eClassroom

A-LEVEL MATHEMATICS • STATISTICS 2

Sampling and Sampling Distributions

Worked Solutions

Pearson Edexcel International A Level (WST02) • Calculator and statistical tables allowed: show every step

Full worked solutions for the free Sampling and Sampling Distributions worksheet, A-Level and International A-Level Maths

A **Examples** & Your turn page 2

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Tap a section to jump straight to it

Full worked solutions for every question on the Statistics 2 Sampling and Sampling Distributions worksheet, in the same order and with the same question numbers.

How to mark your work. Tick each step you got right, not just the final answer. At A-level most marks are for method, so a wrong final answer can still earn most of the marks. If you lost marks, find the *first* line where your working is different from ours. That is the thing to fix.

In Section E: **M** = method mark, **dM** = method mark that depends on the previous M mark, **A** = accuracy mark (needs the M mark before it), **B** = independent mark.

Question sheet | Exam questions by topic | Statistics 2 exam questions | More worksheets

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A Your turn answers

Compare each line of your working with the solution.

CORE

A1 Population: all 6000 members; unit: one member; frame: the membership database; **a sample is quicker and cheaper**

A2 $\frac{\bar{X}}{\sigma}$ contains the unknown σ, so it is not a statistic: $\frac{1}{3}\sum X_i^2$ **and** $X_1 - X_2$

A3 0.7^2, $2(0.7)(0.3)$, 0.3^2: **total 4, 7, 10 with 0.49, 0.42, 0.09**

STRETCH

A4 $M = 5$: $3(0.3)^2(0.7) + 0.3^3 = \frac{27}{125}$: $M = 5$**, 10 with** $\frac{27}{125}, \frac{98}{125}$

A5 $\mathrm{E}(\bar{X}) = 1.5$, $\mathrm{Var}(\bar{X}) = \frac{0.75}{5}$: **1.5, 0.15**

A6 $R = 0$: $0.25 + 0.09 + 0.04$; $R = 1$: $2(0.15) + 2(0.06)$; $R = 2$: $2(0.1)$: **0.38, 0.42, 0.2**

A7 $\mathrm{E} = 0.15$, $\mathrm{Var} = \frac{0.15(0.85)}{40} \approx 0.00319$; $\mathrm{P}(Y \leqslant 4)$: **0.2633**

B Fluency solutions

One mark for each correct part. The notes in italics explain why each sequence was chosen.

CORE

B1 **(a)** **all 12 000 households** **(b)** **a household** **(c)** **the list of addresses** **(d)** **too costly and slow**

The frame is the list from which the sample is chosen.

B2 **(a)** **yes** **(b)** **no**: contains μ **(c)** **no**: contains σ **(d)** **yes**

A statistic is calculated from the sample alone.

B3 **(a)** $(1,1),(1,2),(1,5),(2,2),(2,5),(5,5)$ **(b)** 0.3^2: **0.09** **(c)** $2(0.5)(0.2)$: **0.2**

(d)

t	2	3	4	6	7	10
$\mathrm{P}(T=t)$	0.25	0.3	0.09	0.2	0.12	0.04

Mixed pairs occur in two orders.

B4 **(a)** $0.5+0.6+1$: **2.1** **(b)** $6.7-2.1^2$: **2.29** **(c)** **2.1** **(d)** $\frac{2.29}{2}$: **1.145**

$\mathrm{E}(\bar{X})=\mu$ *and* $\mathrm{Var}(\bar{X})=\frac{\sigma^2}{n}$

B5 **(a)** **sample**: testing destroys the bulbs **(b)** **census**: the population is small **(c)** **sample**: a census is too large and costly

Size, cost and destructive testing decide the choice.

B6 **(a)** 0.5^2: **0.25** **(b)** $0.8^2-0.25$: **0.39** **(c)** $1-0.8^2$: **0.36** **(d)** $0.25+0.78+1.08$: **2.11**

$\mathrm{P}(M\leqslant m)$ *is easiest: both values must be at most* m

STRETCH

B7 **(a)** $3(0.5)^2(0.5)+0.5^3$: **0.5** **(b)** $3(0.2)^2(0.8)+0.2^3$: **0.104** **(c)** $1-0.5-0.104$: **0.396** **(d)** **1.604**

The median is 1 when at least two of the three values are 1

B8 **(a)** **10** **(b)** $\frac{0.4(0.6)}{25}$: **0.0096** **(c)** $\mathrm{P}(Y\geqslant 13)=1-0.8462$: **0.1538** **(d)** **no**: it contains the unknown p

A count from a sample of fixed size is binomial.

B9 **(a)** $\binom{5}{2}$: **10** **(b)** pairs $(1,3)$: $2\times 2=4$: $\frac{2}{5}$

(c)

s	2	3	4	5	6
$\mathrm{P}(S=s)$	$\frac{1}{10}$	$\frac{1}{5}$	$\frac{2}{5}$	$\frac{1}{5}$	$\frac{1}{10}$

(d) **4**

Without replacement, count the pairs of counters, not the numbers.

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B10 **(a)** **0.8**

(b)

$\bar{x}$	0	0.5	1	1.5	2
$P(\bar{X} = \bar{x})$	0.16	0.32	0.32	0.16	0.04

(c) $0.16 + 0.32 + 0.24 + 0.08 = 0.8 = \mu$, as required **(d)** $0.32 + 0.16 + 0.04$: **0.52**

The sample mean is centred on the population mean.

C Reasoning solutions

Your wording can be different. Check that your reason makes the same point.

CORE

C1 "A student" is a sampling unit; the frame is a list of the units: **a list of all 900 students**

C2 **It contains the unknown parameters μ and σ**, so it cannot be calculated from the sample.

C3 The probabilities add to 0.64; totals 6, 7, 10 are missing: $P(T = 6) = 0.2$, $P(T = 7) = 0.12$, $P(T = 10) = 0.04$

C4 A total of 5 can be $(1, 4)$ or $(4, 1)$: $2(0.6)(0.4)$: **0.48**

STRETCH

C5 The median is also 2 when all three are 2: $0.288 + 0.4^3$: **0.352**

C6 Both coins from bag B are at least 2, so at least two of the three values are 2 or more: **the median is 2 or 5**

C7 $\text{Var}(\bar{X}) = \frac{\sigma^2}{n}$: $\frac{6}{4}$: **1.5**

C8 **(a)** $R = 1$ for $(0, 1), (1, 0), (1, 2), (2, 1)$, so $4 \times \frac{1}{9} = \frac{4}{9}$, as required

(b) $P(R = 0) = \frac{3}{9}$, $P(R = 2) = \frac{2}{9}$; $E(R) = \frac{4}{9} + \frac{4}{9} = \frac{8}{9}$; $E(R^2) = \frac{4}{9} + \frac{8}{9} = \frac{12}{9}$; $\frac{12}{9} - \frac{64}{81} = \frac{44}{81}$, as required

D Problem solving solutions

There is often more than one correct method. Any complete, correct method scores full marks.

CORE

D1 **(a)** **a list of all 1800 members** **(b)** **the members** **(c)** **B**: it is calculated from the sample only

D2 Number of blue counters $\mathrm{B}\left(3, \frac{1}{4}\right)$; $X = 6, 10, 14, 18$ for 0, 1, 2, 3 blue

x	6	10	14	18
$\mathrm{P}(X = x)$	$\frac{27}{64}$	$\frac{27}{64}$	$\frac{9}{64}$	$\frac{1}{64}$

D3 $1 - \left(\frac{63}{64}\right)^n > 0.9$, $n > \frac{\ln 0.1}{\ln(63/64)} = 146.2$: $n = 147$

D4 **(a)** **5.25** **(b)** $30.75 - 5.25^2$: **3.1875** **(c)** $\frac{3.1875}{5}$: **0.6375**

STRETCH

D5 $R = 0$: $0.5^3 + 0.3^3 + 0.2^3 = 0.16$; $R = 1$: only 1s and 2s, at least one of each: $0.8^3 - 0.5^3 - 0.3^3 = 0.36$

$R = 3$: only 2s and 5s, at least one of each: $0.5^3 - 0.3^3 - 0.2^3 = 0.09$; $R = 4$: the rest

r	0	1	3	4
$\mathrm{P}(R = r)$	0.16	0.36	0.09	0.39

D6 **(a)** $q^3 = 0.064$: $q = 0.4$

(b) $\bar{X} = 40$ from $(20, 50, 50)$: $3(0.6)(0.4)^2$: **0.288**; $\bar{X} = 30$: $3(0.6)^2(0.4)$: **0.432**

D7 **(a)** One pair of 1s out of $\binom{6}{2} = 15$ pairs, $\frac{1}{15}$, as required

(b)

m	1	1.5	2	2.5	3
$\mathrm{P}(M = m)$	$\frac{1}{15}$	$\frac{2}{5}$	$\frac{1}{5}$	$\frac{2}{15}$	$\frac{1}{5}$

D8 **(a)** $8(0.3)+12(0.5)+20(0.2)$: **£12.40**

(b) median 8: $3(0.3)^2(0.7)+0.3^3 = 0.216$; median 20: $3(0.2)^2(0.8)+0.2^3 = 0.104$

median	£8	£12	£20
probability	0.216	0.68	0.104

E Exam practice solutions

Mark-scheme notes are shown on the right of each line.

CORE *Exam questions for everyone*

E1 **(a)** $T = 8$ means 2 red and 2 green: $\binom{4}{2}(0.4)^2(0.6)^2$ *M1*

$= 0.3456$, as required *A1**

(b) $T = 2Y + 4$, where $Y \sim B(4, 0.4)$ is the number of red *M1*

values 4, 6, 8, 10, 12 *B1*

t	4	6	8	10	12
$P(T = t)$	0.1296	0.3456	0.3456	0.1536	0.0256

A1 A1

(c) $\sum t\,P(T = t)$ or $2(1.6)+4$ *M1*

7.2 *A1*

(d) $p = 0.1536 + 0.0256 = 0.1792$ *B1*

$\binom{5}{2}(0.1792)^2(0.8208)^3$ *M1*

0.178 *A1*

E2 **(a)** **all 1500 students at the college** *B1*

(b) **the college office's list of students** *B1*

(c) Advantage: quicker and cheaper; disadvantage: it may not represent the population *B1 B1*

(d) **B**, as it uses only the sample *B1*

(e) $B(60, 0.7) \approx N(42, 12.6)$ *B1*

$P(W < 35.5)$ *M1*

$$z = \frac{35.5 - 42}{\sqrt{12.6}} \approx -1.83$$ *M1*

0.0336 *A1*

(f) **mean 0.7** *B1*

$\frac{0.7(0.3)}{60}$: **0.0035** *B1*

STRETCH

E3 **(a)** $\binom{6}{3}$: **20** *B1*

(b) The median is 1 only if both 1s are taken *M1*

with any one of the other 4 counters: 4 samples *A1*

$P(\text{median} = 1) = \frac{4}{20}$ *A1*

otherwise the median is 2 (it cannot be 5) *B1*

median	1	2
probability	$\frac{1}{5}$	$\frac{4}{5}$

A1

(c) Samples containing the 5: $\binom{5}{2} = 10$ *M1 A1*

the largest is otherwise 2 (two 1s alone cannot make three counters) *B1*

largest	2	5
probability	$\frac{1}{2}$	$\frac{1}{2}$

A1

(d) $1\left(\frac{1}{5}\right) + 2\left(\frac{4}{5}\right)$: $\frac{9}{5} = 1.8$ *B1*

less than the median, 2, of all six counters *B1*

(e) Largest 5 and median 2: the 5 with two counters not both 1: $10 - 1 = 9$ samples *M1*

$\frac{9/20}{16/20}$: $\frac{9}{16}$ *A1*