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INTERNATIONAL A LEVEL MATHEMATICS • PURE 1 (WMA11)

Pure 1 Practice Paper 1

Worked Solutions

AIMING FOR B

Written in the style of the Pearson Edexcel International Advanced Level paper (WMA11/01) • 75 marks

Every question solved in full on the paper itself, the way your teacher would write it out

How to read these solutions. Each page is the real question page, with a full answer written on the answer lines.

Red pen

M1 A1

boxed

the working a full-marks answer shows, with a short note on why each step is taken

the mark earned on that line: M = method, A = accuracy, B = independent mark

the final answer to each part

Sit the paper first. These solutions are most useful after you have tried every question yourself under timed conditions. Mark your paper with the mark scheme, then use this booklet to see exactly how each lost mark could have been earned.

Q	Topic	Marks	Page
1	Indices and surds	7	2
2	Completing the square, roots	7	3
3	Line and curve, regions	7	4
4	Perpendicular bisector, area	7	6
5	Sine and cosine rules	8	8
6	Tangents and normals	8	10
7	Integration, normals	8	11
8	Radians: arcs and segments	7	12
9	Graph transformations	7	13
10	Cubics and the discriminant	9	15

Question paper & mark scheme | Exam questions by topic | More practice papers

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1. In this question you must show all stages of your working.

Solutions relying on calculator technology are not acceptable.

(a) Given that

$$4^x \times 2^y = 16\sqrt{2} \qquad \text{and} \qquad \frac{9^x}{27^y} = \sqrt{3}$$

find the value of x and the value of y.

(4)

(b) Write

$$\frac{(3-\sqrt{5})^2}{\sqrt{5}-1}$$

in the form $a+b\sqrt{5}$, where a and b are integers.

(3)

(a) Write everything in the first equation as a power of 2:

$4^x \times 2^y = (2^2)^x \times 2^y = 2^{2x+y}$ and $16\sqrt{2} = 2^4 \times 2^{1/2} = 2^{9/2}$ M1

so $2x + y = \frac{9}{2}$ ...(1)

Write everything in the second equation as a power of 3:

$9^x \div 27^y = 3^{2x} \div 3^{3y} = 3^{2x-3y}$ and $\sqrt{3} = 3^{1/2}$, so $2x - 3y = \frac{1}{2}$ ...(2) M1

Subtract (2) from (1): $4y = \frac{9}{2} - \frac{1}{2} = 4$, so $y = 1$ dM1

Substitute into (1): $2x + 1 = \frac{9}{2} \Rightarrow 2x = \frac{7}{2}$

$\boxed{x = \frac{7}{4},\ y = 1}$ A1

Check: $4^{7/4} \times 2 = 2^{7/2} \times 2 = 2^{9/2} = 16\sqrt{2}$ ✓

(b) Expand the numerator first: $(3 - \sqrt{5})^2 = 9 - 6\sqrt{5} + 5 = 14 - 6\sqrt{5}$ M1

Rationalise: multiply the top and bottom by $(\sqrt{5} + 1)$, so the bottom becomes $5 - 1 = 4$

Top: $(14 - 6\sqrt{5})(\sqrt{5} + 1) = 14\sqrt{5} + 14 - 30 - 6\sqrt{5} = 8\sqrt{5} - 16$ M1

$\frac{8\sqrt{5} - 16}{4} = \boxed{-4 + 2\sqrt{5}}$ so $a = -4,\ b = 2$ A1

(Total for Question 1 is 7 marks)

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2. In this question you must show all stages of your working.

Solutions relying on calculator technology are not acceptable.

$$f(x) = 7 + 12x - 3x^2 \qquad x \in \mathbb{R}$$

(a) Express $f(x)$ in the form $a + b(x + c)^2$, where a, b and c are integers to be found.

(3)

The curve C has equation $y = f(x)$.

(b) Sketch C, showing clearly the coordinates of the maximum point and the coordinates of the point where C crosses the y-axis.

(2)

(c) Using your answers to parts (a) and (b), find the set of values of the constant k for which the equation $f(x) = k$ has two distinct **positive** roots.

(2)

(a) Take out −3 from the x terms: $f(x) = -3(x^2 - 4x) + 7$

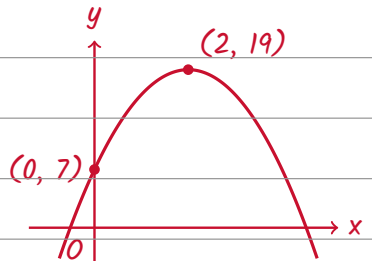
Complete the square inside the bracket: $x^2 - 4x = (x - 2)^2 - 4$ M1

$f(x) = -3[(x - 2)^2 - 4] + 7 = -3(x - 2)^2 + 12 + 7$ $(b = -3)$ B1

$\boxed{f(x) = 19 - 3(x - 2)^2}$ so $a = 19$, $b = -3$, $c = -2$ A1

(b) From (a), the maximum point is (2, 19).

When x = 0, y = 7, so C crosses the y-axis at (0, 7). B1 B1



(c) The roots of f(x) = k are where the line y = k meets C.

Two roots need the line below the maximum point, so k < 19 M1

Both roots positive: both points are right of the y-axis, so the line is above (0, 7).

(At k = 7 the roots are 0 and 4, and 0 is not positive, so k = 7 is not allowed.)

$\boxed{7 < k < 19}$ A1

(Total for Question 2 is 7 marks)

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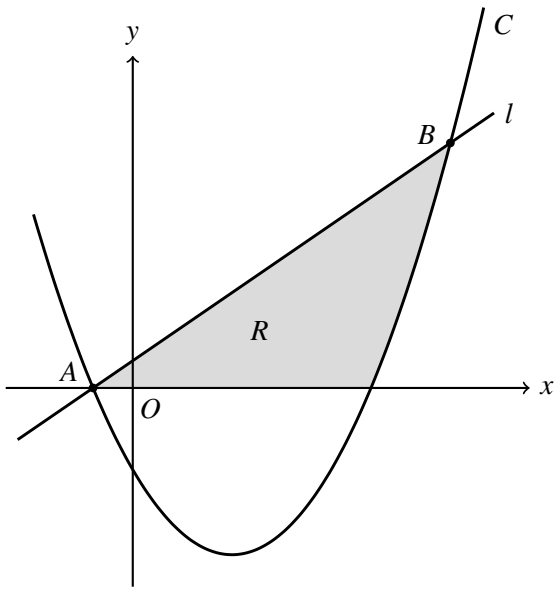

Figure 1

Figure 1 shows a sketch of the curve C with equation $y = 2x^2 - 5x - 3$ and the straight line l with equation $y = 2x + 1$

The line l meets C at the points A and B, as shown.

In this question you must show all stages of your working.

Solutions relying on calculator technology are not acceptable.

(a) Show that the x coordinates of A and B satisfy $2x^2 - 7x - 4 = 0$

(2)

(b) Hence find the coordinates of A and the coordinates of B.

(2)

The region R, shown shaded in Figure 1, is bounded by C, l and the x-axis.

(c) Use inequalities to define the region R.

(3)

(a) At A and B the curve and the line have the same y value: $2x^2 - 5x - 3 = 2x + 1$ M1

$2x^2 - 7x - 4 = 0$ as required A1*

(b) Factorise: $(2x + 1)(x - 4) = 0$, so $x = -\frac{1}{2}$ or $x = 4$ M1

$x = -\frac{1}{2}$: $y = 2(-\frac{1}{2}) + 1 = 0$ $x = 4$: $y = 2(4) + 1 = 9$

$A(-\frac{1}{2}, 0)$ and $B(4, 9)$ A1

(c) R is on or below the line l: $y \leqslant 2x + 1$ B1

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Question 3 continued

R is on or above the curve C: $y \geqslant 2x^2 - 5x - 3$ B1

R is on or above the x-axis: $y \geqslant 0$ B1

(Without $y \geqslant 0$, the region would also include the part below the x-axis between A and (3, 0).)

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(Total for Question 3 is 7 marks)

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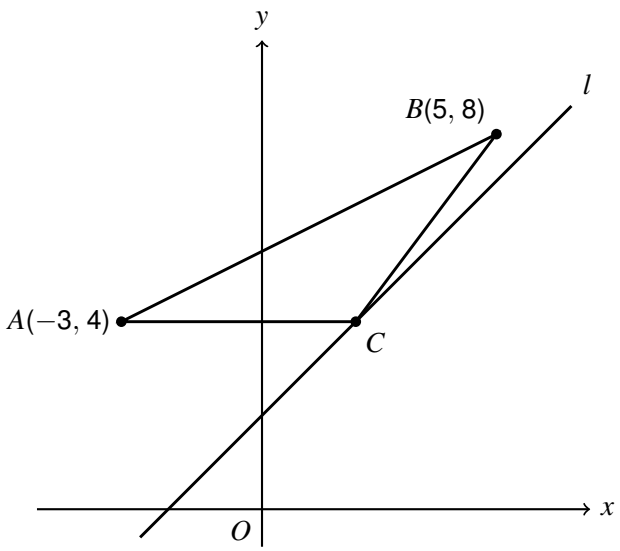

Diagram not drawn to scale

Figure 2

The points A and B have coordinates $(-3, 4)$ and $(5, 8)$ respectively.

In this question you must show all stages of your working.

Solutions relying on calculator technology are not acceptable.

(a) Show that the perpendicular bisector of AB has equation $2x + y - 8 = 0$

(3)

The line l has equation $y = x + 2$

The perpendicular bisector of AB meets l at the point C, as shown in Figure 2.

(b) Find the exact area of triangle ABC.

(4)

(a) Gradient of AB $= \frac{8-4}{5-(-3)} = \frac{4}{8} = \frac{1}{2}$, so the perpendicular gradient is -2 M1

Midpoint of AB: $\left(\frac{-3+5}{2}, \frac{4+8}{2}\right) = (1, 6)$

Line through $(1, 6)$ with gradient -2: $y - 6 = -2(x - 1) \Rightarrow y - 6 = -2x + 2$ M1

$2x + y - 8 = 0$ as required A1*

(b) C is on both lines, so substitute $y = x + 2$: $2x + (x + 2) - 8 = 0$ M1

$3x = 6$, so $x = 2$ and $y = 4$: $C(2, 4)$ A1

A$(-3, 4)$ and C$(2, 4)$ have the same y coordinate, so AC is horizontal, with length 5.

The height is the vertical distance from B to AC: $8 - 4 = 4$ M1

Area of triangle ABC $= \frac{1}{2} \times 5 \times 4 =$ 10 A1

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Question 4 continued

Check: C is on the perpendicular bisector, so area $= \frac{1}{2} \times AB \times MC = \frac{1}{2} \times 4\sqrt{5} \times \sqrt{5} = 10$ ✓

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(Total for Question 4 is 7 marks)

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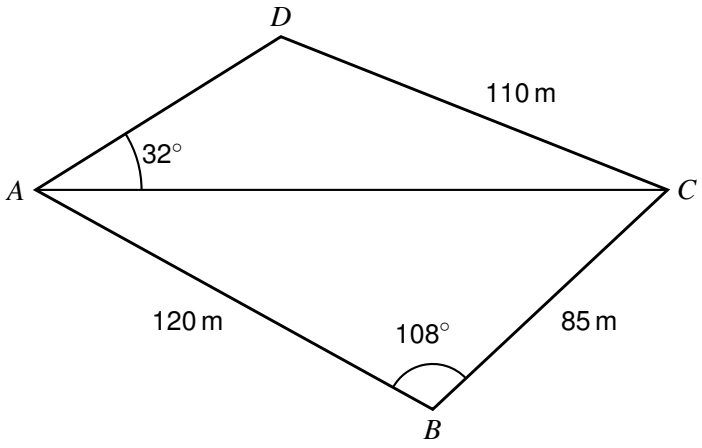

Diagram not drawn to scale

Figure 3

Figure 3 shows the plan of a field $ABCD$.

$AB = 120$ m, $BC = 85$ m, $CD = 110$ m, angle $ABC = 108°$ and angle $CAD = 32°$

Angle ADC is obtuse.

(a) Find the length of AC, in metres to one decimal place.

(2)

(b) Find the size of angle ADC, in degrees to one decimal place.

(3)

(c) Find the area of the field, in m^2 to 3 significant figures.

(3)

(a) Cosine rule in triangle ABC: $AC^2 = 120^2 + 85^2 - 2 \times 120 \times 85 \times \cos 108°$ M1

$AC^2 = 14\,400 + 7225 + 6303.9... = 27\,928.9...$

$AC = 167.1$ m (1 d.p.) A1

(b) Sine rule in triangle ACD: $\frac{\sin ADC}{167.1} = \frac{\sin 32°}{110}$ M1

$\sin ADC = \frac{167.12... \times \sin 32°}{110} = 0.8051...$, giving $53.6°$ A1

But angle ADC is obtuse, so use the other angle with the same sine:

$ADC = 180° - 53.6° = 126.4°$ A1

(c) Area of ABC $= \frac{1}{2} \times 120 \times 85 \times \sin 108° = 4850.4\ m^2$ M1

Angle $ACD = 180° - 32° - 126.4° = 21.6°$

Area of ACD $= \frac{1}{2} \times 167.1 \times 110 \times \sin 21.6° = 3386.5\ m^2$ M1

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Question 5 continued

Total = 4850.4 + 3386.5 = 8236.9, so the area of the field is $\boxed{8240\ m^2}$ (3 s.f.) A1

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(Total for Question 5 is 8 marks)

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6. In this question you must show all stages of your working.

Solutions relying on calculator technology are not acceptable.

The curve C has equation

$$y = x\sqrt{x} - \frac{16}{x} - 4\sqrt{x} + 10 \qquad x > 0$$

The point $P(4, 6)$ lies on C.

(a) Find $\frac{dy}{dx}$, giving each term in its simplest form.

(3)

(b) Show that the tangent to C at P has equation $y = 3x - 6$

(2)

The tangent to C at P meets the x-axis at the point R.

The normal to C at P meets the x-axis at the point Q.

(c) Find the area of triangle PQR.

(3)

(a) Write each term as a power of x: $y = x^{3/2} - 16x^{-1} - 4x^{1/2} + 10$ M1

Multiply by the power, then take 1 off the power (the constant 10 disappears):

$dy/dx = \frac{3}{2}x^{1/2} + 16x^{-2} - 2x^{-1/2}$ A1 A1

(b) At P, x = 4: $\frac{3}{2} \times \sqrt{4} + \frac{16}{16} - \frac{2}{\sqrt{4}} = 3 + 1 - 1 = 3$ M1

Tangent through P(4, 6) with gradient 3: $y - 6 = 3(x - 4) \Rightarrow y = 3x - 6$ as required A1*

(c) The tangent meets the x-axis when $3x - 6 = 0$: R(2, 0)

The normal at P has gradient $-\frac{1}{3}$: $y - 6 = -\frac{1}{3}(x - 4)$ M1

It meets the x-axis when y = 0: $-6 = -\frac{1}{3}(x - 4) \Rightarrow x - 4 = 18$, so Q(22, 0) A1

Base QR = 22 − 2 = 20, and the height is the y coordinate of P, which is 6.

Area of triangle PQR = $\frac{1}{2} \times 20 \times 6 = 60$ A1

(Total for Question 6 is 8 marks)

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7. In this question you must show all stages of your working.

Solutions relying on calculator technology are not acceptable.

The curve C has equation $y = f(x)$, $x > 0$, where

$$f'(x) = \frac{5x^2 - 9x - 4}{2\sqrt{x}}$$

The point $(1, -3)$ lies on C.

(a) Find $f(x)$, giving each term in its simplest form.

(5)

(b) Find an equation of the normal to C at the point where $x = 4$

Give your answer in the form $ax + by + c = 0$, where a, b and c are integers.

(3)

(a) Split the fraction into separate powers of x (remember $\sqrt{x} = x^{1/2}$):

$f'(x) = \frac{5x^2}{2x^{1/2}} - \frac{9x}{2x^{1/2}} - \frac{4}{2x^{1/2}} = \frac{5}{2}x^{3/2} - \frac{9}{2}x^{1/2} - 2x^{-1/2}$ M1

Integrate each term: add 1 to the power, then divide by the new power.

$f(x) = \frac{5}{2}x^{5/2} \div \frac{5}{2} - \frac{9}{2}x^{3/2} \div \frac{3}{2} - 2x^{1/2} \div \frac{1}{2} + c$ M1

$f(x) = x^{5/2} - 3x^{3/2} - 4x^{1/2} + c$ A1

$(1, -3)$ lies on C: $1 - 3 - 4 + c = -3$, so $c = 3$ M1

$\boxed{f(x) = x^{5/2} - 3x^{3/2} - 4x^{1/2} + 3}$ A1

(b) At $x = 4$: $f(4) = 32 - 24 - 8 + 3 = 3$ and $f'(4) = \frac{5(16) - 9(4) - 4}{2 \times 2} = \frac{40}{4} = 10$ M1

The normal is perpendicular to the tangent, so its gradient is $-\frac{1}{10}$: $y - 3 = -\frac{1}{10}(x - 4)$ M1

Multiply by 10: $10y - 30 = -x + 4$

$\boxed{x + 10y - 34 = 0}$ A1

(Total for Question 7 is 8 marks)

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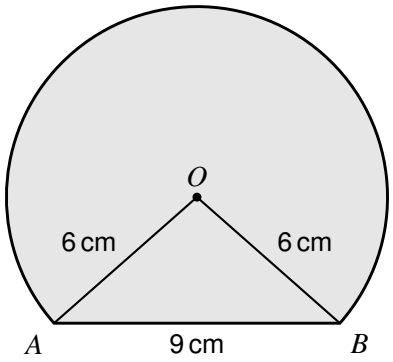

Figure 4

Figure 4 shows the design for a badge.

The badge is the shaded shape bounded by the major arc AB of a circle with centre O and radius 6 cm, and by the chord AB.

$AB = 9$ cm.

(a) Show that the size of angle AOB is 1.696 radians, correct to 3 decimal places.

(2)

(b) Find the perimeter of the badge, in cm to 3 significant figures.

(2)

(c) Find the area of the badge, in cm^2 to 3 significant figures.

(3)

(a) Cosine rule in triangle AOB: $\cos AOB = \frac{6^2 + 6^2 - 9^2}{2 \times 6 \times 6} = \frac{-9}{72} = -\frac{1}{8}$ M1

$AOB = \cos^{-1}(-\frac{1}{8}) = 1.69612... = 1.696$ radians (3 d.p.) as required A1*

(b) The curved edge is the major arc, so its angle is $2\pi - 1.6961... = 4.5871...$

Major arc $= r\theta = 6 \times 4.5871... = 27.52...$ cm M1

Perimeter $= 27.52... + 9 =$ **36.5 cm** (3 s.f.) A1

(c) The badge is the whole circle with the minor segment cut off.

Minor segment $= \frac{1}{2}r^2(\theta - \sin\theta) = \frac{1}{2} \times 36 \times (1.6961... - \sin 1.6961...) = 12.67...$ M1

Area of badge $= \pi \times 6^2 - 12.67... = 113.10... - 12.67... = 100.4...$ M1

Area = 100 cm^2 (3 s.f.) (calculator in radians for sin 1.6961) A1

(Total for Question 8 is 7 marks)

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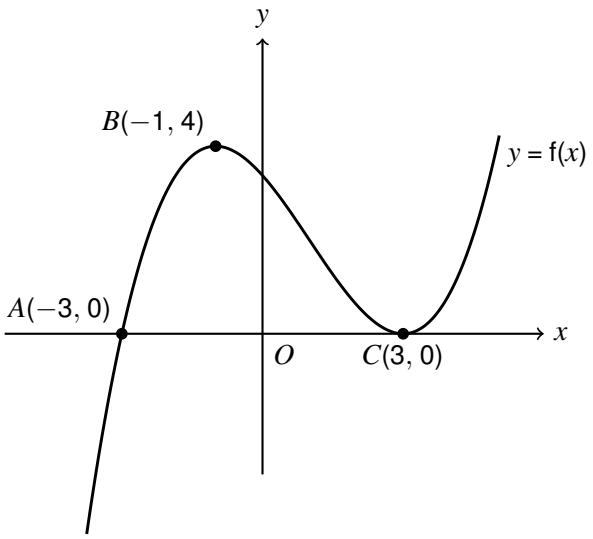

Figure 5

Figure 5 shows a sketch of the curve with equation $y = f(x)$.

The curve crosses the x-axis at the point $A(-3, 0)$, has a maximum point at $B(-1, 4)$ and touches the x-axis at the point $C(3, 0)$.

On separate diagrams, sketch the curve with equation

(a) $y = f(x+2)$

(3)

(b) $y = f\left(\frac{1}{2}x\right)$

(2)

On each sketch, show clearly the coordinates of the maximum point and of any points where the curve meets the x-axis.

(c) The curve with equation $y = f(x) + k$, where k is a constant, meets the x-axis at exactly one point.

Find the set of possible values of k.

(2)

(a) $y = f(x + 2)$ is a translation 2 units to the LEFT: subtract 2 from every x coordinate.

$A(-3, 0) \rightarrow (-5, 0)$, $B(-1, 4) \rightarrow (-3, 4)$, $C(3, 0) \rightarrow (1, 0)$

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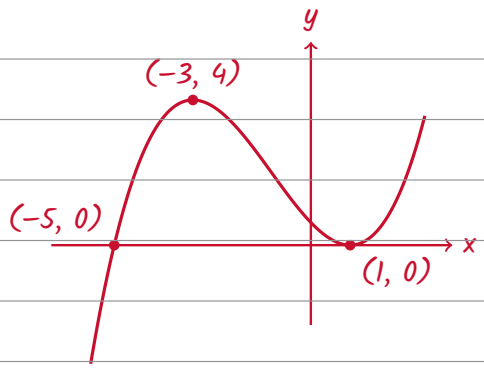
Question 9 continued



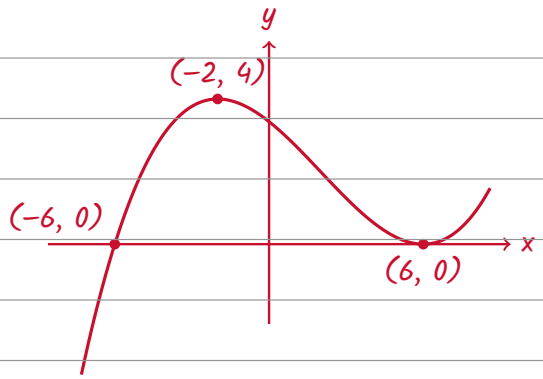
B1 B1 B1

same shape, moved 2 left; touches at (1, 0)

(b) $y = f(\frac{1}{2}x)$ is a stretch parallel to the x-axis, scale factor 2.

DOUBLE every x coordinate. The y coordinates do not change.

A $\rightarrow (-6, 0)$, B $\rightarrow (-2, 4)$, C $\rightarrow (6, 0)$



B1 B1

maximum stays at height 4

(c) $y = f(x) + k$ moves the curve up by k (or down, if k is negative). M1

Up: once $k > 0$, the minimum at C lifts off the axis: only the left crossing is left.

Down: once $k < -4$, the maximum at B drops below the axis: only the right crossing is left.

($k = 0$ and $k = -4$ give two points, and $-4 < k < 0$ gives three.)

$k < -4$ or $k > 0$ A1

(Total for Question 9 is 7 marks)

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10. In this question you must show all stages of your working.

Solutions relying on calculator technology are not acceptable.

$$f(x) = 2x^3 + x^2 - 15x$$

(a) Factorise $f(x)$ completely.

(2)

(b) Sketch the curve with equation $y = f(x)$, showing the coordinates of the points where the curve meets the coordinate axes.

(2)

The line with equation $y = kx$, where k is a constant, meets the curve $y = f(x)$ at the origin.

(c) Show that the x coordinates of any other points where the line meets the curve satisfy

$$2x^2 + x - (15 + k) = 0$$

(2)

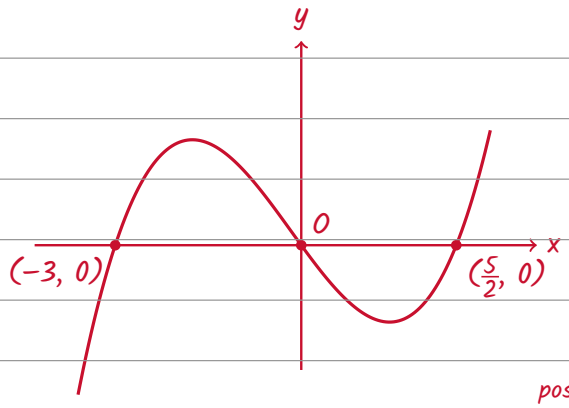
(d) Find the set of values of k for which the line meets the curve at three distinct points.

(3)

(a) Take out the common factor x: $f(x) = x(2x^2 + x - 15)$ M1

Factorise the quadratic ($2 \times -15 = -30$, and $6 - 5 = 1$): $f(x) = x(2x - 5)(x + 3)$ A1

(b) Roots at $x = -3$, 0 and $\frac{5}{2}$. The x^3 term is positive, so the curve rises on the right.



B1 B1

positive cubic through all three roots

(c) Where the line meets the curve: $2x^3 + x^2 - 15x = kx$ M1

$2x^3 + x^2 - (15 + k)x = 0 \Rightarrow x(2x^2 + x - (15 + k)) = 0$

$x = 0$ is the origin, so the other points satisfy $2x^2 + x - (15 + k) = 0$ as required A1*

(d) Three distinct points needs two distinct roots of the quadratic, so $b^2 - 4ac > 0$:

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Question 10 continued

$1^2 - 4(2)(-(15 + k)) > 0 \Rightarrow 1 + 120 + 8k > 0 \Rightarrow 121 + 8k > 0$ M1

$k > -\frac{121}{8}$ A1

But neither root may be $x = 0$, because that is the origin again.

$x = 0$ is a root when $-(15 + k) = 0$, i.e. $k = -15$

(Check: when $k = -15$, $2x^2 + x = 0$ gives $x = 0$ or $x = -\frac{1}{2}$, so only two points in total.)

$k > -\frac{121}{8}$, $k \neq -15$ B1

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(Total for Question 10 is 9 marks)

TOTAL FOR PAPER IS 75 MARKS

END

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