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A-LEVEL MATHEMATICS • PURE 1

Trigonometric Ratios

Worked Solutions

Pearson Edexcel International A Level (WMA11) & A Level (9MA0) • Calculator allowed: show every step

Full worked solutions for the free Trigonometric Ratios worksheet, A-Level and International A-Level Maths



Tap a section to jump straight to it

Full worked solutions for every question on the Pure 1 Trigonometric Ratios worksheet, in the same order and with the same question numbers.

How to mark your work. Tick each step you got right, not just the final answer. At A-level most marks are for method, so a wrong final answer can still earn most of the marks. If you lost marks, find the *first* line where your working is different from ours. That is the thing to fix.

In Section E: **M** = method mark, **dM** = method mark that depends on the previous M mark, **A** = accuracy mark (needs the M mark before it), **B** = independent mark.

Question sheet

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A**Your turn answers**

Compare each line of your working with the solution.

CORE

A1 $QR^2 = 7^2 + 11^2 - 2 \times 7 \times 11 \cos 115^\circ \approx 235.08$, so $QR \approx 15.3 \text{ cm}$

The angle is obtuse, so $\cos 115^\circ$ is negative and the third side is longer than either given side.

A2 BC is opposite A and AB is opposite C : $\frac{BC}{\sin 102^\circ} = \frac{8}{\sin 36^\circ}$, so $BC \approx 13.3 \text{ cm}$

A3 $\frac{1}{2} \times 5.4 \times 7.2 \times \sin 128^\circ \approx 15.3 \text{ cm}^2$

STRETCH

A4 The largest angle is opposite 11: $\cos \theta = \frac{16 + 81 - 121}{2 \times 4 \times 9} = -\frac{24}{72} = -\frac{1}{3}$, so $\theta \approx 109.5^\circ$

A5 $\frac{\sin C}{13} = \frac{\sin 35^\circ}{10}$, so $\sin C \approx 0.7456$: $C \approx 48.2^\circ$ or $180^\circ - 48.2^\circ$

Both fit, because $35^\circ + 131.8^\circ < 180^\circ$: $C \approx 48.2^\circ$ or $C \approx 131.8^\circ$

A6 The graph of $\sin x$ moves 45° to the left. y intercept $\sin 45^\circ$: $\left(0, \frac{\sqrt{2}}{2}\right)$

$x + 45^\circ = 180^\circ$ or 360° : $x = 135^\circ$, $x = 315^\circ$; $x + 45^\circ = 90^\circ$: maximum $(45^\circ, 1)$

A7 $\cos 3x$ completes a cycle every $\frac{360^\circ}{3}$: period 120° . Three full waves, and $y = 1$ crosses each twice:
6 solutions



B**Fluency solutions**

One mark for each correct part. The notes in *italics* explain why each sequence was chosen.

CORE

B1 (a) $\sqrt{25+64-40} = \sqrt{49} = \boxed{7 \text{ cm}}$ (b) $\sqrt{25+64+40} = \sqrt{129} \approx \boxed{11.4 \text{ cm}}$ (c) $\sqrt{89} \approx \boxed{9.43 \text{ cm}}$
 (d) $\sqrt{89-80\cos 30^\circ} \approx \boxed{4.44 \text{ cm}}$

At 90° the cosine rule becomes Pythagoras; as the angle grows, so does the third side.

B2 (a) $AC = \frac{10 \sin 60^\circ}{\sin 40^\circ} \approx \boxed{13.5 \text{ cm}}$ (b) angle $C = 80^\circ$: $AB = \frac{10 \sin 80^\circ}{\sin 40^\circ} \approx \boxed{15.3 \text{ cm}}$
 (c) $BC = \frac{10 \sin 40^\circ}{\sin 60^\circ} \approx \boxed{7.42 \text{ cm}}$ (d) $\frac{\sin A}{7} = \frac{\sin 40^\circ}{10}$, so $\sin A \approx 0.450$: $\boxed{A \approx 26.7^\circ}$

Always pair a side with the angle opposite it; in (d) the unknown is an angle, so put the sines on top.

B3 (a) $\boxed{15 \text{ cm}^2}$ (b) $\boxed{15 \text{ cm}^2}$ (c) $\boxed{30 \text{ cm}^2}$ (d) $30 \times \frac{\sqrt{2}}{2} = \boxed{15\sqrt{2} \text{ cm}^2}$

$\sin 150^\circ = \sin 30^\circ$, so (a) and (b) give the same area; the area is greatest at 90°

B4 (a) $\cos \theta = \frac{9+25-49}{30} = -\frac{1}{2}$: $\boxed{120^\circ}$ (b) $\frac{25+64-49}{80} = \frac{1}{2}$: $\boxed{60^\circ}$ (c) $\boxed{90^\circ}$ (d) $\frac{16+25-36}{40} = \frac{1}{8}$:
 $\boxed{82.8^\circ}$

The sign of the cosine tells you the type of angle before you find it.

B5 (a) $\boxed{30^\circ, 150^\circ}$ (b) $\boxed{60^\circ, 300^\circ}$ (c) $\boxed{45^\circ, 225^\circ}$

$\sin$ is symmetrical about 90° , $\cos$ about 180° , and $\tan$ repeats every 180°

B6 (a) $\boxed{2}$ (b) $\boxed{1}$ (c) $\boxed{0}$ (d) $\boxed{2}$

A line through a maximum touches once; a value outside -1 to 1 is never reached; $\tan$ takes every value once per 180°

STRETCH

B7 $\sin B = \frac{6 \sin 30^\circ}{BC} = \frac{3}{BC}$

(a) $\sin B = 0.75$: 48.6° or 131.4° , and $30^\circ + 131.4^\circ < 180^\circ$: $\boxed{2}$ (b) $\sin B = 1$: $B = 90^\circ$: $\boxed{1}$

(c) $\sin B = 1.5 > 1$: $\boxed{0}$ (d) $\sin B = 0.375$: 22.0° or 158.0° , but $30^\circ + 158.0^\circ > 180^\circ$: $\boxed{1}$

Two triangles are only possible when the side opposite the given angle is the shorter one, and longer than $6 \sin 30^\circ$



- B8** (a) $\boxed{3}$, $\boxed{360^\circ}$ (b) $\boxed{1}$, $\boxed{120^\circ}$ (c) $\boxed{3}$, $\boxed{360^\circ}$ (d) $\boxed{2}$, $\boxed{720^\circ}$

A number outside changes the height; a number multiplying x changes the period; a translation changes neither the period nor the height of a wave.

- B9** (a) $\boxed{4}$ (b) $\boxed{2}$ (c) $\boxed{1}$ (d) $\boxed{6}$

Count the cycles in the interval: two for $\sin 2x$, half for $\sin \frac{x}{2}$, one maximum for (c), six branches for $\tan 3x$

B10 (a) $\frac{1}{2} \times 4 \times 6 \times \frac{\sqrt{3}}{2} = \boxed{6\sqrt{3} \text{ cm}^2}$ (b) $\sqrt{16+36-24} = \sqrt{28} = \boxed{2\sqrt{7} \text{ cm}}$

(c) $\frac{1}{2} \times 4 \times 6 \times \frac{\sqrt{2}}{2} = \boxed{6\sqrt{2} \text{ cm}^2}$ (d) $\cos 135^\circ = -\frac{\sqrt{2}}{2}$: $\sqrt{16+36+24\sqrt{2}} = \boxed{\sqrt{52+24\sqrt{2}} \text{ cm}}$

Exact values of $\sin$ and $\cos$ at 45° , 60° and 135° keep the answers exact.

C

Reasoning solutions

Your wording can be different. Check that your reason makes the same point.

CORE

- C1** $\frac{1}{2}ab \sin C$ needs the angle *between* the two sides. The angle between AB and BC is angle B , not angle A : 50° is between AB and AC

the 50° angle is not between the 6 cm and 8 cm sides

- C2** $y = \sin x$ is symmetrical about $x = 90^\circ$, so the line $y = \frac{1}{2}$ meets it twice between 0° and 180°

$\boxed{x = 30^\circ}$ and $\boxed{x = 150^\circ}$

- C3** The largest angle is opposite 13: $\cos \theta = \frac{49+64-169}{2 \times 7 \times 8} = -\frac{56}{112} = -\frac{1}{2}$, so $\theta = 120^\circ$

Area = $\frac{1}{2} \times 7 \times 8 \times \sin 120^\circ = 28 \times \frac{\sqrt{3}}{2} = \boxed{14\sqrt{3} \text{ cm}^2}$

- C4** Multiplying x by 2 squashes the graph horizontally: the period is $\frac{360^\circ}{2}$, not $360^\circ \times 2$ **period 180°**

There are two full waves in 0° to 360° , and $y = 0.3$ crosses each twice: **4 solutions**



STRETCH

C5 $\frac{\sin B}{11} = \frac{\sin 38^\circ}{8}$, so $\sin B \approx 0.8465$, and $B \approx 57.8^\circ$ or $180^\circ - 57.8^\circ$

$38^\circ + 122.2^\circ < 180^\circ$, so the obtuse angle also gives a triangle: $B \approx 57.8^\circ$ or $B \approx 122.2^\circ$

C6 The perpendicular from A to BC makes a right-angled triangle with hypotenuse $AC = b$ and angle C : $\sin C = \frac{h}{b}$, so $h = b \sin C$

Area = $\frac{1}{2} \times \text{base} \times \text{height} = \frac{1}{2} \times a \times b \sin C$ **as required**

C7 The largest angle would be opposite 8: $\cos \theta = \frac{9+16-64}{24} = -\frac{39}{24}$

This is less than -1 , and $\cos \theta$ is never less than -1 , so no such angle exists.

$3+4 < 8$: the two shorter sides must add to more than the longest side

C8 (a) Area = $\frac{1}{2} \times 6 \times 10 \sin \theta = 30 \sin \theta$. The greatest value of $\sin \theta$ is 1, at $\theta = 90^\circ$: greatest area **30 cm^2**

(b) $30 \sin \theta = 15\sqrt{3}$, so $\sin \theta = \frac{\sqrt{3}}{2}$: $\theta = 60^\circ$ or $\theta = 120^\circ$

(c) $c^2 = 36 + 100 - 120 \cos 120^\circ = 136 + 60 = 196$: **14 cm**

D

Problem solving solutions

There is often more than one correct method. Any complete, correct method scores full marks.

CORE

D1 (a) $BC^2 = 7^2 + 5^2 - 2 \times 7 \times 5 \cos 60^\circ = 49 + 25 - 35 = 39$, so $BC = \sqrt{39} \text{ cm}$

as required

(b) $\frac{1}{2} \times 7 \times 5 \times \frac{\sqrt{3}}{2} = \frac{35\sqrt{3}}{4} \text{ cm}^2$

D2 (a) $\frac{\sin R}{9} = \frac{\sin 35^\circ}{6}$, so $\sin R \approx 0.8604$ and $R \approx 59.36^\circ$ or $180^\circ - 59.36^\circ$

The figure says angle PRQ is obtuse: **120.64°**

(b) Angle $PQR \approx 180^\circ - 35^\circ - 120.64^\circ = 24.36^\circ$; $PR = \frac{6 \sin 24.36^\circ}{\sin 35^\circ} \approx \textbf{4.31 cm}$



- D3** (a) $x = 90^\circ$ and $x = 270^\circ$
 (b) The line $y = 2$ crosses each of the two rising branches once: 2
 (c) $\tan$ repeats every 180° : $63.4^\circ + 180^\circ = 243.4^\circ$

- D4** (a) $\frac{1}{2} \times 8 \times 6 \times \sin A = 12\sqrt{3}$, so $24 \sin A = 12\sqrt{3}$ and $\sin A = \frac{\sqrt{3}}{2}$ **as required**
 (b) A is obtuse, so $A = 120^\circ$: $BC^2 = 64 + 36 - 96 \cos 120^\circ = 100 + 48 = 148$
 $BC = \sqrt{148} = 2\sqrt{37} \text{ m}$

STRETCH

- D5** (a) $AC^2 = 25 + 64 - 80 \cos 120^\circ = 129$: $AC \approx 11.4 \text{ m}$ (keep $\sqrt{129}$)
 (b) $\cos D = \frac{100 + 81 - 129}{2 \times 10 \times 9} = \frac{52}{180}$: $D \approx 73.2^\circ$
 (c) $\frac{1}{2} \times 5 \times 8 \sin 120^\circ + \frac{1}{2} \times 10 \times 9 \sin D \approx 17.32 + 43.08$: 60.4 m^2
- D6** (a) At B the bearing back to A is $060^\circ + 180^\circ = 240^\circ$, so angle $ABC = 240^\circ - 170^\circ = 70^\circ$
 $AC^2 = 144 + 64 - 192 \cos 70^\circ \approx 142.33$: $AC \approx 11.9 \text{ km}$
 (b) $\frac{\sin A}{8} = \frac{\sin 70^\circ}{11.93}$, so angle $BAC \approx 39.06^\circ$ (acute, opposite the shorter side)
 Bearing = $60^\circ + 39.06^\circ$: 099.1°
- D7** (a) $3 \cos 30^\circ = \frac{3\sqrt{3}}{2}$: $P\left(0, \frac{3\sqrt{3}}{2}\right)$
 (b) minimum when $x + 30^\circ = 180^\circ$: $Q(150^\circ, -3)$
 (c) 2 : $y = 2$ lies between -3 and $\frac{3\sqrt{3}}{2} \approx 2.6$, so it crosses the curve once on the way down from P and once on the way back up after Q
 (d) The maximum of $3 \cos(x + 30^\circ)$ is 3 , so $3 + k = 7$: $k = 4$; the minimum is $-3 + 4 = 1$
- D8** (a) The maximum value is 3 , and two cycles in 360° means a period of 180° : $A = 3$, $b = 2$
 (b) $y = 1.5$ crosses each of the two waves twice: 4
 (c) $\sin 2x = \frac{1}{2}$, with $0^\circ \leq 2x \leq 720^\circ$: $2x = 30^\circ, 150^\circ, 390^\circ, 510^\circ$
 $x = 15^\circ, 75^\circ, 195^\circ, 255^\circ$



E

Exam practice solutions

Mark-scheme notes are shown on the right of each line.

CORE

Exam questions for everyone

E1 (a) Angle $ACB = 180^\circ - 38^\circ - 72^\circ = 70^\circ$

$$AC = \frac{120 \sin 38^\circ}{\sin 70^\circ} \approx \boxed{78.6 \text{ m}}$$

M1: angle C and the sine rule; A1

(b) Area = $\frac{1}{2} \times 120 \times AC \times \sin 72^\circ \approx 4486$

M1: $\frac{1}{2}ab \sin C$ with the included angle

$$\boxed{4490 \text{ m}^2}$$

A1

$$4486 \div 250 \approx 17.9, \text{ so } \boxed{18 \text{ bags}}$$

B1ft: rounded up

E2 (a) Maximum when $x - 30^\circ = 90^\circ$: $\boxed{P(120^\circ, 4)}$

B1

$$x - 30^\circ = 180^\circ: \boxed{Q(210^\circ, 0)}$$

B1

(b) $4 \sin(-30^\circ) = -2$: $\boxed{R(0, -2)}$

B1

(c) The minimum value is -4 , so $-4 + k = 0$: $\boxed{k = 4}$

B1

(d) $\boxed{2}$: the curve goes down from Q to its minimum -4 at $x = 300^\circ$ and back up to -2 at $x = 360^\circ$, crossing $y = -3$ once each way

M1 A1

STRETCH

E3 (a) $(2\sqrt{7})^2 = x^2 + (x+2)^2 - 2x(x+2) \cos 60^\circ$

M1: cosine rule with the 60° angle

$$28 = x^2 + x^2 + 4x + 4 - x^2 - 2x$$

dM1: expands, $\cos 60^\circ = \frac{1}{2}$

$$x^2 + 2x - 24 = 0 \quad \boxed{\text{as required}}$$

A1*

(b) $(x+6)(x-4) = 0$; a length is positive: $\boxed{x = 4}$

B1

(c) $\frac{1}{2} \times 4 \times 6 \times \sin 60^\circ = 12 \times \frac{\sqrt{3}}{2} = \boxed{6\sqrt{3} \text{ cm}^2}$

M1 A1

(d) $\frac{1}{2} \times 2\sqrt{7} \times h = 6\sqrt{3}$, so $h = \frac{6\sqrt{3}}{\sqrt{7}} = \boxed{\frac{6\sqrt{21}}{7} \text{ cm}}$

M1 A1

