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INTERNATIONAL A LEVEL MATHEMATICS • PURE 3 (WMA13)

Pure 3 Practice Paper 1

Edexcel IAL Pure Mathematics 3

AIMING FOR B

Written in the style of the Pearson Edexcel International Advanced Level paper (WMA13/01) • Calculator allowed

A full 75-mark paper of multi-step exam questions covering every Pure 3 chapter, with a separate mark scheme

Time	Total	You must have
1 hour 30 minutes	**75 marks**	**Mathematical Formulae and Statistical Tables, calculator**

Instructions

- Use black ink or ball-point pen. Pencil may be used for diagrams and sketches.
- Answer **all** questions, in the spaces provided.
- Show enough working to make your method clear. Answers without working may not gain full credit.
- Give inexact answers to **three significant figures** unless the question says otherwise.

Information

- There are 10 questions in this paper. The total mark is 75.
- The marks for each question are shown in brackets.

Advice

- Sit the paper in one go, in 1 hour 30 minutes, with the mark scheme closed.
- Read each question carefully and try every question. Check your answers if you have time.
- Then mark it line by line with the mark scheme and fill in your score.

Track your score

Q	Topic	Marks	Score
1	Algebraic fractions and integration	8	
2	Sign change and iteration	7	
3	Reciprocal trigonometric equation	7	
4	Harmonic form in a tide model	8	
5	Exponential graph and equation	7	
6	Exponential models and rates	8	
7	Product rule and a tangent	8	
8	Differentiating using $\frac{dx}{dy}$	7	
9	Integration and area	7	
10	Modulus graph and transformations	8	
	Total	**75**	

Rough grade guide (out of 75): A* **65** • A **58** • B **52** • C **46**

Based on recent WMA13 grade boundaries. Real boundaries change every series, so treat this as a guide, not a grade.

Mark scheme | Exam questions by topic | More practice papers

Name ..... Date .....

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1. In this question you must show all stages of your working.

Solutions relying on calculator technology are not acceptable.

$$f(x) = \frac{2x}{x-3} - \frac{x+27}{x^2-x-6} \qquad x \in \mathbb{R},\ x \neq -2,\ x \neq 3$$

(a) Show that

$$f(x) = \frac{2x+9}{x+2}$$

(3)

(b) Solve the equation $f(x) = x$

(2)

(c) Find the exact value of

$$\int_4^8 f(x)\,dx$$

giving your answer in the form $a + b\ln c$, where a, b and c are rational constants.

(3)

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Question 1 continued

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(Total for Question 1 is 8 marks)

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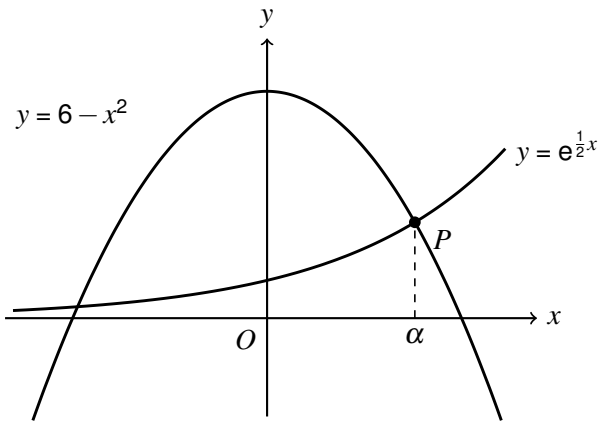

Figure 1

Figure 1 shows part of the curve with equation $y = e^{\frac{1}{2}x}$ and part of the curve with equation $y = 6 - x^2$

The curves meet at the point P, where $x = \alpha$, as shown in Figure 1.

(a) Show that α lies between 1.8 and 1.9

(2)

(b) Show that α is a root of the equation

$$x = \sqrt{6 - e^{\frac{1}{2}x}}$$

(1)

(c) Use the iteration formula

$$x_{n+1} = \sqrt{6 - e^{\frac{1}{2}x_n}}$$

with $x_1 = 1.8$ to find the values of x_2, x_3 and x_4, giving your answers to 4 decimal places.

(2)

A student tries instead to use the iteration formula

$$x_{n+1} = 2\ln\left(6 - x_n^2\right)$$

with $x_1 = 1.8$

(d) Find x_2 and x_3, and explain why this formula cannot be used to find α.

(2)

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Question 2 continued

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(Total for Question 2 is 7 marks)

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3. **(a)** Show that the equation

$$2\cot^2\theta + 5\operatorname{cosec}\theta = 10$$

can be written in the form

$$2\operatorname{cosec}^2\theta + 5\operatorname{cosec}\theta - 12 = 0$$

(2)

(b) Hence solve, for $-180^\circ < \theta < 180^\circ$, the equation

$$2\cot^2\theta + 5\operatorname{cosec}\theta = 10$$

giving your answers to one decimal place.

(5)

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Question 3 continued

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(Total for Question 3 is 7 marks)

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4. The depth of water, D metres, at the entrance to a harbour is modelled by the equation

$$D = 8 + 3\sin\left(\frac{t}{2}\right) - 4\cos\left(\frac{t}{2}\right) \qquad 0 \leqslant t \leqslant 12$$

where t is the number of hours after midnight.

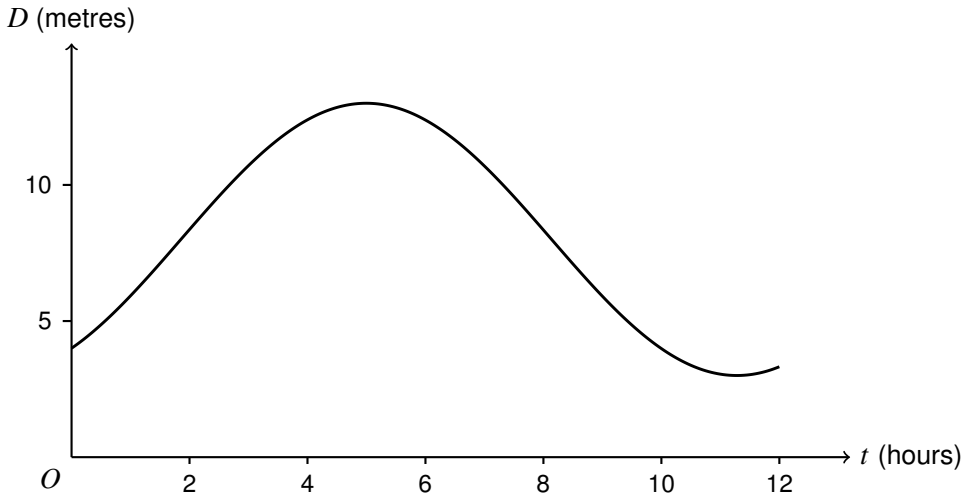

Figure 2

Figure 2 shows the graph of D against t.

(a) Express $3\sin\left(\frac{t}{2}\right) - 4\cos\left(\frac{t}{2}\right)$ in the form $R\sin\left(\frac{t}{2} - \alpha\right)$, where $R > 0$ and $0 < \alpha < \frac{\pi}{2}$

Give the exact value of R, and the value of α in radians to 4 decimal places.

(3)

(b) Find the maximum depth of the water, according to the model, and the value of t at which it occurs.

(2)

(c) Find, according to the model, the times between midnight and midday when the depth of the water is exactly 10 metres. Give each value of t to 2 decimal places.

(3)

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Question 4 continued

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(Total for Question 4 is 8 marks)

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5. In this question you must show all stages of your working.

Solutions relying on calculator technology are not acceptable.

(a) Sketch the curve with equation

$$y = 3 - 2e^{-x}$$

On your sketch, show the coordinates of each point where the curve meets the coordinate axes, and state the equation of the asymptote to the curve.

(3)

(b) Solve the equation

$$3 - 2e^{-x} = e^{x}$$

giving your answers in exact form.

(4)

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Question 5 continued

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(Total for Question 5 is 7 marks)

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6. Two cups of tea, A and B, are made at the same time and left to cool in a room.

The temperature of cup A, $\theta_A\,^\circ$C, t minutes after it is made, is modelled by the equation

$$\theta_A = 20 + 70e^{-0.05t} \qquad t \geqslant 0$$

The temperature of cup B, $\theta_B\,^\circ$C, t minutes after it is made, is modelled by the equation

$$\theta_B = 20 + 60e^{-kt} \qquad t \geqslant 0$$

where k is a positive constant.

Given that the temperature of cup B is 50 °C when $t = 15$

(a) show that $k = \dfrac{\ln 2}{15}$

(2)

(b) Find, according to the models, the value of t when the two cups of tea are at the same temperature.

Give your answer to 3 significant figures.

(3)

(c) Find the rate at which the temperature of each cup is falling at this time, and state which cup is cooling faster.

(3)

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Question 6 continued

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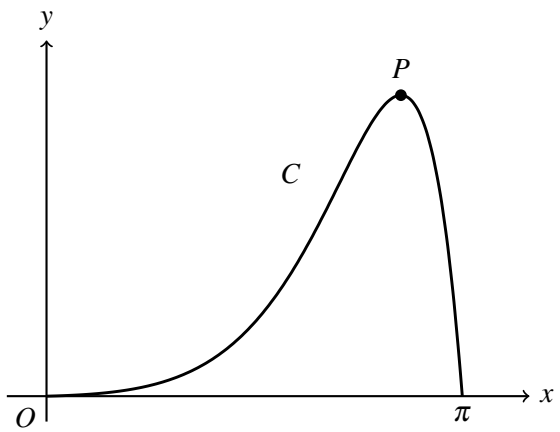

Figure 3

Figure 3 shows a sketch of the curve C with equation

$$y = e^{2x} \sin x \qquad 0 \leqslant x \leqslant \pi$$

The point P is the maximum point on C.

(a) Find $\dfrac{dy}{dx}$

(2)

(b) Show that the x coordinate of P satisfies the equation

$$\tan x = -\frac{1}{2}$$

and hence find the x coordinate of P, giving your answer to 3 significant figures.

(3)

(c) Find the equation of the tangent to C at the point where $x = \dfrac{\pi}{2}$, giving your answer in the form $y = mx + c$, where m and c are exact constants to be found.

(3)

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Question 7 continued

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8. The curve C has equation

$$x = y\mathrm{e}^{y}$$

(a) Find $\dfrac{\mathrm{d}x}{\mathrm{d}y}$

(2)

The point A on C has y coordinate 1

(b) Find an equation of the normal to C at A, giving your answer in the form $y = mx + c$, where m and c are exact constants to be found.

(3)

(c) Find the exact coordinates of the point on C at which the tangent to C is parallel to the y-axis.

(2)

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Question 8 continued

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(Total for Question 8 is 7 marks)

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9. In this question you must show all stages of your working.

Solutions relying on calculator technology are not acceptable.

(a) Find

$$\int \left(\frac{3}{2x+1} + e^{1-x} \right) dx$$

(2)

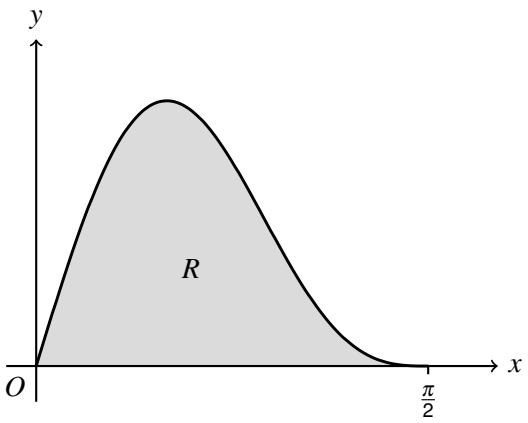

Figure 4

Figure 4 shows part of the curve with equation

$$y = 8 \sin x \cos^3 x \qquad 0 \leqslant x \leqslant \frac{\pi}{2}$$

The region R, shown shaded in Figure 4, is bounded by the curve and the x-axis.

(b) Use calculus to find the exact area of R.

(3)

(c) Show that

$$\int_0^{\frac{\pi}{4}} \tan^2 x \, dx = 1 - \frac{\pi}{4}$$

(2)

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Question 9 continued

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10. In this question you must show all stages of your working.

Solutions relying on calculator technology are not acceptable.

The function f is defined by

$$f(x) = 2|x - 3| - 4 \qquad x \in \mathbb{R}$$

(a) Sketch the graph with equation $y = f(x)$, showing the coordinates of the vertex and of each point where the graph meets the coordinate axes.

(3)

(b) Solve the equation

$$f(x) = \frac{1}{2}x$$

(3)

(c) Find the coordinates of the vertex of the graph with equation $y = 3f\left(\frac{1}{2}x\right)$

(2)

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Question 10 continued

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(Total for Question 10 is 8 marks)

TOTAL FOR PAPER IS 75 MARKS

END

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