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INTERNATIONAL A LEVEL MATHEMATICS • PURE 4 (WMA14)

Pure 4 Practice Paper 3

Edexcel IAL Pure Mathematics 4

AIMING FOR A/A*

Written in the style of the Pearson Edexcel International Advanced Level paper (WMA14/01) • Calculator allowed

A full 75-mark paper set above the hardest recent series: long questions, unfamiliar settings and very few steps given

Time	Total	You must have
1 hour 30 minutes	**75 marks**	**Mathematical Formulae and Statistical Tables, calculator**

Instructions

- Use black ink or ball-point pen. Pencil may be used for diagrams and sketches.
- Answer **all** questions, in the spaces provided.
- Show enough working to make your method clear. Answers without working may not gain full credit.
- Give inexact answers to **three significant figures** unless the question says otherwise.

Information

- There are 8 questions in this paper. The total mark is 75.
- The marks for each question are shown in brackets.

Advice

- Sit the paper in one go, in 1 hour 30 minutes, with the mark scheme closed.
- Read each question carefully and try every question. Check your answers if you have time.
- Then mark it line by line with the mark scheme and fill in your score.

Track your score

Q	Topic	Marks	Score
1	The general term of a series	9	
2	Parametric area and volume	10	
3	A reaction modelled by a differential equation	10	
4	An equilateral triangle on a line	9	
5	Proofs about rational roots	9	
6	Tangents to the folium $x^3 + y^3 = 3xy$	9	
7	An equiangular spiral	9	
8	Filling a leaking container	10	
	Total	**75**	

Rough grade guide (out of 75): A* **57** • A **50** • B **42** • C **34**

Based on recent WMA14 grade boundaries. Real boundaries change every series, so treat this as a guide, not a grade.

Mark scheme | Exam questions by topic | More practice papers

Name ..... Date .....

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1. In this question you must show all stages of your working.

Solutions relying on calculator technology are not acceptable.

$$f(x) = \frac{3-x}{(1-x)^2(2-x)}$$

(a) Find, in its simplest form, the coefficient of x^n in the series expansion of $f(x)$ in ascending powers of x.

(5)

(b) State the range of values of x for which the expansion is valid.

(1)

(c) Hence find the exact value of

$$\sum_{n=0}^{\infty} \frac{2n+1}{2^n}$$

(3)

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Question 1 continued

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(Total for Question 1 is 9 marks)

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2. In this question you must show all stages of your working.

Solutions relying on calculator technology are not acceptable.

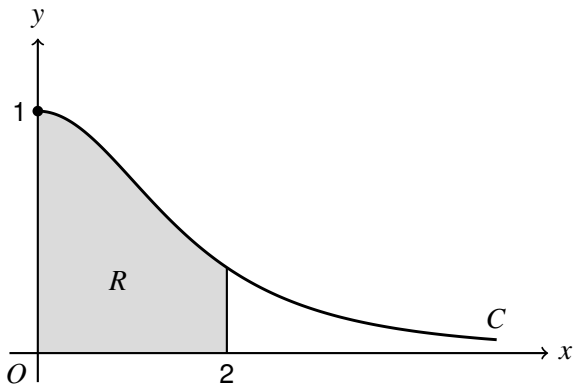

Figure 1

Figure 1 shows the curve C with parametric equations

$$x = 2\tan\theta, \qquad y = \cos^3\theta \qquad 0 \leqslant \theta < \frac{\pi}{2}$$

The region R, shown shaded in Figure 1, is bounded by C, the coordinate axes and the line $x = 2$

(a) Find the exact area of R.

(4)

The region R is rotated through 2π radians about the x-axis to form a solid of revolution.

(b) Find the exact volume of the solid.

(6)

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Question 2 continued

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(Total for Question 2 is 10 marks)

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3. In this question you must show all stages of your working.

Solutions relying on calculator technology are not acceptable.

In a chemical reaction, x grams of a product have formed t minutes after the reaction starts.

The reaction is modelled by the differential equation

$$\frac{dx}{dt} = \frac{(4-x)(2-x)}{8} \qquad 0 \leqslant x < 2$$

At the start of the reaction no product has formed.

(a) Find x in terms of t.

(6)

(b) Sketch the graph of x against t, stating the equation of its asymptote.

(2)

(c) Find the exact time at which 1 gram of the product has formed.

(2)

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Question 3 continued

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(Total for Question 3 is 10 marks)

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4. In this question you must show all stages of your working.

Solutions relying on calculator technology are not acceptable.

The line l has equation

$$\mathbf{r} = \begin{pmatrix} 1 \\ -2 \\ 3 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ 1 \\ -1 \end{pmatrix}$$

The point A has coordinates (3, 2, 5).

The points P and Q lie on l and the triangle APQ is equilateral.

(a) Find the coordinates of P and of Q.

(7)

(b) Find the exact area of the triangle APQ.

(2)

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Question 4 continued

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(Total for Question 4 is 9 marks)

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5. In this question you must show all stages of your working.

Solutions relying on calculator technology are not acceptable.

(a) Given that a, b and c are odd integers, prove by contradiction that the equation

$$ax^2 + bx + c = 0$$

has no rational solutions.

(6)

(b) The equation $x^3 + x + 1 = 0$ has exactly one real root α.

Prove by contradiction that α is irrational.

(3)

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Question 5 continued

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(Total for Question 5 is 9 marks)

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6. In this question you must show all stages of your working.

Solutions relying on calculator technology are not acceptable.

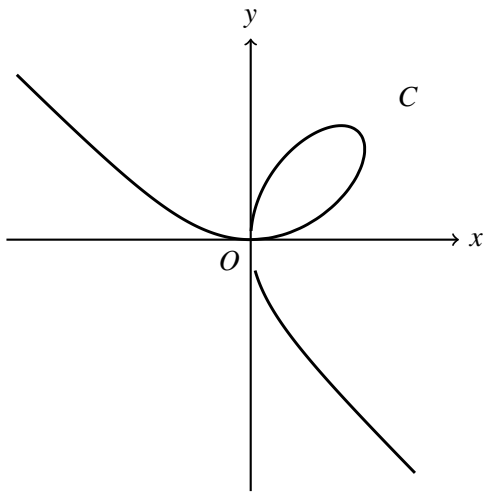

Figure 2

Figure 2 shows the curve C with equation

$$x^3 + y^3 = 3xy$$

(a) Find $\frac{dy}{dx}$ in terms of x and y.

(3)

(b) Find the exact coordinates of the point on C, other than O, at which the tangent to C is parallel to the x-axis.

(4)

(c) Hence write down the exact coordinates of the point on C, other than O, at which the tangent to C is parallel to the y-axis, giving a reason for your answer.

(2)

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Question 6 continued

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7. In this question you must show all stages of your working.

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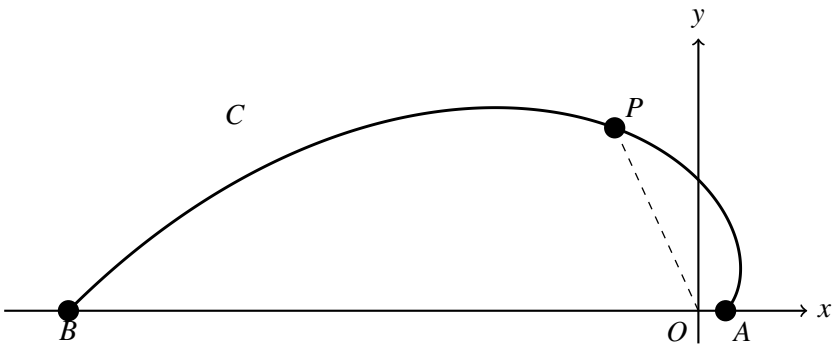

Figure 3

Figure 3 shows the curve C with parametric equations

$$x = e^t \cos t, \qquad y = e^t \sin t \qquad 0 \leqslant t \leqslant \pi$$

The curve starts at A, where $t = 0$, and ends at B, where $t = \pi$

The point P on C has parameter t.

(a) Find $\frac{dy}{dx}$ in terms of t.

(2)

(b) Prove that, for every point P on C, the angle between the tangent to C at P and the line OP is 45°

(4)

(c) Find the exact coordinates of the point on C at which the tangent is parallel to the x-axis.

(3)

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Question 7 continued

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(Total for Question 7 is 9 marks)

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8. In this question you must show all stages of your working.

Solutions relying on calculator technology are not acceptable.

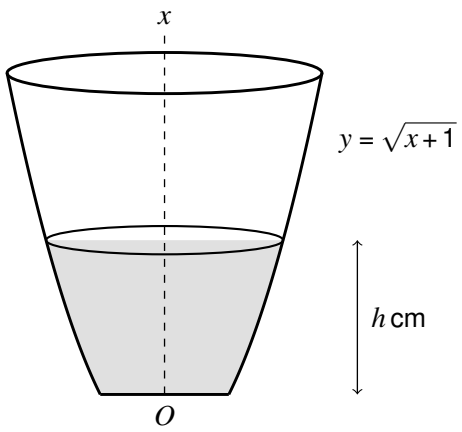

Figure 4

A container is formed by rotating the curve with equation $y = \sqrt{x+1}$, $0 \leqslant x \leqslant 5$, through 2π radians about the x-axis and closing the end at $x = 0$

The container stands on this flat end with the x-axis vertical, as shown in Figure 4. Lengths are in centimetres.

When the depth of water in the container is h cm, the volume of water is V cm^3.

(a) Show that $V = \pi\left(\frac{1}{2}h^2 + h\right)$

(2)

The container is initially empty. Water is poured in at a constant rate of 4π cm^3 s^{-1}, and at the same time water leaks out at a rate of πh cm^3 s^{-1}.

(b) Find the time t seconds, in terms of h, for the depth of the water to reach h cm.

(6)

(c) Find the time taken for the depth to reach 3 cm, and explain why, according to the model, the depth never reaches 4 cm.

(2)

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Question 8 continued

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(Total for Question 8 is 10 marks)

TOTAL FOR PAPER IS 75 MARKS

END

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