eClassroom

A-LEVEL MATHEMATICS • STATISTICS 2

Poisson Distributions

Worked Solutions

Pearson Edexcel International A Level (WST02) • Calculator and statistical tables allowed: show every step

Full worked solutions for the free Poisson Distributions worksheet, A-Level and International A-Level Maths

A **Examples** & Your turn page 2

B **Fluency** core skills page 2

C **Reasoning** explain it page 4

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Tap a section to jump straight to it

Full worked solutions for every question on the Statistics 2 Poisson Distributions worksheet, in the same order and with the same question numbers.

How to mark your work. Tick each step you got right, not just the final answer. At A-level most marks are for method, so a wrong final answer can still earn most of the marks. If you lost marks, find the *first* line where your working is different from ours. That is the thing to fix.

In Section E: **M** = method mark, **dM** = method mark that depends on the previous M mark, **A** = accuracy mark (needs the M mark before it), **B** = independent mark.

Question sheet | Exam questions by topic | Statistics 2 exam questions | More worksheets

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A Your turn answers

Compare each line of your working with the solution.

CORE

A1 $\dfrac{e^{-2.5}(2.5)^4}{4!}$: **0.1336**

A2 $1 - P(X \leqslant 6) = 1 - 0.8311$: **0.1689**; $P(X \leqslant 6) - P(X \leqslant 1) = 0.8311 - 0.0611$: **0.7700**

A3 45 minutes: Po(6); $P(X \leqslant 2)$: **0.0620**

STRETCH

A4 $X + Y \sim$ Po(6); $P(X + Y \leqslant 4)$: **0.2851**

A5 $\dfrac{e^{-\lambda}\lambda^2}{2} = 2e^{-\lambda}\lambda$, so $\lambda^2 = 4\lambda$: $\lambda = 4$

A6 $\dfrac{P(X \leqslant 1)}{P(X \leqslant 3)} = \dfrac{0.1991}{0.6472}$: **0.3076**

A7 p = P(2 or more errors) $= 1 - P(X \leqslant 1) = 1 - 0.8088 = 0.1912$; $1 - (0.8088)^{12}$: **0.9216**

B Fluency solutions

One mark for each correct part. The notes in italics explain why each sequence was chosen.

CORE

B1 **(a)** **0.3423** **(b)** $P(X \leqslant 2)$: **0.1736** **(c)** $1 - P(X \leqslant 2)$: **0.8264** **(d)** $1 - P(X \leqslant 3)$: **0.6577**

Rewrite every inequality as "$\leqslant$" before using the tables.

B2 **(a)** **0.0613** **(b)** **0.1804** **(c)** **0.2240** **(d)** **0.1954**

$P(X = 3)$ *is largest when* $\lambda = 3$

B3 **(a)** Po(2): **0.2707** **(b)** Po(1): **0.3679** **(c)** Po(9): $1 - 0.4557$: **0.5443** **(d)** Po(0.5): **0.9098**

Scale the rate to the interval first: 6 per hour is 0.1 per minute.

B4 **(a)** **3** **(b)** $P(6 \leqslant X \leqslant 12) = 0.8758 - 0.1157$: **0.7601**

(c) **mean 2.25, standard deviation 1.5** **(d)** $P(X > 8) = 1 - 0.9786$: **0.0214**

The mean and the variance are both λ; the standard deviation is $\sqrt{\lambda}$

B5 **(a)** **Yes**: cars pass singly, independently and at a constant mean rate on a quiet road

(b) **No**: there is a fixed number of eggs, so a binomial model fits

(c) **No**: the rate is not constant (the lunchtime rush)

Events occur singly, independently, at random and at a constant rate.

B6 **(a)** $X + Y \sim \text{Po}(4)$: **0.1954** **(b)** **0.2381** **(c)** $e^{-1.5}e^{-2.5}$: **0.0183**

(d) $\dfrac{P(X = 1)P(Y = 3)}{P(X + Y = 4)}$: **0.3662**

Given the total, the split follows $B(4, \frac{1.5}{4})$

STRETCH

B7 **(a)** $e^{-\lambda} = 0.2$: **1.61** **(b)** $e^{-\lambda} = 0.05$: **3.00** **(c)** $\lambda e^{-\lambda} = 3e^{-\lambda}$: **3** **(d)** $\dfrac{\lambda^3}{6} = \dfrac{\lambda^4}{24}$: **4**

Cancel $e^{-\lambda}$ when it appears on both sides; take logarithms when it does not.

B8 **(a)** $1 - P(X \leqslant 4) = 1 - 0.8153$: **0.1847** **(b)** $\binom{8}{2}(0.1847)^2(0.8153)^6$: **0.2805**

(c) $40(0.1847)$: **7.39** **(d)** $(0.8153)^2(0.1847)$: **0.1228**

The Poisson probability becomes the p of the hours.

B9 **(a)** e^{-2t} **(b)** $e^{-2t} \geqslant 0.5$, $t \leqslant \dfrac{\ln 2}{2}$: **0.347** **(c)** $e^{-2t} < 0.01$, $t > \dfrac{\ln 100}{2}$: **2.30**

(d) $e^{-2t}(1 + 2t) < 0.1$; by trial $2t > 3.890$: $t = 1.94$

The mean $2t$ depends on the length of the interval.

B10 **(a)** $2(4) + 3$: **11** **(b)** $4(4)$: **16** **(c)** $P(X > 4) = 1 - 0.6288$: **0.3712** **(d)** $P(X \leqslant 2)$: **0.2381**

Translate a statement about Y into one about X

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C Reasoning solutions

Your wording can be different. Check that your reason makes the same point.

CORE

C1 A Poisson model needs a constant *rate*, not a constant probability.

Customers enter singly, independently and at a constant mean rate

C2 0.4457 is $P(X \leqslant 5)$. $P(X = 5) = P(X \leqslant 5) - P(X \leqslant 4) = 0.4457 - 0.2851$: **0.1606**

C3 The number of items is not fixed: plastic items occur at $6 \times 0.25 = 1.5$ per square metre, so Po(6) in 4 m^2

$1 - P(X \leqslant 4) = 1 - 0.2851$: **0.7149**

C4 The rate must be scaled: 20 minutes is Po(3); $P(X \leqslant 2)$: **0.4232**

STRETCH

C5 $X = 1$, $Y = 1$ gives a total of 2, not 1. $X + Y \sim$ Po(5): $5e^{-5}$: **0.0337**

C6 For a Poisson distribution the *variance* equals the mean. The variance is $2^2 = 4$, equal to the mean, so **the data support a Poisson model**

C7 The mean is small and X is discrete; the Poisson tables give the exact value: **0.3423**

C8 **(a)** $P(X = 1)P(Y = 0) + P(X = 0)P(Y = 1) = \lambda e^{-\lambda} e^{-\mu} + e^{-\lambda} \mu e^{-\mu}$, which is $(\lambda + \mu)e^{-(\lambda+\mu)}$, as required

(b) $\dfrac{P(X = 1)P(Y = 1)}{P(X + Y = 2)} = \dfrac{\lambda \mu e^{-(\lambda+\mu)}}{\frac{1}{2}(\lambda + \mu)^2 e^{-(\lambda+\mu)}}$: $\dfrac{2\lambda\mu}{(\lambda + \mu)^2}$

D Problem solving solutions

There is often more than one correct method. Any complete, correct method scores full marks.

CORE

D1 **(a)** Po(3.2): $\dfrac{e^{-3.2}(3.2)^3}{3!}$: **0.2226**

(b) $1 - e^{-3.2}(1 + 3.2 + 5.12)$: **0.6201**

D2 **(a)** Po(7.5); calls arrive singly and independently, at a constant mean rate during the morning

(b) $P(X \leqslant 4)$: **0.1321**

D3 2 cakes and 0 muffins: $\dfrac{e^{-1.5}(1.5)^2}{2!} \times e^{-1} = 0.2510 \times 0.3679$: **0.0923**

D4 $p = 1 - e^{-0.6} = 0.4512$; $W \sim B(10, p)$: $(0.5488)^{10} + 10(0.4512)(0.5488)^9$: **0.0229**

STRETCH

D5 **(a)** $\dfrac{P(Y = 4)}{P(Y = 3)} = \dfrac{\lambda}{4} = \dfrac{0.1898}{0.1687}$: $\lambda = 4.5$

(b) $1 - P(Y \leqslant 5) = 1 - 0.7029$: **0.2971**

D6 **(a)** $e^{-\lambda/2} = 0.0498$, $\dfrac{\lambda}{2} = 2.9998$: $\lambda = 6$

(b) 40 minutes: Po(4); $P(X \leqslant 3)$: **0.4335**

D7 $p = 1 - P(X \leqslant 5) = 1 - 0.7851 = 0.2149$; $B \sim B(8, p)$

$1 - (0.7851)^8 - 8(0.2149)(0.7851)^7$: **0.5396**

D8 **(a)** $P(X > 4) = 1 - 0.8912$: **0.1088**

(b) $P(X > 5) = 0.0420$, $P(X > 6) = 1 - 0.9858 = 0.0142 < 0.02$: $k = 6$

(c) $(0.9858)^4$: **0.944**

E Exam practice solutions

Mark-scheme notes are shown on the right of each line.

CORE *Exam questions for everyone*

E1 **(a)** Po(7) *B1*

Orders are received singly and independently *B1*

at a constant mean rate during the afternoon *B1*

(b) $\frac{e^{-7}(7)^5}{5!}$ *M1*

0.1277 *A1*

(c) $1 - P(X \leqslant 8) = 1 - 0.7291$ *M1*

0.2709 *A1*

(d) Po(1.75): $e^{-1.75}(1 + 1.75)$ *M1*

0.4779 *A1*

(e) B(6, 0.4779): $\binom{6}{4}(0.4779)^4(0.5221)^2$ *M1 M1*

0.2133 *A1*

E2 **(a)** $\Sigma fx = 179$, $\Sigma fx^2 = 441$ *M1*

mean $\frac{179}{120}$: **1.49** *A1*

variance $\frac{441}{120} - 1.4917^2$: **1.45** *A1*

(b) **The mean and the variance are close**, as they are equal for a Poisson distribution *B1*

(c) $1 - P(X \leqslant 2) = 1 - 0.8088$ *M1*

0.1912 *A1*

(d) 4 pages: Po(6); $P(T \leqslant 4)$ *M1*

0.2851 *A1*

(e) $\frac{(1.5e^{-1.5})^4}{e^{-6}6^4/4!} = \frac{4!(1.5)^4}{6^4}$ *M1*

$= \frac{24}{256} = \frac{3}{32}$, as required *A1**

STRETCH

E3 **(a)** 2 months: $e^{-2\lambda} = 0.0907$ *M1*

$2\lambda = -\ln 0.0907 = 2.4002$ *A1*

$\boxed{\lambda = 1.2}$ *A1**

(b) 3 months: Po(3.6) *B1*

$1 - e^{-3.6}\left(1 + 3.6 + \frac{3.6^2}{2} + \frac{3.6^3}{6}\right)$ *M1*

$\boxed{\mathbf{0.4848}}$ *A1*

(c) $\text{Po}(1.2 + 0.8) = \text{Po}(2)$; $\text{P}(T \leqslant 1)$ *M1*

$\boxed{\mathbf{0.4060}}$ *A1*

(d) $\frac{\text{P}(X = 3)\text{P}(Y = 0)}{\text{P}(T = 3)}$ *M1*

$= \left(\frac{1.2}{2}\right)^3$ *A1*

$\boxed{\mathbf{0.216}}$ *A1*

(e) $1 - e^{-1.2n} > 0.999$ *M1*

$n > \frac{\ln 1000}{1.2} = 5.76$ *dM1*

$\boxed{n = 6}$ *A1*