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A-LEVEL MATHEMATICS • STATISTICS 2

Hypothesis Testing

Worksheet

Pearson Edexcel International A Level (WST02) • Calculator and statistical tables allowed: show every step

Free Hypothesis Testing worksheet for A-Level and International A-Level Maths, with full worked solutions

A → **B** → **C** → **D** → **E**

Examples & Your turn page 2

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Tap a section to jump straight to it

By the end of this worksheet you can: write hypotheses for p or λ • find critical regions and actual significance levels • carry out binomial tests • carry out Poisson tests • *Stretch:* two-tailed tests, tests by normal approximation and the sample size a test needs.

Which questions are for me?

CORE The methods every A-level student must secure. Start here.

STRETCH Two-tailed regions, Poisson rates over different intervals, tests by approximation and the size of sample a test needs.

Every section has both, clearly labelled. Questions get harder as you go, and every skill in Sections A to D is needed in Section E.

Track your progress

Section	B	C	D	E
Core marks	23	10	13	21
Stretch marks	16	14	17	14
My score				

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Name ..... Class ..... Date .....

A Worked examples & Your turn

Read each example carefully. Cover it up, then answer the question beside it using the same method.

CORE *Secure the method*

Worked example A1: a test by probability

$H_0: p = 0.2$, $H_1: p > 0.2$, $n = 25$, 5% level. 9 successes are observed. Carry out the test.

Find the probability of the observed value or more extreme

$P(X \geqslant 9) = 1 - P(X \leqslant 8) = 1 - 0.9532 = 0.0468 < 0.05$

reject H_0

Your turn A1

$H_0: p = 0.25$, $H_1: p > 0.25$, $n = 30$, 5% level. 12 successes are observed. Carry out the test.

Worked example A2: a critical region

$X \sim B(20, 0.35)$ under H_0, and $H_1: p < 0.35$. Find the critical region at the 5% level and its actual significance.

$P(X \leqslant 3) = 0.0444 < 0.05$, $P(X \leqslant 4) = 0.1182$

$X \leqslant 3$, 0.0444

Your turn A2

$X \sim B(50, 0.2)$ under H_0, and $H_1: p < 0.2$. Find the critical region at the 5% level and its actual significance.

Worked example A3: a Poisson test

Faults occur at 4 per week. After a change, 8 faults occur in one week. Test at 5% whether the rate has increased.

$H_0: \lambda = 4$, $H_1: \lambda > 4$; $P(X \geqslant 8) = 1 - 0.9489 = 0.0511 > 0.05$

do not reject H_0

Your turn A3

Calls arrive at 2.5 per hour. After a change, 10 calls arrive in a 2-hour period. Test at 5% whether the rate has increased.

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STRETCH

Worked example A4: two tails

$X \sim \text{B}(20, 0.5)$ under H_0, and $H_1: p \neq 0.5$. Find the critical region at the 5% level.

2.5% in each tail

$P(X \leqslant 5) = 0.0207$, $P(X \leqslant 6) = 0.0577$; by symmetry $P(X \geqslant 15) = 0.0207$

$X \leqslant 5$ or $X \geqslant 15$

Your turn A4

$X \sim \text{B}(30, 0.4)$ under H_0, and $H_1: p \neq 0.4$. Find the critical region at the 5% level and its actual significance.

Worked example A5: a two-tailed Poisson region

$X \sim \text{Po}(6)$ under H_0, and $H_1: \lambda \neq 6$. Find the critical region at the 5% level.

$P(X \leqslant 1) = 0.0174$, $P(X \leqslant 2) = 0.0620$; $P(X \geqslant 12) = 0.0201$, $P(X \geqslant 11) = 0.0426$

$X \leqslant 1$ or $X \geqslant 12$

Your turn A5

$X \sim \text{Po}(8)$ under H_0, and $H_1: \lambda \neq 8$. Find the critical region at the 10% level and its actual significance.

Worked example A6: by normal approximation

$H_0: p = 0.4$, $H_1: p < 0.4$, $n = 150$. 48 successes are observed. Test at 5%

N(60, 36): $P(Y < 48.5)$, $z = -1.92$, $0.0274 < 0.05$

reject H_0

Your turn A6

Complaints arrive at 3 per day. In 20 days there are 72 complaints. Test at 5% whether the rate has increased.

Worked example A7: a 1% critical value

$X \sim \text{Po}(3.5)$ under H_0, and $H_1: \lambda > 3.5$. Find the critical region at the 1% level.

$P(X \geqslant 9) = 0.0099 < 0.01$, $P(X \geqslant 8) = 0.0267$

$X \geqslant 9$

Your turn A7

$X \sim \text{Po}(7.5)$ under H_0, and $H_1: \lambda < 7.5$. Find the critical region at the 1% level.

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B Fluency

Each question is a designed sequence. From one part to the next, ask yourself: what changed, and what did it do to the answer?

CORE *Secure the method*

B1 Write down suitable null and alternative hypotheses for each test.

(a) A coin is suspected of landing heads more often than tails.

(b) A machine's defect rate of 10% is believed to have increased.

(c) Calls average 6 per hour; the manager thinks this has changed.

(d) A proportion of 0.35 is believed to have fallen.

[4 marks]

B2 $H_0: p = 0.25$, $H_1: p > 0.25$, with a sample of 20 at the 5% level. Find the probability needed, and state the conclusion, when the number of successes is

(a) 8

(b) 9

(c) 10

(d) 11

[4 marks]

B3 $X \sim B(30, 0.3)$ under H_0 and $H_1: p > 0.3$. Find the critical region at

(a) the 10% level

(b) the 5% level

(c) the 1% level

(d) Find the actual significance level for part (b).

[4 marks]

B4 $X \sim Po(5)$ under H_0 and $H_1: \lambda < 5$, at the 5% level.

(a) 1 event is observed: carry out the test.

(b) 2 events are observed: carry out the test.

(c) Find the critical region.

(d) Find its actual significance level.

[4 marks]

B5 Breakdowns occur at 1.2 per day. Over a 5-day period after repairs there are 2 breakdowns. A test is made of whether the rate has decreased.

(a) State the distribution of the number of breakdowns in 5 days under H_0

(b) Carry out the test at the 5% level.

(c) State the conclusion at the 10% level instead.

[3 marks]

B6 $X \sim B(15, 0.4)$ under H_0 and $H_1: p \neq 0.4$, at the 5% level.

(a) Find the lower critical region.

(b) Find the upper critical region.

(c) Find the actual significance level.

(d) Is 10 successes significant?

[4 marks]

STRETCH

B7 $H_0: p = 0.45$, $H_1: p > 0.45$, with a sample of 200 at the 5% level.

(a) State the approximate distribution under H_0

(b) Test 100 successes.

(c) Test 105 successes.

(d) Find the critical region.

[4 marks]

B8 Faults occur at 4 per day. Over 30 days there are Y faults. $H_1: \lambda < 4$ per day, at the 5% level.

(a) State the approximate distribution of Y under H_0

(b) Test 105 faults.

(c) Find the critical region.

(d) Find its approximate actual significance level.

[4 marks]

B9 $X \sim B(25, 0.2)$ under H_0 and $H_1: p < 0.2$

(a) Find the critical region at 10%

(b) Find its actual significance level.

(c) Find the critical region at 5%

(d) Find its actual significance level.

[4 marks]

B10 A firm claims that at least 30% of its customers recommend it. A rival suspects that fewer do, and finds that 3 of a random sample of 20 customers recommend it.

(a) State the hypotheses.

(b) Find the probability needed.

(c) State the conclusion at the 5% level.

(d) Find the critical region at the 5% level.

[4 marks]

C Reasoning

Show your working and explain your thinking. A correct answer without a reason does not score full marks.

CORE *Secure the method*

C1 $H_0: p = 0.25$, $H_1: p > 0.25$, $n = 25$, and 11 successes are observed. Amir writes $P(X \geqslant 11) = 1 - P(X \leqslant 11)$. Explain his error, and find $P(X \geqslant 11)$ **[2 marks]**

C2 For a test with $X \sim B(20, 0.35)$ and $H_1: p < 0.35$, Bea writes "critical region = 0.0444". Explain her error, and give the critical region. **[2 marks]**

C3 A council believes 30% of residents cycle to work. In a sample of 50, 20 cycle. Cal writes $H_0: p = 0.4$, $H_1: p > 0.4$

Explain his error, and write down the correct hypotheses. **[3 marks]**

C4 A test of whether a coin is biased towards heads rejects H_0. Dev concludes "the coin is biased".

Explain why this conclusion should be changed, and write a suitable conclusion. **[3 marks]**

STRETCH

C5 $X \sim B(25, 0.3)$ under H_0 and $H_1: p \neq 0.3$, at the 5% level. Eve uses 5% in each tail and gives $X \leqslant 3$ or $X \geqslant 12$

Explain her error, and find the critical region and its actual significance. **[3 marks]**

C6 Requests arrive at 2 per hour. In a 3-hour period after a website change there are 11 requests. Fay tests whether the rate has increased using Po(2)

Explain her error, and carry out the test at the 5% level. **[3 marks]**

C7 $H_0: p = 0.5$, $H_1: p > 0.5$, $n = 100$, at the 2.5% level. 60 successes are observed. Gus uses N(50, 25) and $P(Y \geqslant 60) = 0.0228$, and rejects H_0

Explain his error, and carry out the test correctly. **[3 marks]**

C8 $X \sim B(12, 0.5)$ under H_0 and $H_1: p \neq 0.5$, at the 5% level.

(a) Show that the critical region is $X \leqslant 2$ or $X \geqslant 10$

(2)

(b) Show that the actual significance level is $\frac{79}{2048}$

(3)

[5 marks]

D Problem solving

You decide the method. Read the whole question first, and show every step.

CORE *Secure the method*

D1 A shop sells umbrellas at a mean rate of 3.5 per day. On the day after an advert appears, the shop sells 8 umbrellas. Test, at the 5% level of significance, whether there is evidence that the advert has increased the rate of sales. State your hypotheses clearly. **[3 marks]**

D2 A gardener believes that 25% of a variety of seeds produce red flowers. To test this belief, the gardener grows 20 of the seeds and uses a two-tailed test at the 5% level of significance.

(a) Find the critical region for the test. You should state the probability in each tail, which should be as close as possible to 2.5%

(3)

(b) Find the actual significance level of the test.

(1)

[4 marks]

D3 Emails arrive at a mean rate of 9 per hour. A test is to be carried out, at the 10% level of significance, of whether the rate has changed, using the number of emails in one hour.

(a) Find the critical region, with each tail as close as possible to 5%

(2)

(b) Find the actual significance level.

(1)

[3 marks]

D4 A supplier claims that 15% of its batteries are faulty. A buyer believes the proportion is lower and tests 15 batteries, none of which is faulty.

(a) Test the buyer's belief at the 5% level of significance.

(2)

(b) Explain why this test can never show that the buyer is right.

(1)

[3 marks]

STRETCH

D5 A newspaper states that 35% of adults read a daily newspaper. In a random sample of 200 adults, 56 read a daily newspaper. Using a suitable approximation, test at the 5% level of significance whether the proportion is lower than the newspaper states. State your hypotheses clearly. **[4 marks]**

D6 A council receives complaints at a mean rate of 6 per week. Over 8 weeks after a new policy is introduced it receives 36 complaints. Using a suitable approximation, test at the 5% level of significance whether the rate of complaints has fallen. State your hypotheses clearly. **[4 marks]**

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D7 In a game, a player wins with probability 0.3. A player claims to have improved and plays 25 games, and a test is carried out at the 10% level of significance.

(a) Find the critical region.

(2)

(b) The player wins 11 games. State the conclusion in context.

(1)

(c) Find the actual significance level.

(1)

[4 marks]

D8 A factory's records show that 12% of its items are defective. After a change in the process the manager tests n items and finds that none is defective.

(a) Find the least value of n for which this result is significant at the 1% level, for a test of whether the proportion of defective items has fallen.

(3)

(b) For this value of n, the critical region is $X = 0$. Find the actual significance level of the test.

(2)

[5 marks]

E Exam practice

Full-size exam questions in the style of WST02. Write your answers in the spaces, and show every step.

CORE *Exam questions for everyone*

E1 During weekday afternoons, a company's website receives enquiries at a mean rate of 4.5 per hour.

(a) State a suitable distribution to model the number of enquiries received in one hour, and give one assumption, in context, needed for the model to be valid.

(2)

After the website is redesigned, the company wants to test whether the rate of enquiries has changed. It monitors the number of enquiries in a randomly chosen one-hour period on a weekday afternoon.

(b) Write down suitable null and alternative hypotheses.

(1)

(c) Using a 10% level of significance, find the critical region for the test. You should state the probability in each tail, which should be as close as possible to 5%

(3)

(d) Find the actual significance level of the test.

(2)

In the chosen hour, 8 enquiries are received.

(e) Comment on this finding in the light of your critical region.

(2)

(Total for Question E1 is 10 marks)

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E2 Past records show that 20% of the students at a large school walk to school. The school introduces a cycle-to-school scheme. A random sample of 30 students is then taken, and 2 of them walk to school.

(a) Write down suitable hypotheses to test whether the proportion of students who walk to school has decreased.

(1)

(b) Carry out the test at the 5% level of significance.

(3)

(c) Find the critical region for this test.

(2)

(d) Find the actual significance level of the test.

(1)

A second random sample of 200 students is taken, and 28 of them walk to school.

(e) Using a suitable approximation, test at the 5% level of significance whether the proportion of students who walk to school has decreased.

(4)

(Total for Question E2 is 11 marks)

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STRETCH

E3 Telephone calls arrive at a help desk at a mean rate of 2 per hour. A manager believes that, after a new online help service was launched, the rate of calls has decreased. The manager plans to test this belief at the 5% level of significance by recording the number of calls received in a period of t hours.

(a) Find the least whole number of hours, t, for which receiving no calls in the period would be significant.

(3)

The manager decides to record the number of calls in a period of 5 hours.

(b) Find the critical region for the test.

(2)

(c) Find the actual significance level of the test.

(1)

(d) In the 5 hours, 4 calls are received. State the conclusion of the test in context.

(2)

Over the following 40 hours, the help desk receives 64 calls.

(e) Using a suitable approximation, test at the 1% level of significance whether the rate of calls has decreased. State your hypotheses clearly.

(4)

(f) Explain why the approximation you used in part (e) is suitable.

(2)

(Total for Question E3 is 14 marks)

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