ax + by + c = 0$, where a , b and c are integers,

(2)

(b) find $f(x)$, writing your answer in simplest form.

(6)

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Question 9 continued

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(Total for Question 9 is 8 marks)



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Solutions relying on calculator technology are not acceptable.

$$f'(x) = \frac{x^2 - 9}{x^2}$$

(a) find $f(x)$.

(4)

(b) Find an equation of the tangent to C at the point where $x = 1$

(3)

(c) Using algebra, show that this tangent does not meet C at any other point.

(3)

Question 10 continued

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Question 10 continued

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(Total for Question 10 is 10 marks)

TOTAL FOR CHAPTER 9 IS 67 MARKS

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