Binomial Expansion

Worked Solutions

Suitable for:

Pearson Edexcel International A-Level (iAL) — Pure Mathematics 2

Question 1 — Worked Solution

Expand $(1 + 2x)^5$ using the binomial theorem

$$(1+2x)^5 = \binom{5}{0}1^5 + \binom{5}{1}1^4(2x) + \binom{5}{2}1^3(2x)^2 + \binom{5}{3}1^2(2x)^3 + \binom{5}{4}1(2x)^4 + \binom{5}{5}(2x)^5$$

$$= 1 + 5(2x) + 10(4x^2) + 10(8x^3) + 5(16x^4) + 32x^5$$

$$\therefore = 1 + 10x + 40x^2 + 80x^3 + 80x^4 + 32x^5$$

Question 2 — Worked Solution

Expand (3 + 2x)^4

$$(3+2x)^4 = \binom{4}{0}3^4 + \binom{4}{1}3^3(2x) + \binom{4}{2}3^2(2x)^2 + \binom{4}{3}3(2x)^3 + \binom{4}{4}(2x)^4$$

$$= 1(81) + 4(27)(2x) + 6(9)(4x^2) + 4(3)(8x^3) + 1(16x^4)$$

$$= 81 + 216x + 216x^2 + 96x^3 + 16x^4$$

$$\therefore (3+2x)^4 = 81 + 216x + 216x^2 + 96x^3 + 16x^4$$

Question 3 — Worked Solution

Expand (2 – 3x)^4

$$(2-3x)^4=\binom{4}{0}2^4+\binom{4}{1}2^3(-3x)+\binom{4}{2}2^2(-3x)^2+\binom{4}{3}2(-3x)^3+\binom{4}{4}(-3x)^4$$

$$=1(16)+4(8)(-3x)+6(4)(9x^2)+4(2)(-27x^3)+1(81x^4)$$

$$=16-96x+216x^2-216x^3+81x^4$$

$$\therefore (2-3x)^4=16-96x+216x^2-216x^3+81x^4$$

Question 4 — Worked Solution

First four terms of (5 – 2x)^5

$$(5-2x)^5 = \binom{5}{0}5^5 + \binom{5}{1}5^4(-2x) + \binom{5}{2}5^3(-2x)^2 + \binom{5}{3}5^2(-2x)^3 + \cdots$$

$$= 1(3125) + 5(625)(-2x) + 10(125)(4x^2) + 10(25)(-8x^3) + \cdots$$

$$= 3125 - 6250x + 5000x^2 - 2000x^3 + \cdots$$

$$\therefore = 3125 - 6250x + 5000x^2 - 2000x^3 + \cdots$$

Question 5 — Worked Solution

Find the coefficient of x^3 in $(2 + 5x)^6$

The general term is:

$$\binom{6}{r}(2)^{6-r}(5x)^r$$

For the x^3 term, set r = 3:

$$\binom{6}{3}(2)^3(5)^3x^3 = 20 \times 8 \times 125 \times x^3$$

$$\therefore \text{Coefficient of } x^3 = 20{,}000$$

Question 6 — Worked Solution

Find k given coefficient of x^2 in $(1 + kx)^8$ is 112

The general term of $(1 + kx)^8$ is:

$$\binom{8}{r}(kx)^r$$

For the x^2 term, set r = 2:

$$\binom{8}{2}(k)^2x^2 = 28k^2x^2$$

Set the coefficient equal to 112:

$$28k^2 = 112 \;\Rightarrow\; k^2 = 4$$

$$\therefore k = 2 \quad \text{or} \quad k = -2$$

Question 7 — Worked Solution

(a) First three terms of $(2 - x)^5$

$$(2-x)^5 = \binom{5}{0}2^5 + \binom{5}{1}2^4(-x) + \binom{5}{2}2^3(-x)^2 + \cdots$$

$$= 32 - 5(16)x + 10(8)x^2 - \cdots$$

$$\therefore = 32 - 80x + 80x^2 - \cdots$$

(b) Coefficient of x^2 in $(1 + 3x)(2 - x)^5$

Multiply $(1 + 3x)$ by the first three terms of $(2 - x)^5$:

$$(1+3x)(32 - 80x + 80x^2 - \cdots)$$

Collect terms in x^2:

$$1 \times 80x^2 \; + \; 3x \times (-80x) = 80x^2 - 240x^2 = -160x^2$$

$$\therefore \text{Coefficient of } x^2 = -160$$

Question 8 — Worked Solution

(a) First four terms of $(1 + x)^8$

$$(1+x)^8 = 1 + \binom{8}{1}x + \binom{8}{2}x^2 + \binom{8}{3}x^3 + \cdots$$

$$\therefore = 1 + 8x + 28x^2 + 56x^3 + \cdots$$

(b) Approximate $(1.02)^8$

Write 1.02 = 1 + 0.02, so substitute x = 0.02:

$$(1.02)^8 \approx 1 + 8(0.02) + 28(0.02)^2 + 56(0.02)^3$$

$$= 1 + 0.16 + 28(0.0004) + 56(0.000008)$$

$$= 1 + 0.16 + 0.0112 + 0.000448$$

$$\therefore (1.02)^8 \approx 1.17165$$

(c) Overestimate or underestimate?

The expansion has been truncated after the x^3 term. Since all remaining terms are positive (even powers of a positive number with positive binomial coefficients), the true value is slightly larger than our approximation.

The approximation is an underestimate, as the omitted terms (x^4 and higher) are all positive.