



eClassroom

A-LEVEL MATHEMATICS • PURE 3

Algebraic Methods

Worked Solutions

Pearson Edexcel International A Level (WMA13) & A Level (9MA0) • Calculator allowed: show every step

Full worked solutions for the free Algebraic Methods worksheet, A-Level and International A-Level Maths



Tap a section to jump straight to it

Full worked solutions for every question on the Pure 3 Algebraic Methods worksheet, in the same order and with the same question numbers.

How to mark your work. Tick each step you got right, not just the final answer. At A-level most marks are for method, so a wrong final answer can still earn most of the marks. If you lost marks, find the *first* line where your working is different from ours. That is the thing to fix.

In Section E: **M** = method mark, **dM** = method mark that depends on the previous M mark, **A** = accuracy mark (needs the M mark before it), **B** = independent mark.

Question sheet

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Page 1 of 7

A**Your turn answers**

Compare each line of your working with the solution.

CORE

$$\text{A1} \quad \frac{3(x-2)(x+2)}{(x+3)(x-2)} = \boxed{\frac{3(x+2)}{x+3}}$$

$$\text{A2} \quad \frac{(x+2)(x+3)}{(x-2)(x+2)} \times \frac{4(x-2)}{x+3} = \boxed{4}$$

$$\text{A3} \quad \frac{5(x-4)+3(2x+1)}{(2x+1)(x-4)} = \boxed{\frac{11x-17}{(2x+1)(x-4)}}$$

STRETCH

$$\text{A4} \quad x^2+x-2=(x+2)(x-1): \frac{3-(x+2)}{(x+2)(x-1)} = \frac{-(x-1)}{(x+2)(x-1)} = \boxed{-\frac{1}{x+2}}$$

$$\text{A5} \quad (x-1)(3x-4)=3x^2-7x+4, \text{ remainder } 1: \boxed{3x-4+\frac{1}{x-1}}$$

$$\text{A6} \quad (x^2-x+1)(2x+1)=2x^3-x^2+x+1, \text{ remainder } 2x+3: \boxed{2x+1+\frac{2x+3}{x^2-x+1}}$$

$$\text{A7} \quad 4x^2-2x+1 \equiv (Ax+B)(2x-1)+C: A=2, \text{ then } B=0; x=\frac{1}{2}: C=1: \boxed{A=2, B=0, C=1}$$



B**Fluency solutions**

One mark for each correct part. The notes in *italics* explain why each sequence was chosen.

CORE

B1 (a) $\frac{x-5}{x}$ (b) $\frac{2(x+2)}{x-2}$ (c) $\frac{(x-3)(x-4)}{-(x-4)(x+4)} = \frac{x-3}{x+4}$ (d) $\frac{3x+2}{2x+1}$

$16 - x^2 = -(x-4)(x+4)$: write the factor -1 out before cancelling.

B2 (a) $\frac{x+2}{x}$ (b) $\frac{(x-3)(x+3)}{2x} \times \frac{4x^2}{x-3} = 2x(x+3)$ (c) 1 (d) $\frac{1}{x}$

Everything can cancel: an answer of 1 is possible.

B3 (a) $\frac{2x+1}{x(x+1)}$ (b) $\frac{3(x+1) - 2(x-2)}{(x-2)(x+1)} = \frac{x+7}{(x-2)(x+1)}$ (c) $\frac{x^2+9}{x^2-9}$ (d) $\frac{2x-1}{x+1}$

A whole number is a fraction over 1: $2 = \frac{2(x+1)}{x+1}$

B4 (a) $x+3+\frac{1}{x+2}$ (b) $2x+3+\frac{2}{x-2}$ (c) $x^2+x-1+\frac{2}{x-1}$ (d) $2x-1+\frac{2}{2x+1}$

Write any missing power with a zero coefficient before dividing.

B5 (a) $3(x+1) = 2(x-1)$: $x = -5$ ($x \neq \pm 1$) (b) $x(x+2) - (x-2) = x^2 - 4$: $x = -6$ ($x \neq \pm 2$) (c) $x+2 = 5$: $x = 3$ ($x \neq 2$)

Multiply through by the common denominator, then check the answer is not excluded.

B6 (a) $A = 2, B = 3$ (b) $A = 1, B = 1, C = 4$ (c) $A = 3, B = 1, C = -2$ (d) $x^3 + 1 = x(x^2 + 1) - x + 1$: $A = 1, B = 0, C = -1, D = 1$

The remainder always has a lower degree than the divisor.

STRETCH

B7 (a) $\frac{x+1}{x} \times \frac{x^2}{(x-1)(x+1)} = \frac{x}{x-1}$ (b) $\frac{(x-2)(x+2)}{x(x+2)} = \frac{x-2}{x}$ (c) $\frac{(x+1) - (x-1) - 2}{x^2 - 1} = 0$ (d) $\frac{x-1}{x}$

Clear the small fractions first by multiplying top and bottom by x or x^2



- B8** (a) $x+1$, remainder 2 (b) x^2-1 , remainder 0 (c) $2x+1$, remainder 6
 (d) x^2+3 , remainder 8

Check each by multiplying back: (divisor) $\times$ (quotient) + (remainder)

- B9** (a) $\frac{x+1}{(x-2)(x+1)}$ (b) $\frac{4+(x-2)}{(x-2)(x+2)} = \frac{x+2}{(x-2)(x+2)}$ (c) $\frac{2x-(x+1)}{(x-1)(x+1)} = \frac{x-1}{(x-1)(x+1)}$
 (d) $\frac{5-(x+3)}{(x+3)(x-2)} = \frac{-(x-2)}{(x+3)(x-2)}$

each simplifies to the given fraction

Factorise the quadratic denominator first: it contains the other denominator.

- B10** (a) $x+2+\frac{3}{x+2}$ (b) $x=-2$ (c) $\frac{3}{x+2} > 0$ when $x > -2$: so $f(x) > x+2$ (d) $2+\frac{3}{2} = \frac{7}{2}$ and $\frac{7}{2} : \frac{7}{2}$

The mixed form shows the behaviour of the function at a glance.

C

Reasoning solutions

Your wording can be different. Check that your reason makes the same point.

CORE

- C1** x^2+9 does not factorise, so $(x+3)$ is not a factor of it: nothing cancels. By division, $\frac{x^2+9}{x+3} = x-3 + \frac{18}{x+3}$

- C2** The whole numerator $x+1$ is subtracted: $3-(x+1) = 2-x$, not $4-x$

$$\frac{2-x}{x-2} = -1$$

- C3** $2x^2-18 = 2(x-3)(x+3)$: Kai lost the factor 2 $\frac{2(x+3)}{x+2}$

- C4** The roots give $(x+1)(3x-2)$; Jo's version has x^2 coefficient 1, not 3

$$\frac{(x+1)(3x-2)}{x(3x-2)} = \frac{x+1}{x}$$



STRETCH

C5 The question asks for $f(x)$ in the form, so it must be written in full. Check: $(x-3)(2x+5)+4=2x^2-x-11$

$$f(x) = 2x + 5 + \frac{4}{x-3}$$

C6 $x^2 - 4 = (x-2)(x+2)$ already contains $(x-2)$, so the lowest common denominator is $(x-2)(x+2)$; Ali's choice gives a cubic.

$$\frac{1+(x+2)}{(x-2)(x+2)} = \frac{x+3}{(x-2)(x+2)}$$

C7 The fraction is undefined at $x = 1$, so the graph is the line $y = x + 1$ with the point $(1, 2)$ missing.

a hole at $(1, 2)$

C8 (a) $\frac{(n+1)-n}{n(n+1)} = \frac{1}{n(n+1)}$ **as required**

(b) $\frac{a^2+b^2-2ab}{ab} = \frac{(a-b)^2}{ab}$ **as required**

(c) With $a = x$, $b = x + 1$: $\frac{(x-(x+1))^2}{x(x+1)} = \frac{1}{x(x+1)} > 0$ for $x > 0$, so the sum exceeds 2 **as required**

D

Problem solving solutions

There is often more than one correct method. Any complete, correct method scores full marks.

CORE

D1 $\frac{4x}{(x-2)(x+2)} - \frac{2(x-2)}{(x-2)(x+2)} = \frac{2x+4}{(x-2)(x+2)} = \frac{2(x+2)}{(x-2)(x+2)} = \frac{2}{x-2}$

D2 $(x^2+2)(x^2+1) = x^4+3x^2+2$, so the remainder is $2x+3$

$$A = 1, B = 0, C = 1, D = 2, E = 3$$

D3 $2(x+2)+3(x-1) = (x-1)(x+2)$, so $5x+1 = x^2+x-2$, $x^2-4x-3 = 0$

$$x = 2 \pm \sqrt{7}; \text{ neither is } 1 \text{ or } -2, \text{ so both are valid.}$$



D4 (a) $(x-1)(3x-1) = 3x^2 - 4x + 1$, remainder 4: $3x - 1 + \frac{4}{x-1}$

(b) $\frac{4}{x-1} > 0$ for $x > 1$, so $g(x) > 3x - 1$

STRETCH

D5 (a) $ff(x) = \frac{2\frac{2x+1}{x-3} + 1}{\frac{2x+1}{x-3} - 3} = \frac{4x+2+x-3}{2x+1-3x+9} = \frac{5x-1}{10-x}$ as required

(b) $5x - 1 = x(10 - x)$, so $x^2 - 5x - 1 = 0$: $x = \frac{5 \pm \sqrt{29}}{2}$

D6 (a) $(x+1)(x+2) = x^2 + 3x + 2$, remainder 4: $y = x + 2 + \frac{4}{x+1}$

(b) $x^2 + 3x + 6 = (2x+2)(x+1) = 2x^2 + 4x + 2$, so $x^2 + x - 4 = 0$; $x > -1$: $x = \frac{-1 + \sqrt{17}}{2}$

D7 (a) $\frac{(x+3)(x-1)}{(x-3)(x+3)} \times \frac{(x-3)^2}{x(x-1)} = \frac{x-3}{x}$ as required

(b) $2(x-3) = x$: $x = 6$, which is not 0, 1 or ± 3

D8 (a) Over $(x+1)(x-2)$: $3(x-2) - 2(x+1) + x + 10 = 2x + 2 = 2(x+1)$, so $f(x) = \frac{2(x+1)}{(x+1)(x-2)} = \frac{2}{x-2}$

as required

(b) the original fractions have $x+1$ in a denominator, so $x = -1$ is excluded from the domain

(c) $x(x-2) = 2$, $x^2 - 2x - 2 = 0$: $x = 1 \pm \sqrt{3}$

E

Exam practice solutions

Mark-scheme notes are shown on the right of each line.



CORE

Exam questions for everyone

- E1** (a) $\frac{(x+2)(x+5)}{(x+2)(x-1)} - \frac{3}{x-1}$ *M1: factorises and cancels*
- $$= \frac{x+5-3}{x-1}$$
- $$= \frac{x+2}{x-1} \quad \text{as required} \quad \text{A1*}$$
- (b) $x+2 = 4x-4$ *M1*
- $$x = 2; \text{ it satisfies } x > 1 \quad \text{A1}$$

- E2** (a) $(x^2 - x + 2)(2x^2 - x + 1) = 2x^4 - 3x^3 + 6x^2 - 3x + 2$ *M1 A1*
- $$A = 2, B = -1, C = 1 \quad \text{A1}$$
- (b) remainder $(-x+5) - (-3x+2) = 2x+3$: $D = 2, E = 3$ *M1 A1*
- (c) $2x+3$ *B1*

STRETCH

- E3** (a) $(x+1)(2x-1) = 2x^2 + x - 1$, remainder 4 *M1 A1*
- $$y = 2x - 1 + \frac{4}{x+1} \quad \text{A1}$$
- (b) $2(u-1) - 1 + \frac{4}{u} = 2u - 3 + \frac{4}{u}$ *B1*
- $$\text{as required}$$
- (c) $\frac{dy}{du} = 2 - \frac{4}{u^2} = 0, u = \sqrt{2} (u > 0)$ *M1*
- $$x = \sqrt{2} - 1, y = 2\sqrt{2} - 3 + 2\sqrt{2}$$
- $$P(\sqrt{2} - 1, 4\sqrt{2} - 3) \quad \text{A1}$$
- (d) $k > 4\sqrt{2} - 3$ *B1*

