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INTERNATIONAL A LEVEL MATHEMATICS • PURE 4 (WMA14)

Pure 4 Practice Paper 1

Mark Scheme

AIMING FOR B

Written in the style of the Pearson Edexcel International Advanced Level paper (WMA14/01) • 75 marks

Every question marked line by line the way an examiner marks it, with notes on the common errors

Mark every line, not just the answer. Method marks are given for a correct method even when the final answer is wrong. If you lost marks, find the *first* line where your working differs from the scheme. That is the thing to fix.

Code	Meaning
M1	Method mark: for knowing a correct method and attempting to apply it. Small slips in the working do not lose this mark.
dM1	Dependent method mark: can only be given if the method mark before it has been given.
A1	Accuracy mark: for a correct answer or line of working. It depends on the method mark before it.
B1	Independent mark: for a correct result or statement, whatever method was used.
*	The answer is given in the question. Every step must be shown, and the answer must follow from correct working (cso).
cao / cso	Correct answer only / correct solution only, with no errors anywhere in the working.
ft / oe	Follow through from your own earlier answer / or any correct equivalent form.

"Solutions relying on calculator technology are not acceptable." Questions 1, 5, 7 and 8 say this. There, a correct final answer with no algebraic working scores **no marks**.

Rough grade guide (out of 75): A* **66** • A **59** • B **51** • C **43**

Based on recent WMA14 grade boundaries. Real boundaries change every series, so treat this as a guide, not a grade.

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What to revise next

Write in your score for each question. Where you dropped marks, tap the chapter to practise ten more exam questions on it.

Q	What it tested	Marks	Practise it	Score
1	Partial fractions with a repeated factor; three binomial series added; the range of validity	8	Ch 2 Partial Fractions Ch 4 Binomial Expansion	
2	Proof by contradiction: an irrational logarithm and an equation with no integer solutions	7	Ch 1 Proof	
3	A parametric curve crossing itself; parametric gradients; a Cartesian equation	7	Ch 3 Coordinate Geometry Ch 5 Differentiation	
4	Implicit differentiation; points where the tangent is horizontal	7	Ch 5 Differentiation	
5	Integration by parts, once and then twice; a volume of revolution	8	Ch 6 Integration	
6	Similar triangles in a cone; connected rates of change	7	Ch 5 Differentiation	
7	Integration by parts inside a separable differential equation	8	Ch 6 Integration	
8	Integration by substitution; an exact area	7	Ch 6 Integration	
9	Intersecting lines; the angle between lines; a point at a given distance	8	Ch 7 Vectors	
10	Partial fractions in a differential equation; a limiting value	8	Ch 2 Partial Fractions Ch 6 Integration	
	Total	**75**		

Where did the marks go? If you lost the first marks of a question, the method needs work: go back to that chapter's key skills page. If you lost the final marks, the method is fine and the gap is accuracy, a missing constant of integration, or a proof without a clear conclusion. Papers 2 and 3 give less help between the parts, so that is the skill to build next.

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Question 1

8 marks

Part	Scheme	Marks
(a)	$\frac{x+8}{(1-x)(2+x)^2} \equiv \frac{A}{1-x} + \frac{B}{2+x} + \frac{C}{(2+x)^2}$, so $x+8 \equiv A(2+x)^2 + B(1-x)(2+x) + C(1-x)$	M1
	$x = 1$: $9 = 9A$, so $A = 1$	B1
	$x = -2$: $6 = 3C$, so $C = 2$; then compares coefficients or substitutes another value for B	M1
	$\boxed{\frac{1}{1-x} + \frac{1}{2+x} + \frac{2}{(2+x)^2}}$	A1
	The form must include both $\frac{B}{2+x}$ and $\frac{C}{(2+x)^2}$. Leaving out the $\frac{B}{2+x}$ term scores M0.	(4)
(b)	$(1-x)^{-1} = 1 + x + x^2$ and $(2+x)^{-1} = \frac{1}{2}\left(1+\frac{x}{2}\right)^{-1} = \frac{1}{2} - \frac{1}{4}x + \frac{1}{8}x^2$	M1
	$2(2+x)^{-2} = \frac{1}{2}\left(1+\frac{x}{2}\right)^{-2} = \frac{1}{2} - \frac{1}{2}x + \frac{3}{8}x^2$	M1
	$\boxed{2 + \frac{1}{4}x + \frac{3}{2}x^2}$	A1
	Taking out the 2 correctly is the key step: $(2+x)^{-2} = \frac{1}{4}\left(1+\frac{x}{2}\right)^{-2}$, not 2^{-2} forgotten or $\frac{1}{2}$ used.	(3)
(c)	$\boxed{\lvert x \rvert < 1}$	B1
	The expansion of $(1-x)^{-1}$ needs $\lvert x \rvert < 1$, which is stricter than $\lvert x \rvert < 2$ for the other two terms. $x < 1$ alone scores B0.	(1)
		Total for Question 1: 8 marks

Question 2

7 marks

Part	Scheme	Marks
(a)	Assume $\log_2 3$ is rational: $\log_2 3 = \frac{p}{q}$ for positive integers p and q	B1
	Then $3 = 2^{\frac{p}{q}}$, so $2^p = 3^q$	M1
	2^p is even (as $p \geqslant 1$) and 3^q is odd, so they cannot be equal	A1
	This contradiction shows that $\log_2 3$ is irrational	A1
	The assumption must be stated, and p, q must be positive (as $\log_2 3 > 0$). The final A1 needs a conclusion that refers back to the assumption.	(4)
(b)	Assume there are integers a and b with $6a + 15b = 2$	B1
	$3(2a + 5b) = 2$, so $2a + 5b = \frac{2}{3}$	M1
	$2a + 5b$ is an integer but $\frac{2}{3}$ is not: a contradiction, so no such integers exist	A1
		(3)
		Total for Question 2: 7 marks

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Question 3

7 marks

Part	Scheme	Marks
(a)	$t^2+1=4 \Rightarrow t=\pm\sqrt{3}$	M1
	$t=\sqrt{3}$: $y=3\sqrt{3}-3\sqrt{3}=0$, and $t=-\sqrt{3}$: $y=-3\sqrt{3}+3\sqrt{3}=0$, so C passes through A twice *	A1*
		(2)
(b)	$\frac{dy}{dx}=\frac{dy/dt}{dx/dt}=\frac{3t^2-3}{2t}$	M1
	Substitutes $t=\sqrt{3}$ and $t=-\sqrt{3}$	M1
	$\sqrt{3}$ and $-\sqrt{3}$	A1
		(3)
(c)	$y=t(t^2-3)$, so $y^2=t^2(t^2-3)^2$, with $t^2=x-1$	M1
	$y^2=(x-1)(x-1-3)^2=(x-1)(x-4)^2$ *	A1*
	Working with $t=\sqrt{x-1}$ is fine but must cover both signs of t, which squaring y does automatically.	
		(2)
	Total for Question 3: 7 marks	

Question 4

7 marks

Part	Scheme	Marks
(a)	Differentiates y^2 to $2y\frac{dy}{dx}$	M1
	Product rule on xy: $y+x\frac{dy}{dx}$	M1
	$2x+y+x\frac{dy}{dx}+2y\frac{dy}{dx}=0$	A1
	$\frac{dy}{dx}=-\frac{2x+y}{x+2y}$	A1
	Differentiating 12 to give 12 instead of 0, and the sign of the final answer, are the common errors.	
		(4)
(b)	$\frac{dy}{dx}=0 \Rightarrow y=-2x$	M1
	Substitutes into the curve: $x^2-2x^2+4x^2=12$, so $x^2=4$	M1
	$(2,-4)$ and $(-2,4)$	A1
	Both points are needed, each with its own y value. $x=\pm2$ with $y=\pm4$ in the wrong pairing scores A0.	
		(3)
	Total for Question 4: 7 marks	

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Question 5

8 marks

Part	Scheme	Marks
(a)	By parts with $u = x$, $\frac{dv}{dx} = e^{-x}$: $\pm xe^{-x} \pm \int e^{-x}\,dx$	M1
	$-xe^{-x} + \int e^{-x}\,dx$	A1
	$\boxed{-xe^{-x} - e^{-x} + c}$	A1
		(3)
(b)	$V = \pi \int_0^2 x^2 e^{-2x}\,dx$ (π, y^2 and the limits all present)	B1
	First integration by parts: $-\frac{1}{2}x^2e^{-2x} + \int xe^{-2x}\,dx$	M1
	Second integration by parts on $\int xe^{-2x}\,dx$	M1
	$-e^{-2x}\left(\frac{1}{2}x^2 + \frac{1}{2}x + \frac{1}{4}\right)$	A1
	$\pi\left[\frac{1}{4} - \frac{13}{4}e^{-4}\right] = \frac{\pi}{4}\left(1 - 13e^{-4}\right)$ *	A1*
	Squaring y gives x^2e^{-2x}, not $x^2e^{-x^2}$. Losing π, or taking the lower limit to give 0 (it gives $-\frac{1}{4}$, since $e^0 = 1$), costs the final mark.	
		(5)
		Total for Question 5: 8 marks

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Question 6

7 marks

Part	Scheme	Marks
(a)	Similar triangles: $\frac{r}{h} = \frac{15}{30}$, so $r = \frac{h}{2}$ $V = \frac{1}{3}\pi\left(\frac{h}{2}\right)^2 h = \frac{\pi h^3}{12}$ *	M1 A1* (2)
(b)	$\frac{dV}{dh} = \frac{\pi h^2}{4}$ $\frac{dh}{dt} = \frac{dV}{dt} \div \frac{dV}{dh} = \frac{-3}{9\pi}$ at $h = 6$ Falling at $\boxed{\frac{1}{3\pi}\text{ cm s}^{-1}}$ *$\frac{dV}{dt} = -3$, as the volume is decreasing. An answer of $-\frac{1}{3\pi}$ for the rate of fall is accepted if its meaning is clear.*	M1 M1 A1 (3)
(c)	$r = \frac{h}{2}$, so $\frac{dr}{dt} = \frac{1}{2}\frac{dh}{dt}$ Decreasing at $\boxed{\frac{1}{6\pi}\text{ cm s}^{-1}}$	M1 A1 (2)
		Total for Question 6: 7 marks

Question 7

8 marks

Part	Scheme	Marks
(a)	By parts with $u = x$, $\frac{dv}{dx} = e^{2x}$: $\frac{1}{2}xe^{2x} - \int \frac{1}{2}e^{2x}\,dx$	M1
	$\frac{1}{2}xe^{2x} - \frac{1}{4}e^{2x}$	A1
	$\boxed{\frac{1}{2}xe^{2x} - \frac{1}{4}e^{2x} + c}$ (constant included)	A1
		(3)
(b)	Separates: $\int (y+3)^{-2}\,dy = \int xe^{2x}\,dx$	M1
	$-\frac{1}{y+3} = \frac{1}{2}xe^{2x} - \frac{1}{4}e^{2x} + c$	B1
	Uses $x = 0$, $y = -2$: $-1 = -\frac{1}{4} + c$	M1
	$c = -\frac{3}{4}$	A1
	$\frac{1}{y+3} = \frac{3+(1-2x)e^{2x}}{4}$, so $y = -3 + \frac{4}{3+(1-2x)e^{2x}}$ *	A1*
	Integrating $(y+3)^{-2}$ as $\ln(y+3)^2$ scores B0. The constant must be found before rearranging for y.	
		(5)
		Total for Question 7: 8 marks

Question 8

7 marks

Part	Scheme	Marks
(a)	$u = e^x + 1 \Rightarrow \frac{du}{dx} = e^x$, so $e^{2x}\,dx = e^x \cdot e^x\,dx = (u-1)\,du$	M1
	$\int \frac{u-1}{u}\,du = \int \left(1 - \frac{1}{u}\right) du$	M1
	$u - \ln u$	A1
	$e^x + 1 - \ln(e^x+1) + c$, and the 1 joins the constant: $e^x - \ln(e^x+1) + c$ *	A1*
	The A1 needs the substitution written in full with du, and a comment on the constant.*	
		(4)
(b)	Uses the limits 0 and $\ln 3$ in the result of part (a)	M1
	$(3 - \ln 4) - (1 - \ln 2)$	M1
	$\boxed{2 - \ln 2}$ oe, such as $2 + \ln\frac{1}{2}$	A1
		(3)
		Total for Question 8: 7 marks

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Question 9

8 marks

Part	Scheme	Marks
(a)	Equates components, for example $1+2\lambda = 5+\mu$ and $2-\lambda = 3+\mu$ Solves two equations: $\lambda = 1$, $\mu = -2$ Checks the third: $-1+3(1) = 2$ and $-2-2(-2) = 2$, so the lines meet $\boxed{P(3, 1, 2)}$ *The check in the third component is needed for the A1. Without it, the lines could be skew.*	M1 M1 A1 A1 (4)
(b)	$\cos\theta = \dfrac{\lvert 2-1-6\rvert}{\sqrt{14}\sqrt{6}} = \dfrac{5}{\sqrt{84}}$ $\boxed{56.9^\circ}$ *Using the position vectors instead of the direction vectors scores M0.* 123.1° *is the obtuse angle.*	M1 A1 (2)
(c)	$AP^2 = 14$; $B = (5+\mu, 3+\mu, -2-2\mu)$: $(4+\mu)^2+(1+\mu)^2+(1+2\mu)^2 = 14$, so $3\mu^2+7\mu+2=0$ $\mu = -2$ gives P, so $\mu = -\frac{1}{3}$: $\boxed{B\left(\frac{14}{3}, \frac{8}{3}, -\frac{4}{3}\right)}$	M1 A1 (2)
		Total for Question 9: 8 marks

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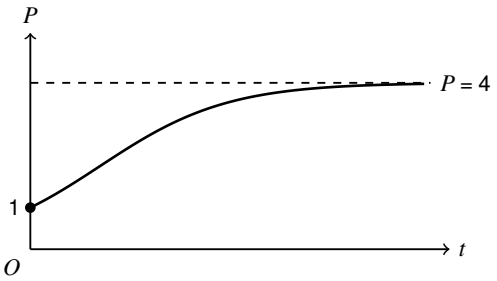
Question 10

8 marks

Part	Scheme	Marks
(a)	$1 \equiv A(4-P)+BP$, with $P=0$ and $P=4$ $\boxed{\frac{1}{4P}+\frac{1}{4(4-P)}}$	M1 A1 (2)
(b)	Separates: $\int \frac{8}{P(4-P)}\,\mathrm{d}P = \int 1\,\mathrm{d}t$ Integrates using part (a): $2\ln P - 2\ln(4-P)$ $2\ln\left(\frac{P}{4-P}\right) = t+c$ $t=0$, $P=1$: $c = 2\ln\frac{1}{3} = -2\ln 3$ $\frac{P}{4-P} = \frac{1}{3}\mathrm{e}^{\frac{1}{2}t}$, so $3P=(4-P)\mathrm{e}^{\frac{1}{2}t}$ and $P=\frac{4}{1+3\mathrm{e}^{-\frac{1}{2}t}}$ * *$\int \frac{1}{4-P}\,\mathrm{d}P = -\ln(4-P)$: the sign is the most common slip. Working in thousands, $P=1$ at $t=0$, not 1000.*	M1 M1 A1 M1 A1* (5)
(c)	 An increasing curve from (0, 1), levelling off below the asymptote $P=4$	B1 (1)
		Total for Question 10: 8 marks

TOTAL FOR PAPER: 75 MARKS

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