



eClassroom

A-LEVEL MATHEMATICS • PURE 2

Exponentials and Logarithms

Worksheet

Pearson Edexcel International A Level (WMA12) & A Level (9MA0) • Calculator allowed: show every step

Free Exponentials and Logarithms worksheet for A-Level and International A-Level Maths, with full worked solutions



Tap a section to jump straight to it

By the end of this worksheet you can: use index and log form • use the laws of logarithms • solve $a^x = b$ and log equations, checking every answer • sketch $y = a^x$ • *Stretch:* hidden quadratics, change of base and exponential models.

Which questions are for me?

CORE The methods every A-level student must secure. Start here.

STRETCH Rejecting invalid roots, hidden quadratics, a log equation that becomes a cubic, and half-life models.

Every section has both, clearly labelled. Questions get harder as you go, and every skill in Sections A to D is needed in Section E.

Track your progress

Section	B	C	D	E
Core marks	23	10	13	11
Stretch marks	16	14	17	8
My score				

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A

Worked examples & Your turn

Read each example carefully. Cover it up, then answer the question beside it using the same method.

CORE

Secure the method

Worked example A1

Find the value of (i) $\log_2 32$ (ii) $\log_9 3$ (iii) $\log_5 \frac{1}{25}$

$\log_a b = x$ means $a^x = b$: ask "which power of a gives b ?"

$$2^5 = 32, \quad 9^{\frac{1}{2}} = 3, \quad 5^{-2} = \frac{1}{25}$$

$$5, \quad \frac{1}{2}, \quad -2$$

Your turn A1

Find the value of (i) $\log_3 81$ (ii) $\log_8 2$ (iii) $\log_2 \frac{1}{8}$

Worked example A2

Write $2\log_3 6 - \log_3 4$ as a single logarithm, and evaluate it.

Power law first, then the subtraction law.

$$\log_3 36 - \log_3 4 = \log_3 \frac{36}{4} = \log_3 9$$

$$2$$

Your turn A2

Write $3\log_2 6 - \log_2 27$ as a single logarithm, and evaluate it.

Worked example A3

Solve $3^x = 20$, giving your answer to 3 significant figures.

Take logs of both sides: $x \log 3 = \log 20$

$$x = \frac{\log 20}{\log 3}$$

$$x = 2.73$$

Your turn A3

Solve $5^{2x} = 40$, giving your answer to 3 significant figures.



STRETCH

Worked example A4: a log equationSolve $\log_3(x+8) - \log_3 x = 2$

$$\log_3 \frac{x+8}{x} = 2, \text{ so } \frac{x+8}{x} = 3^2 = 9$$

Then $x+8 = 9x$. Check: both logs are of positive numbers.

$$x = 1$$

Your turn A4Solve $\log_2(3x+4) - \log_2(x-1) = 3$ **Worked example A5: a hidden quadratic**Solve $2^{2x} - 6 \times 2^x + 8 = 0$

$$2^{2x} = (2^x)^2. \text{ Let } u = 2^x: u^2 - 6u + 8 = 0$$

$$(u-2)(u-4) = 0, \text{ so } 2^x = 2 \text{ or } 2^x = 4$$

$$x = 1, x = 2$$

Your turn A5Solve $9^x - 4 \times 3^x + 3 = 0$ **Worked example A6: change of base**Solve $\log_2 x + \log_4 x = 6$

$$\log_4 x = \frac{\log_2 x}{\log_2 4} = \frac{1}{2} \log_2 x$$

$$\frac{3}{2} \log_2 x = 6, \text{ so } \log_2 x = 4$$

$$x = 16$$

Your turn A6Solve $\log_3 x + \log_9 x = 9$ **Worked example A7: a model $N = ab^t$**

$N = ab^t$. When $t = 0$, $N = 500$, and when $t = 2$, $N = 1280$. Find a and b

$t = 0$ gives a directly. Then $500b^2 = 1280$

$$b^2 = 2.56$$

$$a = 500, b = 1.6$$

Your turn A7

$N = ab^t$. When $t = 1$, $N = 360$, and when $t = 3$, $N = 810$. Find a and b



B**Fluency**

Each question is a designed sequence. From one part to the next, ask yourself: what changed, and what did it do to the answer?

CORE*Secure the method***B1** Find the value of

(a) $\log_2 16$

(b) $\log_3 \frac{1}{9}$

(c) $\log_{25} 5$

(d) $\log_7 1$

[4 marks]**B2** Write each as a single logarithm, simplified as far as possible.

(a) $\log 4 + \log 5$

(b) $\log 24 - \log 3$

(c) $3\log 2 + 2\log 3$

(d) $\frac{1}{2}\log 81 - 2\log 3$

[4 marks]**B3** Solve, giving each answer to 3 significant figures.

(a) $2^x = 10$

(b) $2^x = 0.1$

(c) $2^{x+1} = 10$

(d) $2^{3x} = 10$

[4 marks]**B4** Solve

(a) $\log_3 x = 4$

(b) $\log_3(2x - 1) = 2$

(c) $\log_3 x + \log_3 4 = 2$

(d) $2\log_3 x = \log_3 25$

[4 marks]

CORE

continued: graphs and laws

B5 For each curve, state the coordinates of the y intercept and the equation of the asymptote.

(a) $y = 3^x$

(b) $y = 3^x + 2$

(c) $y = 3^{-x}$

[3 marks]

B6 Given that $p = \log_a 2$ and $q = \log_a 3$, write in terms of p and q

(a) $\log_a 6$

(b) $\log_a 18$

(c) $\log_a \frac{8}{3}$

(d) $\log_a (a\sqrt{2})$

[4 marks]

STRETCH

B7 Solve, giving exact answers.

(a) $4^x - 5 \times 2^x + 4 = 0$

(b) $4^x - 2^{x+1} - 8 = 0$

(c) $3^{2x+1} - 10 \times 3^x + 3 = 0$

(d) $2^{2x} - 3 \times 2^x - 10 = 0$

[4 marks]

B8 Solve, explaining any value you reject.

(a) $\log_2 x + \log_2(x-2) = 3$

(b) $\log_6 x + \log_6(x+1) = 1$

(c) $2\log_3 x - \log_3(x+6) = 1$

(d) $\log_2(x+1) = 1 + \log_2 x$

[4 marks]

B9

(a) Find the exact value of $\log_4 8$

(b) Find the exact value of $\log_8 32$

(c) Show that $\log_2 3 \times \log_3 4 = 2$

(d) Solve $\log_2 x = \log_8 64$

[4 marks]

B10 A population is modelled by $N = 200 \times 1.05^t$, where t is the time in years.

(a) State the initial population.

(b) Find the population after 10 years.

(c) Find the time taken for the population to double.

(d) Explain what the 1.05 tells you about the population.

[4 marks]



C**Reasoning**

Show your working and explain your thinking. A correct answer without a reason does not score full marks.

CORE

Secure the method

- C1** Sam writes $\log(x+3) = \log x + \log 3$
Explain Sam's mistake, and write $\log x + \log 3$ as a single logarithm. [2 marks]
- C2** Lily writes $3\log_2 x = \log_2(3x)$
Explain Lily's mistake, and write $3\log_2 x$ correctly as a single logarithm. [2 marks]
- C3** Kai solves $\log_2(x+4) + \log_2(x-2) = 4$ and gives the answers $x = 4$ and $x = -6$
Explain which answer must be rejected and why. [3 marks]
- C4** Jo solves $\log_3(1-x) - \log_3(x+3) = 1$ and rejects her answer $x = -2$ because it is negative.
Explain why Jo is wrong. [3 marks]



STRETCH

- C5** Ali sketches $y = 2^x + 3$ and labels the y intercept as $(0, 3)$
Explain Ali's mistake, and give the correct intercept and the equation of the asymptote. [3 marks]
- C6** Max solves $\log_3(x+1) = 2 + \log_3 x$ by writing 2 as $\log_3 6$
Explain Max's mistake, and solve the equation correctly. [3 marks]
- C7** A car worth £18 000 loses 15% of its value each year. Jo says its value after 4 years is $18\,000 \times 0.85^3$
Explain Jo's mistake, and find the correct value to the nearest pound. [3 marks]
- C8** (a) Prove that $\log_a b = \frac{1}{\log_b a}$ for positive a and b , $a, b \neq 1$ (2)
- (b) Hence solve $\log_2 x = 4 \log_x 2$ (3)
- [5 marks]



D**Problem solving**

You decide the method. Read the whole question first, and show every step.

CORE*Secure the method*

- D1** *In this question you must show all stages of your working. Solutions relying on calculator technology are not acceptable.*

Using the laws of logarithms, solve the equation

$$\log_3(x+5) + \log_3(x-1) = 3$$

You should combine the logarithms into a single logarithm first, then form and solve a quadratic equation in x . Explain clearly why any value you find must be rejected. **[3 marks]**

- D2** *In this question you must show all stages of your working. Solutions relying on calculator technology are not acceptable.*

Using the laws of logarithms, solve the equation

$$\log_2(10-x) = 3 + 2\log_2(x-1)$$

Show clearly how you write the number 3 as a logarithm to base 2, and give a reason for rejecting any value that does not satisfy the original equation. **[4 marks]**

- D3** The curve C has equation $y = 5 - 2^x$, $x \in \mathbb{R}$

(a) Sketch C . On your sketch, show clearly the coordinates of the point where C crosses the y -axis and the equation of the asymptote of C . **(2)**

(b) Find the exact x coordinate of the point where C crosses the x -axis, giving your answer as a logarithm to base 2 **(1)**

[3 marks]

- D4** The curves C_1 and C_2 have equations $y = 4^x$ and $y = 5 \times 2^x - 6$ respectively. The curves meet at two points.

By forming a quadratic equation in 2^x , find the exact x coordinates of the two points where C_1 and C_2 meet, writing any logarithm in the form $\log_2 k$, where k is an integer.

[3 marks]

STRETCH

D5 In this question you must show all stages of your working. Solutions relying on calculator technology are not acceptable.

(a) Using the laws of logarithms, show that the equation $2\log_3 x - \log_3(x - 2) = 2$ can be written as $x^2 - 9x + 18 = 0$

(2)

(b) Hence solve the equation $2\log_3 x - \log_3(x - 2) = 2$, checking in the original equation that each of your answers is valid.

(2)

[4 marks]

D6 The mass, M grams, of a radioactive sample t days after it was prepared is modelled by

$$M = 80 \times 2^{-\frac{t}{k}}$$

where k is a positive constant. When the sample is first prepared its mass is 80 grams, and after 12 days its mass is 20 grams. The sample is safe to dispose of once its mass is less than 1 gram.

(a) Find the value of k , and explain what k represents.

(2)

(b) Find, according to the model, the number of whole days after which the sample is first safe to dispose of.

(2)

[4 marks]



D7 In this question you must show all stages of your working. Solutions relying on calculator technology are not acceptable.

(a) Show that $\log_2(4x^2) - \log_2\left(\frac{x}{2}\right) = 3 + \log_2 x$ for $x > 0$ (2)

(b) Hence solve $\log_2(4x^2) - \log_2\left(\frac{x}{2}\right) = 7$ (1)

(c) Hence find the set of values of x for which $\log_2(4x^2) - \log_2\left(\frac{x}{2}\right) \leq 5$, remembering that the logarithms are only defined for $x > 0$ (1)

[4 marks]

D8 The curve C_1 has equation $y = 3^x$ and the curve C_2 has equation $y = 2 \times 3^{-x} + 1$. The curves meet at exactly one point, P

(a) By letting $u = 3^x$, show that the x coordinate of P satisfies $u^2 - u - 2 = 0$ (2)

(b) Find the exact x coordinate of P , explaining why one solution of the quadratic is rejected. (2)

(c) Find the y coordinate of P (1)

[5 marks]



E**Exam practice**

Exam-style questions in the style of WMA12. Write your answers in the spaces, and show every step.

CORE

Exam questions for everyone

E1 In this question you must show all stages of your working. Solutions relying on calculator technology are not acceptable.

(i) Using the laws of logarithms, solve the equation

$$\log_2(x+1) + \log_2(x-5) = 4$$

showing each step of your working and giving a reason for rejecting any value that you find.

.....
(3)

(ii) Given that $\log_a 2 = p$, where a is a positive constant greater than 1, write $\log_a (8a^2)$ in terms of p , giving your answer in its simplest form and showing each law of logarithms that you use.

.....
(2)

(Total for Question E1 is 5 marks)



E2 The curve C has equation

$$y = 3^{-x} + 2 \quad x \in \mathbb{R}$$

(a) Sketch the curve C . On your sketch, show clearly the coordinates of the point where C crosses the y -axis and the equation of the asymptote of C .

(3)

The line with equation $y = 11$ meets C at the point P

(b) Using algebra, find the exact x coordinate of P , showing your working.

(1)

(c) Solve the equation $3^{-x} + 2 = 7$, giving your answer to 3 significant figures and showing each step of your method.

(2)

(Total for Question E2 is 6 marks)



STRETCH

E3 In this question you must show all stages of your working. Solutions relying on calculator technology are not acceptable.

(a) Given that x satisfies the equation $\log_2(x^3 + 6) - \log_2 x = \log_2(4x - 1)$, use the laws of logarithms to show that

$$x^3 - 4x^2 + x + 6 = 0$$

(3)

(b) Given that $(x + 1)$ is a factor of $x^3 - 4x^2 + x + 6$, use algebra to factorise $x^3 - 4x^2 + x + 6$ completely.

(2)

(c) Hence solve the equation $\log_2(x^3 + 6) - \log_2 x = \log_2(4x - 1)$, explaining clearly why any value of x must be rejected, and checking that each value you keep makes every logarithm defined.

(3)

(Total for Question E3 is 8 marks)

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