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INTERNATIONAL A LEVEL MATHEMATICS • FURTHER PURE 1 (WFM01)

Further Pure 1 Practice Paper 1

Mark Scheme

AIMING FOR B

Written in the style of the Pearson Edexcel International Advanced Level paper (WFM01/01) • 75 marks

Every question marked line by line the way an examiner marks it, with notes on the common errors

Mark every line, not just the answer. Method marks are given for a correct method even when the final answer is wrong. If you lost marks, find the *first* line where your working differs from the scheme. That is the thing to fix.

Code	Meaning
M1	Method mark: for knowing a correct method and attempting to apply it. Small slips in the working do not lose this mark.
dM1	Dependent method mark: can only be given if the method mark before it has been given.
A1	Accuracy mark: for a correct answer or line of working. It depends on the method mark before it.
B1	Independent mark: for a correct result or statement, whatever method was used.
*	The answer is given in the question. Every step must be shown, and the answer must follow from correct working (cso).
cao / cso	Correct answer only / correct solution only, with no errors anywhere in the working.
ft / oe	Follow through from your own earlier answer / or any correct equivalent form.

"Solutions relying on calculator technology are not acceptable." Questions 4 and 8 say this. There, a correct final answer with no algebraic working scores **no marks**. Elsewhere a calculator may be used, but the method must still be seen.

Rough grade guide (out of 75): A **63** • B **56** • C **49**

Based on recent WFM01 grade boundaries. Real boundaries change every series, so treat this as a guide, not a grade.

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What to revise next

Write in your score for each question. Where you dropped marks, tap the chapter to practise ten more exam questions on it.

Q	What it tested	Marks	Practise it	Score
1	A determinant with a parameter; non-singular for all k; the inverse	6	Ch 5 Matrices	
2	A sign change, one Newton–Raphson step and linear interpolation	7	Ch 3 Numerical Solutions	
3	Sum and product of roots; a new equation with integer coefficients	7	Ch 2 Roots of Quadratic Equations	
4	An equation with z^*; modulus, argument and an Argand diagram	8	Ch 1 Complex Numbers	
5	Standard series results; a sum from $n+1$ to $2n$	8	Ch 7 Series	
6	Rotation and enlargement matrices; a combined transformation; area	9	Ch 6 Transformations; Ch 5 Matrices	
7	The normal to a rectangular hyperbola; intercepts and an area	9	Ch 4 Coordinate Systems	
8	Complex roots of a quartic with real coefficients	10	Ch 1 Complex Numbers	
9	Proof by induction: a series and divisibility	11	Ch 8 Proof	
	Total	**75**		

Where did the marks go? In Further Pure 1 most lost marks are about presentation, not knowledge: a conclusion not written ("so $\mathbf{A}$ is non-singular"), a centre of rotation left out, a sign of a root sum reversed, or an induction without a closing statement. If you lost the first mark of a question, revise that chapter's key skills page. If you lost a final mark, check the wording the scheme asked for.

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Question 1

6 marks

Part	Scheme	Marks
(a)	$k(k+1)-3(k-2) = \boxed{k^2-2k+6}$	B1 (1)
(b)	$k^2-2k+6 = (k-1)^2+5$ (or discriminant $4-24=-20<0$) $(k-1)^2+5 \geqslant 5 > 0$, so $\det\mathbf{A} \neq 0$ and $\mathbf{A}$ is non-singular for all real k *A conclusion is needed for the A mark.*	M1 A1 (2)
(c)	$\frac{1}{\det\mathbf{A}} \times \ldots$ $\begin{pmatrix} k+1 & -3 \\ 2-k & k \end{pmatrix}$ $\boxed{\mathbf{A}^{-1} = \frac{1}{k^2-2k+6}\begin{pmatrix} k+1 & -3 \\ 2-k & k \end{pmatrix}}$	M1 A1 A1 (3)
		Total for Question 1: 6 marks

Question 2

7 marks

Part	Scheme	Marks
(a)	$f(1.8) = -1.279\ldots$ and $f(2) = 1$ Change of sign and f is continuous on [1.8, 2], so α lies in [1.8, 2] *The A mark needs both the sign change and continuity.*	M1 A1 (2)
(b)	$f'(x) = 3x^2 + 2x^{-2}$ $2 - \frac{f(2)}{f'(2)} = 2 - \frac{1}{12.5}$ $\boxed{1.92}$	B1 M1 A1 (3)
(c)	$\frac{\alpha-1.8}{1.279\ldots} = \frac{2-\alpha}{1}$ (or $1.8+0.2\times\frac{1.279\ldots}{2.279\ldots}$) $\boxed{1.912}$ *Using -1.279 in place of 1.279 gives a value outside the interval: M0.*	M1 A1 (2)
		Total for Question 2: 7 marks

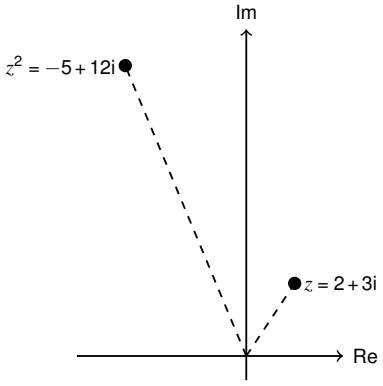
Question 3

7 marks

Part	Scheme	Marks
(a)	$\alpha+\beta=\frac{3}{2},\ \alpha\beta=\frac{5}{2}$	B1
		(1)
(b)	$(\alpha+\beta)^2-2\alpha\beta=\frac{9}{4}-5$	M1
	$\boxed{-\frac{11}{4}}$	A1
		(2)
(c)	Sum: $\alpha+\beta+\dfrac{\alpha+\beta}{\alpha\beta}=\frac{3}{2}+\frac{3}{5}=\frac{21}{10}$	M1 A1
	Product: $\alpha\beta+2+\dfrac{1}{\alpha\beta}=\frac{5}{2}+2+\frac{2}{5}=\frac{49}{10}$	M1
	$\boxed{10x^2-21x+49=0}$	A1
	The equation must have integer coefficients and "= 0".	
		(4)
		Total for Question 3: 7 marks

Question 4

8 marks

Part	Scheme	Marks
(a)	$z=a+b\text{i},\ z^*=a-b\text{i}$: $a+b\text{i}+2\text{i}(a-b\text{i})=8+7\text{i}$	M1
	Real: $a+2b=8$; imaginary: $2a+b=7$	M1 A1
	$\boxed{z=2+3\text{i}}$	A1
		(4)
(b)	$\lvert z\rvert=\sqrt{13}$	B1
	$\arg z=\arctan\frac{3}{2}=\boxed{0.983}$	B1
		(2)
(c)	$z^2=-5+12\text{i}$	B1
	Both points plotted in the correct quadrants	
		B1
		(2)
		Total for Question 4: 8 marks

Question 5

8 marks

Part	Scheme	Marks
(a)	$\sum r^2 + 3\sum r = \frac{1}{6}n(n+1)(2n+1) + \frac{3}{2}n(n+1)$	M1
	$= \frac{1}{6}n(n+1)[(2n+1)+9]$	M1
	$= \frac{1}{6}n(n+1)(2n+10)$	A1
	$= \frac{1}{3}n(n+1)(n+5)$	A1*
		(4)
(b)	$\sum_1^{2n} - \sum_1^{n}$: $\frac{1}{3}(2n)(2n+1)(2n+5) - \frac{1}{3}n(n+1)(n+5)$	M1
	$\frac{1}{3}n\left[2(4n^2+12n+5) - (n^2+6n+5)\right]$	M1
	$\frac{1}{3}n(7n^2+18n+5)$	A1
	$\boxed{a=7,\ b=18,\ c=5}$	A1
	Using $\sum_1^{2n} - \sum_1^{n+1}$ loses the last two marks: the lower limit $n+1$ means subtracting the sum to n.	
		(4)
		Total for Question 5: 8 marks

Question 6

9 marks

Part	Scheme	Marks
(a)	Rotation 90° anticlockwise about (0, 0)	B1 B1 (2)
(b)	$\begin{pmatrix} 3 & 0 \\ 0 & 3 \end{pmatrix}$	B1 (1)
(c)	$\mathbf{R} = \mathbf{QP}$ $\begin{pmatrix} 0 & -3 \\ 3 & 0 \end{pmatrix}$ *$\mathbf{PQ}$ happens to give the same matrix here, but the order must be shown as $\mathbf{QP}$ for the method mark.*	M1 A1 (2)
(d)	$\det \mathbf{R} = 9$ $45\,\text{cm}^2$	M1 A1 (2)
(e)	$\begin{pmatrix} 0 & -3 \\ 3 & 0 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -6 \\ 12 \end{pmatrix}$, so $-3y = -6$, $3x = 12$ (or use $\mathbf{R}^{-1}$) (4, 2)	M1 A1 (2)
		Total for Question 6: 9 marks

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Question 7

9 marks

Part	Scheme	Marks
(a)	$y = 16x^{-1}$, $\dfrac{dy}{dx} = -16x^{-2} = -\dfrac{1}{t^2}$ at P	M1 A1
	Normal gradient t^2: $y - \dfrac{4}{t} = t^2(x - 4t)$	M1
	$ty - 4 = t^3x - 4t^4$, so $t^3x - ty = 4t^4 - 4$	A1*
		(4)
(b)	$t = 2$: $8x - 2y = 60$	M1
	$\boxed{A\left(\frac{15}{2}, 0\right)}$	A1
	$\boxed{B(0, -30)}$	A1
		(3)
(c)	$\frac{1}{2} \times \frac{15}{2} \times 30$	M1
	$\boxed{112.5}$	A1
	Lengths are positive: using -30 gives a negative area.	
		(2)
		Total for Question 7: 9 marks

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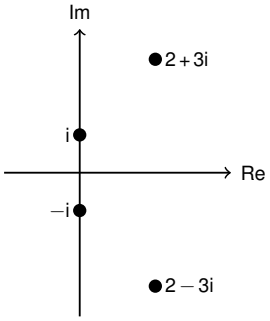
Question 8

10 marks

Part	Scheme	Marks
(a)	$\boxed{2+3\text{i}}$	B1 (1)
(b)	$(z-(2-3\text{i}))(z-(2+3\text{i})) = (z-2)^2+9$ $\boxed{z^2-4z+13}$	M1 A1 (2)
(c)	$z^4-4z^3+14z^2-4z+13 = (z^2-4z+13)(z^2+az+b)$ z^3: $a-4=-4$, so $a=0$; constant: $13b=13$, so $b=1$ Second factor z^2+1 $z^2=-1$ $\boxed{z=\text{i},\ z=-\text{i}}$ *Long division is acceptable but must be complete. A calculator solution with no working scores no marks.*	M1 A1 A1 M1 A1 (5)
(d)	$2\pm3\text{i}$ plotted symmetrically about the real axis $\pm\text{i}$ on the imaginary axis, closer to the origin  	B1 B1 (2)
		Total for Question 8: 10 marks

Question 9

11 marks

Part	Scheme	Marks
(i)	$n = 1$: LHS = 1, RHS = $\frac{1}{3}(1)(1)(3) = 1$, so true for $n = 1$	B1
	Assume true for $n = k$: $\sum_{r=1}^{k+1}(2r-1)^2 = \frac{1}{3}k(2k-1)(2k+1) + (2k+1)^2$	M1
	$= \frac{1}{3}(2k+1)\left[2k^2 - k + 6k + 3\right] = \frac{1}{3}(2k+1)(2k^2+5k+3)$	dM1
	$= \frac{1}{3}(k+1)(2k+1)(2k+3) = \frac{1}{3}(k+1)\left(2(k+1)-1\right)\left(2(k+1)+1\right)$	A1
	True for $n = 1$, and true for $n = k$ implies true for $n = k+1$, so true for all positive integers n	A1
		(5)
(ii)	$n = 1$: $3^2 + 7 = 16 = 2 \times 8$, so true for $n = 1$	B1
	Assume $f(k) = 3^{2k} + 7$ is divisible by 8: $f(k+1) - f(k) = 9 \times 3^{2k} - 3^{2k}$	M1
	$= 8 \times 3^{2k}$	A1
	$f(k+1) = f(k) + 8 \times 3^{2k}$	M1
	Both terms are divisible by 8, so $f(k+1)$ is divisible by 8	A1
	True for $n = 1$, and true for $n = k$ implies true for $n = k+1$, so true for all positive integers n	A1
		(6)
		Total for Question 9: 11 marks

TOTAL FOR PAPER: 75 MARKS

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