Measures of Location and Spread

Worked Solutions

Suitable for:

Pearson Edexcel International A-Level (iAL) — Statistics 1

Question 1 — Worked Solution

(a) Mean

$$\bar{x} = \frac{12 + 15 + 15 + 18 + 20 + 22 + 25 + 25 + 25 + 28}{10} = \frac{205}{10} = 20.5$$

(b) Median

n=10, so median = average of 5th and 6th values.

Ordered: 12, 15, 15, 18, **20, 22**, 25, 25, 25, 28

$$\text{Median} = \frac{20 + 22}{2} = 21$$

(c) Mode

Mode = 25 (appears 3 times)

Question 2 — Worked Solution

$$\bar{x} = \frac{\sum fx}{\sum f} = \frac{6(3) + 7(8) + 8(11) + 9(6) + 10(2)}{30}$$

$$= \frac{18 + 56 + 88 + 54 + 20}{30} = \frac{236}{30}$$

$$\therefore \bar{x} = 7.87 \text{ (3 s.f.)}$$

Question 3 — Worked Solution

(a) Mean

Heights recorded to nearest cm so true class boundaries are 149.5, 154.5, 159.5, 164.5, 169.5, 174.5.

Use midpoints of true boundaries: 152, 157, 162, 167, 172

$$\bar{x} = \frac{152(4) + 157(10) + 162(14) + 167(8) + 172(4)}{40}$$

$$= \frac{608 + 1570 + 2268 + 1336 + 688}{40} = \frac{6470}{40}$$

$$\therefore = 161.75 \text{ cm}$$

(b) Median

n=40, so the median lies at the 20th value.

Cumulative frequencies: 4, 14, 28, 36, 40. The 20th value lies in the class 160–164, which has true boundaries [159.5, 164.5).

$$\text{Median} = 159.5 + \frac{20 - 14}{14} \times 5 = 159.5 + \frac{30}{14}$$

$$\therefore \approx 161.64 \text{ cm}$$

Question 4 — Worked Solution

(a) Q_1

Q_1 = 10th value. CF: 4, 14, ... → 10th value in the class 155–159, with true boundaries [154.5, 159.5).

$$Q_1 = 154.5 + \frac{10-4}{10} \times 5 = 154.5 + 3 = 157.5 \text{ cm}$$

(b) Q_3

Q_3 = 30th value. CF: ..., 28, 36 → 30th value in the class 165–169, with true boundaries [164.5, 169.5).

$$Q_3 = 164.5 + \frac{30-28}{8} \times 5 = 164.5 + 1.25 = 165.75 \text{ cm}$$

(c) IQR

$$\therefore IQR = Q_3 - Q_1 = 165.75 - 157.5 = 8.25 \text{ cm}$$

Question 5 — Worked Solution

(a) Mean

$$\bar{x} = \frac{\sum x}{n} = \frac{80}{10} = 8$$

(b) Variance

$$s^2 = \frac{\sum x^2}{n} - \bar{x}^2 = \frac{680}{10} - 8^2 = 68 - 64 = 4$$

(c) Standard deviation

$$\therefore s = \sqrt{4} = 2$$

Question 6 — Worked Solution

(a) Mean

$$\sum fx = 0(2) + 1(6) + 2(10) + 3(8) + 4(4) = 0 + 6 + 20 + 24 + 16 = 66$$

$$\bar{x} = \frac{66}{30} = 2.2$$

(b) Standard deviation

$$\sum fx^2 = 0(2) + 1(6) + 4(10) + 9(8) + 16(4) = 0 + 6 + 40 + 72 + 64 = 182$$

$$s^2 = \frac{182}{30} - 2.2^2 = 6.0\overline{6} - 4.84 = 1.22\overline{6}$$

$$\therefore s = \sqrt{1.22\overline{6}} \approx 1.108$$

Question 7 — Worked Solution

(a) Mean

Midpoints: 4.5, 14.5, 24.5, 34.5, 44.5

$$\sum fx = 4.5(3) + 14.5(8) + 24.5(14) + 34.5(10) + 44.5(5)$$

$$= 13.5 + 116 + 343 + 345 + 222.5 = 1040$$

$$\bar{x} = \frac{1040}{40} = 26 \text{ min}$$

(b) Standard deviation

$$\sum fx^2 = 4.5^2(3) + 14.5^2(8) + 24.5^2(14) + 34.5^2(10) + 44.5^2(5)$$

$$= 60.75 + 1682 + 8408.5 + 11902.5 + 9901.25 = 31954$$

$$s^2 = \frac{31954}{40} - 26^2 = 798.85 - 676 = 122.85$$

$$\therefore s = \sqrt{122.85} \approx 11.08 \text{ min}$$

Question 8 — Worked Solution

If y = (x–50)/5 then x = 5y + 50.

$$\bar{x} = 5\bar{y} + 50 = 5(2.4) + 50 = 12 + 50 = 62$$

$$s_x = 5 \times s_y = 5 \times 1.3 = 6.5$$

$$\bar{x} = 62 \qquad s_x = 6.5$$

Question 9 — Worked Solution

y = (x–100)/5

$$\bar{y} = \frac{\bar{x} - 100}{5} = \frac{120 - 100}{5} = \frac{20}{5} = 4$$

$$s_y = \frac{s_x}{5} = \frac{15}{5} = 3$$

$$\bar{y} = 4 \qquad s_y = 3$$

Question 10 — Worked Solution

Location: Dataset A has a higher mean ($42 > 38$), so on average the values in A are larger than those in B.

Spread: Dataset B has a larger standard deviation ($12 > 8$), so the values in B are more spread out and variable than those in A.

Question 11 — Worked Solution

(a) Mean

Midpoints: 24.5, 34.5, 44.5, 54.5, 64.5

$$\sum fx = 24.5(5) + 34.5(12) + 44.5(18) + 54.5(10) + 64.5(5)$$

$$= 122.5 + 414 + 801 + 545 + 322.5 = 2205$$

$$\bar{x} = \frac{2205}{50} = 44.1 \text{ years}$$

(b) Standard deviation

$$\sum fx^2 = 24.5^2(5) + 34.5^2(12) + 44.5^2(18) + 54.5^2(10) + 64.5^2(5)$$

$$= 3001.25 + 14283 + 35653.5 + 29702.5 + 20801.25 = 103441.5$$

$$s^2 = \frac{103441.5}{50} - 44.1^2 = 2068.83 - 1944.81 = 124.02$$

$$\therefore s = \sqrt{124.02} \approx 11.14 \text{ years}$$

(c) Median

n=50, 25th value. CF: 5, 17, 35 $\rightarrow$ 25th value in [40, 50).

$$\text{Median} = 40 + \frac{25 - 17}{18} \times 10 = 40 + \frac{80}{18}$$

$$\therefore \approx 44.44 \text{ years}$$

Question 12 — Worked Solution

(a) Find n

$$\bar{x} = \frac{\sum x}{n} \Rightarrow 25 = \frac{300}{n} \Rightarrow n = 12$$

(b) Variance

$$s^2 = \frac{\sum x^2}{n} - \bar{x}^2 = \frac{7608}{12} - 25^2 = 634 - 625 = 9$$

(c) Standard deviation

$$\therefore s = \sqrt{9} = 3$$

Question 13 — Worked Solution

(a) Find

$$\sum x$$

$\sum x = n\bar{x} = 25 \times 12 = 300$

(b) Find

$$\sum x^2$$

$s^2 = \dfrac{\sum x^2}{n} - \bar{x}^2 \;\Rightarrow\; \sum x^2 = n(s^2 + \bar{x}^2)$

$= 25(4^2 + 12^2) = 25(16 + 144) = 25 \times 160$

$\therefore \sum x^2 = 4000$

Question 14 — Worked Solution

(a) Combined mean

$$\sum x_{\text{combined}} = 150 + 270 = 420 \qquad n_{\text{combined}} = 10 + 15 = 25$$

$$\bar{x} = \frac{420}{25} = 16.8$$

(b) Combined standard deviation

$$\sum x^2_{\text{combined}} = 2410 + 5130 = 7540$$

$$s^2 = \frac{7540}{25} - 16.8^2 = 301.6 - 282.24 = 19.36$$

$$\therefore s = \sqrt{19.36} = 4.4$$

Question 15 — Worked Solution

(a) Mean of x

$$\bar{y} = \frac{\sum y}{n} = \frac{100}{20} = 5$$

$$\bar{x} = \bar{y} + 30 = 5 + 30 = 35$$

(b) Standard deviation of x

$$s_y^2 = \frac{\sum y^2}{n} - \bar{y}^2 = \frac{2120}{20} - 25 = 106 - 25 = 81$$

Since y = x–30, subtracting a constant does not change the spread:

$$s_x = s_y = \sqrt{81}$$

$$\therefore s_x = 9$$