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INTERNATIONAL A LEVEL MATHEMATICS • FURTHER PURE MATHEMATICS 3

Further Pure 3

Chapter 1: Hyperbolic Functions

Exam Questions by Topic

Pearson Edexcel International Advanced Level (WFM03/01) • Calculator allowed

Multi-step exam questions at the difficulty of the real WFM03 paper, with a full mark scheme

Chapters (tap a chapter to jump to it)		Qs	Marks	Page	My score
1	**Hyperbolic Functions** Definitions, graphs, inverse functions, identities and equations	10	80	2	
2	**Further Coordinate Systems** Ellipses and hyperbolas, foci and directrices, tangents, normals and loci	10	80	separate file	
3	**Differentiation** Hyperbolic and inverse trigonometric and hyperbolic functions	10	80	separate file	
4	**Integration** Substitutions, reduction formulae, arc length and surface area	10	80	separate file	
5	**Vectors** Vector product, triple products, lines and planes, distances and angles	10	80	separate file	
6	**Further Matrix Algebra** 3×3 matrices, inverses, eigenvalues and eigenvectors, diagonalisation	10	80	separate file	

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Name ..... Class ..... Date .....

CHAPTER 1 • KEY SKILLS

1 Hyperbolic Functions

Specification FP3: Hyperbolic functions • 10 questions • 80 marks • about 96 minutes

1 Definitions

$\sinh x = \frac{1}{2}\left(e^{x} - e^{-x}\right)$,

$\cosh x = \frac{1}{2}\left(e^{x} + e^{-x}\right)$, $\tanh x = \dfrac{\sinh x}{\cosh x}$; sech, cosech, coth are reciprocals.

2 Graphs and ranges

$\cosh x \geqslant 1$; $\sinh x$ takes all real values; $-1 < \tanh x < 1$; $0 < \operatorname{sech} x \leqslant 1$

3 Proving identities

When told to use the definitions, write them in full and show a line of working before the result.

4 Osborn's rule

From a trigonometric identity, change the sign of any product of two sines:

$\cosh 2x = 2\cosh^2 x - 1 = 1 + 2\sinh^2 x$

5 Inverse functions

$\operatorname{arsinh} x = \ln\left(x + \sqrt{x^2 + 1}\right)$,

$\operatorname{arcosh} x = \ln\left(x + \sqrt{x^2 - 1}\right)$ $(x \geqslant 1)$,

$\operatorname{artanh} x = \frac{1}{2}\ln\dfrac{1+x}{1-x}$

6 Exponential equations

For $a\cosh x + b\sinh x = c$, substitute the definitions and multiply by e^{x}: a quadratic in e^{x}

7 Quadratics in cosh or sinh

Use an identity to get one function, solve, then reject $\cosh x < 1$ (and $|\tanh x| \geqslant 1$).

8 Both signs

$\cosh x = k > 1$ gives $x = \pm\ln\left(k + \sqrt{k^2 - 1}\right)$: two answers.

9 Checking roots

$e^{x} > 0$ always. Squaring an equation can introduce extra roots, so substitute back.

Skills map every skill above is tested at least once; interleaved skills come from GCSE or other chapters

Skill	Q1	Q2	Q3	Q4	Q5	Q6	Q7	Q8	Q9	Q10
Marks	6	6	6	6	6	10	10	10	10	10
1 Definitions	•	•	•			•	•	•	•	•
2 Graphs and ranges					•					•
3 Proving identities	•	•				•		•	•	•
4 Osborn's rule				•		•		•		
5 Inverse functions		•	•	•	•	•		•	•	•
6 Exponential equations			•				•			
7 Quadratics in cosh or sinh				•		•	•	•	•	
8 Both signs				•		•				•
9 Checking roots			•	•	•	•	•		•	•
Interleaved: exponentials and logarithms (P2, P3)	•	•	•	•		•	•	•	•	•
Interleaved: quadratic equations and surds (P1)		•	•	•		•	•	•	•	•
Interleaved: binomial expansion and $R\cos(x - \alpha)$ (P2, P3)						•				•

Exam difficulty. Questions 1 to 5 check each skill on its own. Questions 6 to 10 are multi-step questions built like the last questions on a real WFM03 paper, where one part leads into the next and skills from different topics are combined.

Must learn. Follow the method the question names: "using the definitions" means exponentials, written in full. Give exact answers as logarithms, and state why any root is rejected.

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1. **(a)** Using the definitions of $\sinh x$ and $\cosh x$ in terms of exponentials, prove that $\cosh^2 x - \sinh^2 x \equiv 1$

(3)

(b) Find the exact values of $\sinh(\ln 3)$, $\cosh(\ln 3)$ and $\tanh(\ln 3)$.

(3)

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Question 1 continued

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(Total for Question 1 is 6 marks)

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2. **(a)** Show that $\operatorname{arcosh} x = \ln\left(x + \sqrt{x^2 - 1}\right)$, $x \geqslant 1$

(4)

(b) Hence write down the exact value of $\operatorname{arcosh} 2$

(1)

(c) Find the exact value of $\operatorname{artanh} \frac{1}{2}$

(1)

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Question 2 continued

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(Total for Question 2 is 6 marks)

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3. Using the definitions of $\cosh x$ and $\sinh x$ in terms of exponentials, solve

$$3\cosh x + \sinh x = 5$$

giving your answers as exact logarithms.

(6)

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Question 3 continued

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4. Solve the equation

$$\cosh 2x - 5\cosh x + 4 = 0$$

giving your answers in exact form.

(6)

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Question 4 continued

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5. **(a)** On the same axes, sketch the graphs of $y = \cosh x$ and $y = \sinh x$.

(2)

(b) Find the range of the function $f(x) = 3 - 2\operatorname{sech} x$, $x \in \mathbb{R}$

(2)

(c) Explain why the equation $\cosh x = \frac{1}{2}$ has no real solutions.

(1)

(d) State the domain and range of $y = \operatorname{arcosh} x$

(1)

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Question 5 continued

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6. **(a)** Using the definition of $\cosh x$ in terms of exponentials, show that

$8\cosh^4 x = \cosh 4x + 4\cosh 2x + 3$

(4)

(b) Hence solve $\cosh 4x + 4\cosh 2x = 15$, giving your answers in the form $\pm \ln k$, where k is an exact surd expression.

(6)

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Question 6 continued

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Question 6 continued

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7. **(a)** Using exponentials, solve $4\tanh x - \operatorname{sech} x = 1$, giving your answer as an exact logarithm.

(6)

(b) A student rearranges to $4\sinh x = 1 + \cosh x$, squares both sides and obtains $\cosh x = \frac{17}{15}$. Explain why this method gives an extra solution, and identify it.

(2)

(c) Verify your answer to part (a) by finding the exact values of $\tanh x$ and $\operatorname{sech} x$.

(2)

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8. **(a)** Show that $\operatorname{arsinh} x = \ln\left(x + \sqrt{x^2 + 1}\right)$

(4)

(b) Solve $2\cosh^2 x - \sinh x = 7$, giving each answer in the form $\ln\left(p + \sqrt{q}\right)$

(4)

(c) Give your answers to part (b) to 3 decimal places.

(2)

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Question 8 continued

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9. **(a)** Starting from the definitions of $\sinh x$ and $\cosh x$ in terms of exponentials, prove that

$$\tanh 2x = \frac{2\tanh x}{1+\tanh^2 x}$$

(4)

(b) Hence solve $\tanh 2x = \frac{4}{5}$, giving your answer as an exact logarithm.

(4)

(c) Check your answer by finding $\text{artanh}\,\frac{4}{5}$ in logarithmic form.

(2)

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Question 9 continued

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10. **(a)** Express $5\cosh x - 3\sinh x$ in the form $R\cosh(x - \alpha)$, where $R > 0$, giving the exact values of R and α.

(4)

(b) Hence state the minimum value of $5\cosh x - 3\sinh x$ and the exact value of x at which it occurs.

(2)

(c) Solve $5\cosh x - 3\sinh x = 5$, giving your answers in exact form.

(4)

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TOTAL FOR CHAPTER 1 IS 80 MARKS

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