eClassroom

A-LEVEL MATHEMATICS • MECHANICS 2

# Variable Acceleration

Worked Solutions

Pearson Edexcel International A Level (WME02) • Calculator allowed: take $g = 9.8\,\text{m}\,\text{s}^{-2}$ and give answers to 2 or 3 s.f.

Full worked solutions for the free Variable Acceleration worksheet, A-Level and International A-Level Maths


A **Examples** & Your turn page 2

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Tap a section to jump straight to it

**Full worked solutions** for every question on the Mechanics 2 Variable Acceleration worksheet, in the same order and with the same question numbers.

**How to mark your work.** Tick each step you got right, not just the final answer. At A-level most marks are for method, so a wrong final answer can still earn most of the marks. If you lost marks, find the *first* line where your working is different from ours. That is the thing to fix.

In Section E: **M** = method mark, **dM** = method mark that depends on the previous M mark, **A** = accuracy mark (needs the M mark before it), **B** = independent mark.


Question sheet | Exam questions by topic | Mechanics 2 exam questions | More worksheets



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## A Your turn answers

Compare each line of your working with the solution.

### CORE

**A1** $v = 3t^2 - 12t + 5$, $a = 6t - 12$: $\boxed{v = -4\,\text{m s}^{-1},\ a = 6\,\text{m s}^{-2}}$

**A2** $v = 3t^2 - 4t - 3$; $x = t^3 - 2t^2 - 3t + 5$: $\boxed{v = 12\,\text{m s}^{-1},\ x = 5\,\text{m}}$

**A3** $(2t - 1)(t - 4) = 0$; $a = 4t - 9$: $\boxed{t = \tfrac{1}{2}\text{: } a = -7\,\text{m s}^{-2};\ t = 4\text{: } a = 7\,\text{m s}^{-2}}$

### STRETCH

**A4** Rest at $t = 1$, $t = 4$. $x = 2t^3 - 15t^2 + 24t$: $x(1) = 11$, $x(4) = -16$, $x(5) = -5$

$11 + 27 + 11$: $\boxed{\textbf{49 m}}$

**A5** $a = 4 - 2t = 0$ at $t = 2$: $v(2) = 19$; $v(8) = -17$: $\boxed{\textbf{greatest velocity } 19\,\text{m s}^{-1}\textbf{, greatest speed } 19\,\text{m s}^{-1}}$

**A6** $\mathbf{v} = (4t - 6)\mathbf{i} + (3t^2 - 12)\mathbf{j}$; parallel to $\mathbf{i}$ when $t = 2$: $\boxed{2\mathbf{i}\,\text{m s}^{-1}}$

$\mathbf{a} = 4\mathbf{i} + 6t\mathbf{j} = 4\mathbf{i} + 12\mathbf{j}$: $\boxed{4\sqrt{10} \approx 12.6\,\text{m s}^{-2}}$

**A7** $\mathbf{v} = (2t - 3)\mathbf{i} + (3t^2 - 6t + 1)\mathbf{j}$; $\mathbf{r} = (t^2 - 3t + 1)\mathbf{i} + (t^3 - 3t^2 + t - 2)\mathbf{j}$

$\mathbf{r}(3) = \mathbf{i} + \mathbf{j}$: $\boxed{\sqrt{2} \approx 1.41\,\textbf{m}}$

## B Fluency solutions

One mark for each correct part. The notes in italics explain why each sequence was chosen.

### CORE

**B1** **(a)** $\boxed{v = 3t^2 - 12t + 9}$ **(b)** $\boxed{a = 6t - 12}$ **(c)** $3(t - 1)(t - 3) = 0$: $\boxed{t = 1,\ t = 3}$ **(d)** $\boxed{-6\textbf{ and } 6\,\text{m s}^{-2}}$

*The sign of $a$ at an instant of rest tells you which way the particle turns.*

**B2** **(a)** $\boxed{v = 3t^2}$ **(b)** $\boxed{v = 3t^2 + 4}$ **(c)** $4 = 3 + c$: $\boxed{v = 3t^2 + 1}$ **(d)** $\boxed{x = t^3 + 4t}$

*The same integral; the constant depends on the condition and on when it is given.*

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**B3** **(a)** $t = 2, t = 3$ **(b)** $2 < t < 3$ **(c)** $2(2.5) - 5$: $0$ **(d)** $v(2.5)$: $-0.25\,\text{m s}^{-1}$

*The least velocity is where $a = 0$, between the two rests.*

**B4** **(a)** $a = 18 - 6t = 0$, $t = 3$: $27\,\text{m s}^{-1}$ **(b)** $v = 0$ at $t = 6$: **108 m** **(c)** **64 m** **(d)** $-48\,\text{m s}^{-1}$

*Greatest values come from $a = 0$ or $v = 0$, but always check the ends of the interval: by $t = 8$ it moves at $48\,\text{m s}^{-1}$.*

## CORE

**B5** **(a)** $x = t^3 - 3t^2$: **0 m** **(b)** rest at $t = 2$, $x(2) = -4$: **8 m** **(c)** $x(4) = 16$; $4 + 20$: **24 m**

*A zero displacement does not mean a zero distance.*

**B6** **(a)** $\mathbf{v} = 2t\mathbf{i} + (3t^2 - 2)\mathbf{j}$ **(b)** $\mathbf{a} = 2\mathbf{i} + 6t\mathbf{j}$

**(c)** $\sqrt{2^2 + 1^2}$: $\sqrt{5} \approx 2.24\,\text{m s}^{-1}$ **(d)** $\sqrt{2^2 + 6^2}$: $2\sqrt{10} \approx 6.32\,\text{m s}^{-2}$

*Differentiate each component; speed is the magnitude of $\mathbf{v}$.*

## STRETCH

**B7** **(a)** $v = 12t^{\frac{1}{2}} - 6t$ **(b)** $12\sqrt{t} = 6t$, $\sqrt{t} = 2$: $t = 4$

**(c)** $a = 6t^{-\frac{1}{2}} - 6 = 0$ at $t = 1$: $6\,\text{m s}^{-1}$ **(d)** $8(8) - 3(16)$: **16 m**

*Fractional powers are differentiated the same way; $t^{\frac{3}{2}}$ at $t = 4$ is 8.*

**B8** **(a)** $t^2 - 4 = 0$: $t = 2$ **(b)** $t^2 - 4 = -3t$, $(t + 4)(t - 1) = 0$: $t = 1$

**(c)** $3(t^2 - 4) = 4(3t)$, $t^2 - 4t - 4 = 0$: $t = 2 + 2\sqrt{2}$

**(d)** $\mathbf{v} = -3\mathbf{i} + 3\mathbf{j}$: $3\sqrt{2} \approx 4.24\,\text{m s}^{-1}$

*Parallel to $p\mathbf{i} + q\mathbf{j}$ means the components are in the ratio $p : q$; reject negative times.*

**B9** **(a)** $\mathbf{v} = (2t + 1)\mathbf{i} + (3 - 3t^2)\mathbf{j}$ **(b)** $\mathbf{r} = (t^2 + t)\mathbf{i} + (3t - t^3)\mathbf{j}$

**(c)** $3 - 3t^2 = 0$: $t = 1$ **(d)** $2\mathbf{i} + 2\mathbf{j}$

*Vector constants: $\mathbf{v}(0) = \mathbf{i} + 3\mathbf{j}$ and $\mathbf{r}(0) = \mathbf{0}$*

**B10** **(a)** $t(t - 2)(t - 4) = 0$: $t = 0$, **2**, **4** **(b)** $x = \frac{1}{4}t^4 - 2t^3 + 4t^2$: **4 m** **(c)** **0 m**

**(d)** $x(5) = \frac{25}{4}$; $4 + 4 + \frac{25}{4}$: **14.25 m**

*Three sections: add the size of each displacement.*

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# C Reasoning solutions

Your wording can be different. Check that your reason makes the same point.

## CORE

**C1** He has left out the constant of integration: $v = 2t^2 + c$ with $v = 3$ at $t = 0$: $\boxed{v = 2t^2 + 3}$

**C2** $v = u + at$ only holds for a constant acceleration, and here $a = 6t$ changes. $v = \int 6t\,\mathrm{d}t = 3t^2$: $\boxed{12\,\mathrm{m\,s^{-1}}}$

**C3** The particle changes direction at $t = 1$ (and $t = 5$), so the end displacement is not the distance.

$x = t^3 - 9t^2 + 15t$: $x(1) = 7$, $x(5) = -25$; $7 + 32$: $\boxed{\textbf{39 m}}$

**C4** The direction of motion is given by $\mathbf{v}$, not $\mathbf{r}$. $\mathbf{v} = (2t - 4)\mathbf{i} + 2\mathbf{j}$, parallel to $\mathbf{j}$ when $2t - 4 = 0$: $\boxed{t = 2}$

## STRETCH

**C5** $\mathbf{v}$ only has to be a multiple of $\mathbf{i} + \mathbf{j}$, so the two components must be *equal*, not both 1.

$5t - 2 = t^2 + 2$, $(t - 1)(t - 4) = 0$: $\boxed{t = 1 \textbf{ and } t = 4}$

**C6** $a = 0$ gives the greatest *velocity* inside the interval; the end points must be checked too. $v(5) = -10$: $\boxed{\textbf{greatest speed } 10\,\mathrm{m\,s^{-1}}}$

**C7** The integral gives the displacement, and the particle turns at $t = 2$

$x = 12t - t^3$: $x(2) = 16$, $x(3) = 9$; $16 + 7$: $\boxed{\textbf{23 m}}$

**C8** **(a)** $x = t^3 - bt^2 = t^2(t - b)$, which is 0 when $t = b$: $\boxed{\textbf{returns to } O \textbf{ at } t = b}$

**(b)** $v = t(3t - 2b) = 0$ at $t = \dfrac{2b}{3}$; $x\left(\dfrac{2b}{3}\right) = \dfrac{8b^3}{27} - \dfrac{4b^3}{9} = -\dfrac{4b^3}{27}$

Out $\dfrac{4b^3}{27}$ and back $\dfrac{4b^3}{27}$: $\boxed{\textbf{distance } \dfrac{8b^3}{27}\textbf{ m}}$

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# D Problem solving solutions

There is often more than one correct method. Any complete, correct method scores full marks.

## CORE

**D1** **(a)** $a = 0.6t - 0.06t^2$; $\dfrac{\mathrm{d}a}{\mathrm{d}t} = 0.6 - 0.12t = 0$ at $t = 5$

$a(5) = 3 - 1.5$: $\boxed{1.5\,\mathrm{m\,s^{-2}}}$

**(b)** $v \geqslant 0$ throughout; $\int_0^{10}(0.3t^2 - 0.02t^3)\,\mathrm{d}t = \left[0.1t^3 - 0.005t^4\right]_0^{10} = 100 - 50$: $\boxed{\textbf{50 m}}$

**D2** **(a)** $v = 6t^2 - 42t + 60 = 6(t-2)(t-5) = 0$: $\boxed{t = 2 \textbf{ and } t = 5}$

**(b)** $x(2) = 52$, $x(5) = 25$; $52 + (52 - 25)$: $\boxed{\textbf{79 m}}$

**D3** **(a)** $\mathbf{v} = 6t^{\frac{1}{2}}\mathbf{i} + (2t - 6)\mathbf{j}$; parallel to $\mathbf{i}$ when $t = 3$: $\boxed{6\sqrt{3}\mathbf{i}\,\mathrm{m\,s^{-1}}}$

**(b)** $\mathbf{a} = 3t^{-\frac{1}{2}}\mathbf{i} + 2\mathbf{j}$: $\boxed{(\sqrt{3}\mathbf{i} + 2\mathbf{j})\,\mathrm{m\,s^{-2}}}$

**D4** $\mathbf{a} = (6t - 4)\mathbf{i} + 2\mathbf{j}$; at $t = 2$, $\mathbf{a} = 8\mathbf{i} + 2\mathbf{j}$, so $\mathbf{F} = 2\mathbf{a} = 16\mathbf{i} + 4\mathbf{j}$

$\sqrt{16^2 + 4^2} = \sqrt{272}$: $\boxed{4\sqrt{17} \approx 16.5\,\mathbf{N}}$

## STRETCH

**D5** **(a)** Continuous at $t = 2$: $2^2 + 2(2) = k - 2(2)^2$, $8 = k - 8$: $\boxed{k = 16}$

**(b)** $16 - 2t^2 = 0$ with $t > 2$: $\boxed{t = 2\sqrt{2}}$

**(c)** $\int_0^2 (t^2 + 2t)\,\mathrm{d}t = \dfrac{20}{3}$; $\int_2^{2\sqrt{2}} (16 - 2t^2)\,\mathrm{d}t = \dfrac{64\sqrt{2} - 80}{3}$

$\boxed{\dfrac{64\sqrt{2} - 60}{3}\,\mathbf{m}}$ $(\approx 10.2\,\mathrm{m})$

**D6** **(a)** $\mathbf{v} = (2t - 4)\mathbf{i} + (3t^2 - 3)\mathbf{j}$; parallel to $\mathbf{j}$ when $t = 2$: $\mathbf{v} = 9\mathbf{j}$: $\boxed{9\,\mathrm{m\,s^{-1}}}$

**(b)** The $\mathbf{i}$ part of $\mathbf{r}$ is 0 only when $t = 4$ $(t > 0)$; then the $\mathbf{j}$ part is $64 - 12 = 52 \neq 0$, so $\boxed{P \textbf{ never returns to } O}$

**D7** **(a)** $\mathbf{r} = (3t^2 - 4t)\mathbf{i} + (t^3 + 2t)\mathbf{j} + (2\mathbf{i} - 12\mathbf{j})$; $t = 2$: $\boxed{6\mathbf{i}\,\mathbf{m}}$

**(b)** $7(6t - 4) = 4(3t^2 + 2)$, so $12t^2 - 42t + 36 = 0$, $2t^2 - 7t + 6 = 0$, $(2t - 3)(t - 2) = 0$

$\boxed{t = 1.5 \textbf{ and } t = 2}$

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**D8** **(a)** $v = \int k(2-t)\,dt = k\left(2t - \frac{1}{2}t^2\right)$ (at rest at $t = 0$); $v(2) = 2k = 6$: $\boxed{k = 3}$

**(b)** $x = \int 3\left(2t - \frac{1}{2}t^2\right)\,dt = 3t^2 - \frac{1}{2}t^3 = \frac{1}{2}t^2(6-t)$, as required

**(c)** Back at $O$ when $t = 6$; at rest when $t = 4$: $x(4) = 16$

$16 + 16$: $\boxed{\mathbf{32\,m}}$

## E Exam practice solutions

Mark-scheme notes are shown on the right of each line.

**CORE** *Exam questions for everyone*

**E1** **(a)** $v = \int (6t - 14)\,dt = 3t^2 - 14t + c$ *M1*

$v = 8$ at $t = 0$: $\boxed{v = 3t^2 - 14t + 8}$ *A1**

**(b)** $(3t - 2)(t - 4) = 0$ *M1*

$\boxed{t = \frac{2}{3} \text{ and } t = 4}$ *A1*

**(c)** $a = 0$ at $t = \frac{7}{3}$: $v = 3\left(\frac{7}{3}\right)^2 - 14\left(\frac{7}{3}\right) + 8$ *M1*

$\boxed{-\frac{25}{3}\,\text{m s}^{-1}}$ *A1*

**(d)** $x = t^3 - 7t^2 + 8t$: $x\left(\frac{2}{3}\right) = \frac{68}{27}$, $x(4) = -16$, $x(5) = -10$ *M1*

$\frac{68}{27} + \left(\frac{68}{27} + 16\right) + 6$ *dM1*

$\boxed{\mathbf{27.0\,m}}$ $\left(\frac{730}{27}\right)$ *A1*

**E2** **(a)** $t^2 - 4t = 0$ with $t > 0$: $t = 4$ *M1*

$\mathbf{v} = 13\mathbf{i}$ *A1*

$\boxed{13\,\text{m s}^{-1}}$ *A1*

**(b)** $\mathbf{a} = 4\mathbf{i} + (2t - 4)\mathbf{j}$; at $t = 1$, $\mathbf{a} = 4\mathbf{i} - 2\mathbf{j}$ *M1*

$\mathbf{F} = 0.5\mathbf{a} = 2\mathbf{i} - \mathbf{j}$ *M1*

$\boxed{\sqrt{5}\,\mathbf{N}}$ *A1*

**(c)** $\mathbf{r} = (2t^2 - 3t)\mathbf{i} + \left(\frac{1}{3}t^3 - 2t^2\right)\mathbf{j}$ *M1*

$\boxed{9\mathbf{i} - 9\mathbf{j}}$ *A1*

**(d)** Due east when $\frac{1}{3}t^3 - 2t^2 = 0$ and the $\mathbf{i}$ part is positive: $t = 6$ *M1*

$2(36) - 18$: $\boxed{\mathbf{54\,m}}$ *A1*

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**STRETCH**

**E3** **(a)** $v = \int \left(6 - 6t^{\frac{1}{2}}\right) \mathrm{d}t = 6t - 4t^{\frac{3}{2}}$ (from rest) *M1*

$v = 2t\left(3 - 2t^{\frac{1}{2}}\right) = 0$ when $\sqrt{t} = \frac{3}{2}$: $\boxed{t = \frac{9}{4}}$ *A1**

**(b)** $a = 0$ when $\sqrt{t} = 1$, $t = 1$ *M1*

$v(1) = 6 - 4$ *dM1*

$\boxed{2\,\mathrm{m\,s^{-1}}}$ *A1*

**(c)** $x = \int \left(6t - 4t^{\frac{3}{2}}\right) \mathrm{d}t = 3t^2 - \frac{8}{5}t^{\frac{5}{2}}$ ($x = 0$ at $t = 0$), as required *M1 A1**

**(d)** $x\left(\frac{9}{4}\right) = \frac{243}{16} - \frac{243}{20} = \frac{243}{80}$ *M1 A1*

$x(4) = 48 - \frac{256}{5} = -\frac{16}{5}$ *M1*

$\frac{243}{80} + \frac{243}{80} + \frac{16}{5}$: $\boxed{\frac{371}{40}\,\mathbf{m}}$ *A1*

**(e)** $v(4) = 24 - 32 = -8$, and $8 > 2$: $\boxed{8\,\mathrm{m\,s^{-1}}}$ *B1*