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A-LEVEL MATHEMATICS • FURTHER PURE 1

Coordinate Systems

Worked Solutions

Pearson Edexcel International A Level (WFM01) • Calculator allowed: show every step

Full worked solutions for the free Coordinate Systems worksheet, A-Level and International A-Level Maths

A **Examples** & Your turn page 2

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Tap a section to jump straight to it

Full worked solutions for every question on the Further Pure 1 Coordinate Systems worksheet, in the same order and with the same question numbers.

How to mark your work. Tick each step you got right, not just the final answer. At A-level most marks are for method, so a wrong final answer can still earn most of the marks. If you lost marks, find the *first* line where your working is different from ours. That is the thing to fix.

In Section E: **M** = method mark, **dM** = method mark that depends on the previous M mark, **A** = accuracy mark (needs the M mark before it), **B** = independent mark.

Question sheet | Exam questions by topic | Further Pure 1 exam questions | More worksheets

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A Your turn answers

Compare each line of your working with the solution.

CORE

A1 $4a = 6$, $a = \frac{3}{2}$: $\boxed{S\left(\frac{3}{2}, 0\right),\ x = -\frac{3}{2}}$

A2 $t = \frac{x}{5}$, $y = \frac{25}{x}$: $\boxed{xy = 25}$

A3 $a = \frac{5}{2}$; $PS = 10 + \frac{5}{2}$: $\boxed{\mathbf{12.5}}$

STRETCH

A4 $y = -\sqrt{12x}$ near P, $\frac{dy}{dx} = \frac{6}{y} = -1$ at $(3, -6)$: $\boxed{y = -x - 3}$

A5 $\frac{dy}{dx} = -\frac{12}{x^2} = -\frac{1}{3}$; normal gradient 3; $y - 2 = 3(x - 6)$: $\boxed{y = 3x - 16}$

A6 $\frac{dy}{dx} = -\frac{25}{x^2} = -\frac{1}{t^2}$; $y - \frac{5}{t} = -\frac{1}{t^2}(x - 5t)$, so $t^2y - 5t = -x + 5t$: $\boxed{x + t^2y = 10t}$

A7 $x(x + 1) = 6$, $x^2 + x - 6 = 0$: $\boxed{(2, 3)\ \textbf{and}\ (-3, -2)}$

B Fluency solutions

One mark for each correct part. The notes in italics explain why each sequence was chosen.

CORE

B1 **(a)** $\boxed{(1, 0),\ x = -1}$ **(b)** $\boxed{(6, 0),\ x = -6}$ **(c)** $\boxed{\left(\frac{3}{4}, 0\right),\ x = -\frac{3}{4}}$ **(d)** $\boxed{y^2 = 28x}$

Always find a from $4a$ first.

B2 **(a)** $\boxed{y^2 = 16x}$ **(b)** $\boxed{xy = 9}$ **(c)** $\boxed{y^2 = x}$ **(d)** $\boxed{xy = 16}$

$(at^2, 2at)$ *gives* $y^2 = 4ax$; $(ct, \frac{c}{t})$ *gives* $xy = c^2$

B3 **(a)** $3+3$: $\boxed{6}$ **(b)** $12+3$: $\boxed{15}$ **(c)** $x=7$: $\boxed{(7,\pm 2\sqrt{21})}$ **(d)** $\boxed{3t^2+3}$

The focal distance is the distance to the directrix.

B4 **(a)** $\boxed{y=x+4}$ **(b)** $\boxed{y=-2x-2}$ **(c)** $\boxed{ty=x+4t^2}$ **(d)** $2y=x+16$: $\boxed{(-16,0)}$

The general tangent includes every numerical case.

B5 **(a)** curve gradient -4: $\boxed{4y=x+45}$ **(b)** curve gradient $-\frac{1}{4}$: $\boxed{y=4x-45}$ **(c)** curve gradient -1: $\boxed{y=x}$

The normal gradient is -1 divided by the curve's gradient.

B6 **(a)** $\boxed{(0,0),\,(9,9)}$ **(b)** $\boxed{\left(\frac{1}{2},2\right)}$ **(c)** $x^2+3x-4=0$: $\boxed{(1,4),\,(-4,-1)}$ **(d)** $\frac{y^3}{4}=2$: $\boxed{(1,2)}$

Substitute to get one equation in one variable.

STRETCH

B7 **(a)** gradient of curve $\frac{1}{t}$, normal $-t$: $y-2at=-t(x-at^2)$, giving the result **(b)** curve gradient $-\frac{1}{t^2}$, normal t^2: $y-\frac{c}{t}=t^2(x-ct)$, so $ty-c=t^3x-ct^4$, giving the result

(c) $y-\frac{c}{t}=-\frac{1}{t^2}(x-ct)$, giving $x+t^2y=2ct$ **(d)** $y=0$: $tx=2at+at^3$, so $x=2a+at^2$

General results save work in every numerical case.

B8 **(a)** $\left(\frac{at^2}{2},at\right)$: $\boxed{y^2=2ax}$ **(b)** $\left(\frac{a(t^2+1)}{2},at\right)$: $\boxed{y^2=2ax-a^2}$ **(c)** $\boxed{xy=\frac{c^2}{4}}$

(d) $SN=2a+at^2-a=a(t^2+1)=SP$, as required

Write x and y in terms of t, then eliminate t

B9 **(a)** $\boxed{y=x+4,\,T(-4,0)}$ **(b)** $\boxed{y=-x+12,\,N(12,0)}$ **(c)** $\frac{1}{2}\times 16\times 8$: $\boxed{64}$ **(d)** $\frac{-4+12}{2}=4$, the focus $(4,0)$

A tangent and normal meet at right angles.

B10 **(a)** $\boxed{x+y=3}$ **(b)** $(3-x)^2=4x$ gives $x^2-10x+9=0$ **(c)** $x=9$: $\boxed{Q(9,-6)}$ **(d)** $\sqrt{64+64}$: $\boxed{8\sqrt{2}}$

One root of the quadratic is always P itself.

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C Reasoning solutions

Your wording can be different. Check that your reason makes the same point.

CORE

C1 $4a = 12$, so $a = 3$, not 12: **focus** $(3, 0)$**, directrix** $x = -3$

C2 $\sqrt{x} = x^{\frac{1}{2}}$, not x^{-1}: $y = 6x^{\frac{1}{2}}$: $\frac{dy}{dx} = 3x^{-\frac{1}{2}}$

C3 t is a parameter, not a gradient. The tangent gradient is $\frac{1}{t}$, so the normal gradient is $-t$: $y + tx = 2at + at^3$

C4 $-\frac{1}{8}$ is the parameter, not x: $\left(2\left(-\frac{1}{8}\right), \frac{2}{-1/8}\right)$: $\left(-\frac{1}{4}, -16\right)$

STRETCH

C5 An exact length was asked for: $\sqrt{9^2 + 18^2} = \sqrt{405}$: $9\sqrt{5}$

C6 She used the tangent gradient -1; the normal has gradient 1: $y = x$

C7 PS equals the distance to the directrix $x = -a$, which is $at^2 + a$: $a(t^2 + 1)$

C8 **(a)** The tangent $x + t^2y = 2ct$ meets the axes at $A(2ct, 0)$ and $B\left(0, \frac{2c}{t}\right)$; the midpoint of AB is $\left(ct, \frac{c}{t}\right) = P$, as required

(b) Area $= \frac{1}{2}(2ct)\left(\frac{2c}{t}\right) = 2c^2$, which does not depend on t, as required

D Problem solving solutions

There is often more than one correct method. Any complete, correct method scores full marks.

CORE

D1 $a = 5$: $PS = 15$, so $x + 5 = 15$, $P(10, 10\sqrt{2})$; $OS = 5$, $OP = \sqrt{100 + 200} = 10\sqrt{3}$

Perimeter $5 + 15 + 10\sqrt{3}$: $\boxed{20 + 10\sqrt{3}}$

D2 **(a)** $\dfrac{dy}{dx} = -\dfrac{64}{x^2} = -\dfrac{1}{t^2}$; normal: $y - \dfrac{8}{t} = t^2(x - 8t)$, so $t^3x - ty = 8(t^4 - 1)$, as required

(b) $x = 0$: $y = \dfrac{8(1 - t^4)}{t}$: $\boxed{A\left(0, \dfrac{8(1 - t^4)}{t}\right)}$; $t = \dfrac{1}{2}$: $\boxed{(0, 15)}$

D3 $A(2ct, 0)$, $B\left(0, \dfrac{2c}{t}\right)$; area $\dfrac{1}{2}(2ct)\left(\dfrac{2c}{t}\right) = 2c^2 = 50$: $\boxed{c = 5}$

D4 $x(4x - 37.5) = 25$, $8x^2 - 75x - 50 = 0$, $(x - 10)(8x + 5) = 0$: $\boxed{\left(-\dfrac{5}{8}, -40\right)}$

STRETCH

D5 $t = 2$: $y + 2x = 36$; $(36 - 2x)^2 = 12x$, $x^2 - 39x + 324 = 0$, $(x - 12)(x - 27) = 0$: $\boxed{Q(27, -18)}$

D6 **(a)** $\dfrac{dy}{dx} = \dfrac{2}{2t} = \dfrac{1}{t}$; normal: $y - 2t = -t(x - t^2)$, so $y + tx = 2t + t^3$, as required

(b) Through $(6, 0)$: $6t = 2t + t^3$, $t(t^2 - 4) = 0$, $t = 0, \pm 2$: $\boxed{y = 0,\ y = -2x + 12,\ y = 2x - 12}$

D7 **(a)** $x + a = 5a$, $x = 4a$, $y^2 = 16a^2$: $\boxed{P(4a, 4a)}$

(b) $\dfrac{1}{2} \times a \times 4a = 2a^2 = 18$: $\boxed{a = 3,\ P(12, 12)}$

D8 **(a)** $a = 4$; $x + 4 = 20$, $x = 16$, $y^2 = 256$: $\boxed{P(16, 16)}$

(b) $M(-4, 16)$, $PM = 20$; height of $S(4, 0)$ below PM is 16; $\dfrac{1}{2} \times 20 \times 16$: $\boxed{\mathbf{160}}$

E Exam practice solutions

Mark-scheme notes are shown on the right of each line.

CORE *Exam questions for everyone*

E1 **(a)** $y = 48x^{-1}$, $\frac{dy}{dx} = -\frac{48}{x^2} = -\frac{4}{3}$ at $x = 6$ *M1 A1*

normal gradient $\frac{3}{4}$: $y - 8 = \frac{3}{4}(x - 6)$, so $4y = 3x + 14$, as required *A1**

(b) $x\left(\frac{3x+14}{4}\right) = 48$, $3x^2 + 14x - 192 = 0$ *M1*

$(x - 6)(3x + 32) = 0$, $x = -\frac{32}{3}$ *M1*

$Q\left(-\frac{32}{3}, -\frac{9}{2}\right)$ *A1*

E2 **(a)** $2y\frac{dy}{dx} = 12$, $\frac{dy}{dx} = \frac{6}{6p} = \frac{1}{p}$ *M1*

$y - 6p = \frac{1}{p}(x - 3p^2)$, so $py = x + 3p^2$, as required *A1**

(b) $py - x = 3p^2$ and $qy - x = 3q^2$; subtract: $(p - q)y = 3(p^2 - q^2)$ *M1*

$y = 3(p + q)$ *A1*

$x = 3p(p + q) - 3p^2 = 3pq$, as required *A1**

(c) $P(3, 6)$, $Q(27, 18)$, $R(9, 12)$; $\frac{1}{2}|3(18 - 12) + 27(12 - 6) + 9(6 - 18)|$ *M1*

36 *A1*

STRETCH

E3 **(a)** $y = c^2x^{-1}$, $\dfrac{dy}{dx} = -\dfrac{c^2}{x^2} = -\dfrac{1}{t^2}$ *M1 A1*

normal: $y - \dfrac{c}{t} = t^2(x - ct)$, so $t^3x - ty = c(t^4 - 1)$, as required *A1**

(b) $c = 4$, $t = 1$: the normal is $y = x$; $x^2 = 9x$ *M1*

$\boxed{Q(9, 9)}$ *A1*

(c) $R\left(\dfrac{9}{4}, 0\right)$, l: $x = -\dfrac{9}{4}$ *B1*

QR: gradient $\dfrac{9}{27/4} = \dfrac{4}{3}$, $y = \dfrac{4}{3}\left(x - \dfrac{9}{4}\right)$ *M1*

$S\left(-\dfrac{9}{4}, -6\right)$ *A1*

$QS = \sqrt{\left(\dfrac{45}{4}\right)^2 + 15^2}$: $\boxed{\dfrac{75}{4}}$ *A1*