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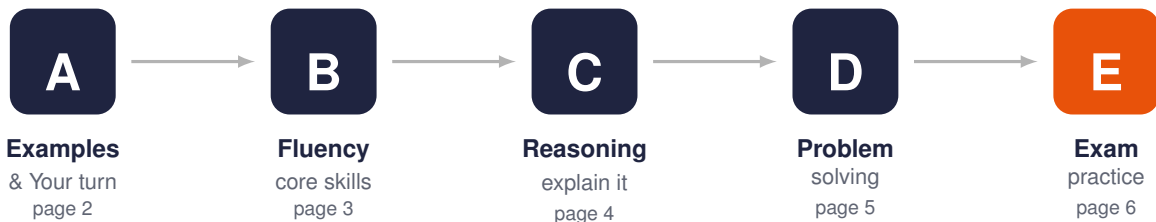
A-LEVEL MATHEMATICS • PURE 3

Functions and Graphs

Worked Solutions

Pearson Edexcel International A Level (WMA13) & A Level (9MA0) • Calculator allowed: show every step

Full worked solutions for the free Functions and Graphs worksheet, A-Level and International A-Level Maths



Tap a section to jump straight to it

Full worked solutions for every question on the Pure 3 Functions and Graphs worksheet, in the same order and with the same question numbers.

How to mark your work. Tick each step you got right, not just the final answer. At A-level most marks are for method, so a wrong final answer can still earn most of the marks. If you lost marks, find the *first* line where your working is different from ours. That is the thing to fix.

In Section E: **M** = method mark, **dM** = method mark that depends on the previous M mark, **A** = accuracy mark (needs the M mark before it), **B** = independent mark.

Question sheet

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A**Your turn answers**

Compare each line of your working with the solution.

CORE

A1 $3x + 1 = 2x + 6$: $x = 5$; $-(3x + 1) = 2x + 6$: $x = -\frac{7}{5}$; both give $2x + 6 > 0$: $x = 5$, $x = -\frac{7}{5}$

A2 $gf(1) = g(5) = 24$; $fg(x) = 3(x^2 - 1) + 2 = 3x^2 - 1$

A3 $y(x - 3) = 2x + 1$, so $x(y - 2) = 3y + 1$; range of f is $y > 2$: $f^{-1}(x) = \frac{3x + 1}{x - 2}$, $x > 2$

STRETCH

A4 Maximum 5 at $x = -1$ (in the domain); least at $x = 2$: $5 - 9 = -4$: $-4 \leq f(x) \leq 5$

A5 $(5, 1)$; $(5, -1)$ and $(-5, -1)$

A6 x : $\div 2$, so -1 ; y : $\frac{1}{2} \times 8 + 1 = 5$: $(-1, 5)$

A7 $2x + 1 = x + 5$: $x = 4$; $-(2x + 1) = x + 5$: $x = -2$. The V is above the line outside these: $x < -2$ or $x > 4$



B**Fluency solutions**

One mark for each correct part. The notes in *italics* explain why each sequence was chosen.

CORE

B1 (a) $x = \pm 7$ (b) $x = 9, x = -5$ (c) $x = \frac{9}{2}, x = -\frac{5}{2}$ (d) **no solutions**, since $|\dots| \geq 0$

A modulus is never negative.

B2 (a) $1 \leq f(x) \leq 7$ (b) $0 \leq f(x) \leq 9$ (c) $0 < f(x) \leq \frac{1}{2}$ (d) $f(x) \geq 4$

In (b) the minimum is at $x = 0$, not at an end of the domain: sketch first.

B3 (a) $f(2) = 5$ (b) $g(4) = 32$ (c) $\frac{1}{x} + 3$ (d) $\frac{1}{2(x+3)^2}$

The function nearest x is applied first.

B4 (a) $\frac{x+3}{4}, x \in \mathbb{R}$ (b) $2x-5, x \in \mathbb{R}$ (c) $1 + \frac{2}{x}, x > 0$ (d) $x^2 + 2, x \geq 0$

The domain of the inverse is the range of the function.

B5 (a) $(5, -6)$ (b) $(2, -18)$ (c) $(2, 6)$

Changes inside the bracket affect x ; changes outside affect y

B6 (a) $x = 1, x = 7$ (b) $1 < x < 7$ (c) $-(x-4) = x: x = 2$ (d) $x < 2$

The line $y = x$ meets only one branch of the V

STRETCH

B7 (a) $\left(-\frac{3}{2}, 5\right)$ (b) $(-5, -4)$ (c) $(-6, 5)$ (d) $(3, 4)$

x changes undo the inside operations in reverse order.

B8 (a) $\frac{x-1}{2}$ (b) $\sqrt{x}, x \geq 0$ (c) $fg(x) = 2x^2 + 1, x \geq 0: \sqrt{\frac{x-1}{2}}, x \geq 1$ (d) $4x+3$

$(fg)^{-1} = g^{-1}f^{-1}$: undo f first.



- B9** (a) roots $x = 1, x = 5$; vertex $(3, -4)$ (b) $(3, 4)$ (c) $(3, -4), (-3, -4)$
 (d) $f(x) = 4$ gives 2 solutions and $f(x) = -4$ gives 1: 3
 Both $f(x) = 4$ and $f(x) = -4$ satisfy $|f(x)| = 4$

- B10** (a) it is many-to-one, e.g. $f(2) = f(-2)$ (b) $k = -2$ (c) $h^{-1}(x) = -2 + \sqrt{x+5}$ (d) $x \geq -5$
 Restricting to one side of the vertex makes a quadratic one-to-one.

C

Reasoning solutions

Your wording can be different. Check that your reason makes the same point.

CORE

- C1** Sam found $fg(2)$. In $gf(2)$, f acts first: $g(f(2)) = g(3) = 27$

- C2** f^{-1} means the inverse function, not the reciprocal. $y = 2x + 3$ gives $x = \frac{y-3}{2}$: $f^{-1}(x) = \frac{x-3}{2}$

- C3** A range needs both ends. On $-1 \leq x \leq 2$ the least value is at $x = 2$: $5 \leq f(x) \leq 9$

- C4** Dividing by x assumes $x \neq 0$. Factorise: $x(x^2 - 4) = 0$: $x = 0$ is also a solution

STRETCH

- C5** $x = -5$ gives $|-6| = 6$ but $2(-5) + 4 = -6$: a modulus cannot equal a negative number. reject $x = -5$; $x = -1$ works.

- C6** The domain is missing: it is the range of f , so values $x < 2$ must be excluded. $x \geq 2$

- C7** Inside the bracket, undo in reverse order: subtract 6 first, then divide by 2: $\frac{4-6}{2} = -1$: $(-1, 3)$

- C8** (a) $ff(x) = \frac{a \frac{ax+b}{cx-a} + b}{c \frac{ax+b}{cx-a} - a} = \frac{a^2x + ab + bcx - ab}{acx + bc - acx + a^2} = \frac{(a^2 + bc)x}{a^2 + bc} = x$ as required

- (b) Here $a = 3, b = 1, c = 2$: $f^{-1}(x) = \frac{3x+1}{2x-3}$ (c) symmetrical in $y = x$



D**Problem solving solutions**

There is often more than one correct method. Any complete, correct method scores full marks.

CORE

D1 (a) $P(-2, -6)$

(b) $3(x+2) - 6 = x: x = 0; -3(x+2) - 6 = x: x = -3$. Checks: $0 = 0, 3 - 6 = -3: x = 0, x = -3$

D2 (a) $f(2) = \frac{3}{2}, g\left(\frac{3}{2}\right) = 5 - \frac{9}{4} = \frac{11}{4}$

(b) $f(0) = 0$ and $\frac{6}{x+2} \rightarrow 0$ as $x \rightarrow \infty: 0 \leq f(x) < 3$

(c) $y(x+2) = 3x$, so $x(3-y) = 2y: f^{-1}(x) = \frac{2x}{3-x}, 0 \leq x < 3$

D3 (a) $(2, -3)$ (b) $\left(\frac{5}{2}, 2\right)$ (c) $(5, 6)$

D4 (a) $f(0) = 3; \frac{12}{x+3} > 0$ always, so $f(x)$ never reaches 7: $3 \leq f(x) < 7$

(b) $\frac{12}{x+3} = 7 - y$, so $x = \frac{12}{7-y} - 3: f^{-1}(x) = \frac{12}{7-x} - 3, 3 \leq x < 7$

STRETCH

D5 (a) $A(-3, -9), B(3, -9)$

(b) For $x \geq 0: x^2 - 6x = 16, (x-8)(x+2) = 0; x = -2$ is rejected since $x \geq 0$ here. By symmetry: $x = \pm 8$

(c) $k > 0$ or $k = -9$

D6 (a) $x = \sqrt{y+2}: f^{-1}(x) = \sqrt{x+2}, x \geq -2$

(b) The graphs are reflections in $y = x$, and f is increasing, so they meet on $y = x: x^2 - 2 = x, (x-2)(x+1) = 0, x \geq 0: x = 2$

D7 (a) $V(4, 2)$ (b) $2 = 4m: m = \frac{1}{2}$

(c) The line must pass above V ($m > \frac{1}{2}$) and be less steep than the right branch (gradient 1): $\frac{1}{2} < m < 1$



D8 (a) $fg(x) = \frac{8}{x-1} + 3$ (b) $\frac{8}{x-1} > 0$: $fg(x) > 3$ (c) $\frac{8}{x-1} = 4$: $x = 3$

(d) $g(2x+3) = \frac{4}{2x+2} = \frac{2}{x+1}$, defined when $2x+3 > 1$: $x > -1$

E

Exam practice solutions

Mark-scheme notes are shown on the right of each line.

CORE

Exam questions for everyone

E1 (a) $V(3, -4)$, $A(0, 2)$ B1 B1

$B(1, 0)$, $C(5, 0)$ B1

(b) $2x - 6 - 4 = x - 1$: $x = 9$; $-(2x - 6) - 4 = x - 1$: $x = 1$ M1

both check: $x = 9, x = 1$ A1

E2 (a) $y(x-1) = 3x+2$, so $x = \frac{y+2}{y-3}$ M1 A1

$f^{-1}(x) = \frac{x+2}{x-3}, x > 3$ B1

(b) $\frac{3\frac{3x+2}{x-1}+2}{\frac{3x+2}{x-1}-1} = \frac{9x+6+2x-2}{3x+2-x+1} = \frac{11x+4}{2x+3}$ M1 A1

(c) $(1, 11)$ B1

STRETCH

E3 (a) $(x-2)^2 - 3$ with $x \geq 2$: $f(x) \geq -3$ B1

(b) $y = (x-2)^2 - 3$, so $x = 2 + \sqrt{y+3}$ (positive root, since $x \geq 2$) M1 A1

$f^{-1}(x) = 2 + \sqrt{x+3}, x \geq -3$ B1

(c) $fg(x) = f(|x-3|)$ needs $|x-3|$ in the domain of f : $|x-3| \geq 2$ B1

$u^2 - 4u + 1 = 6, u = 5$ ($u \geq 2$), so $|x-3| = 5$ M1

$x = 8, x = -2$ A1

(d) $gf(x) = |f(x) - 3| \geq 0$, so **it can never equal -1** B1

