



eClassroom

INTERNATIONAL A LEVEL MATHEMATICS • PURE 3

Pure Mathematics 3

Exam Questions by Topic

Pearson Edexcel International Advanced Level (WMA13/01) • Calculator allowed

Multi-step exam questions at the difficulty of the real WMA13 paper, with a full mark scheme

Chapters (tap a chapter to jump to it)	Qs	Marks	Page	My score
1 Algebraic Methods Algebraic fractions, improper fractions and algebraic division	10	69	own PDF	
2 Functions and Graphs Modulus, composite and inverse functions, transformations	10	76	own PDF	
3 Trigonometric Functions sec, cosec, cot, identities, inverse trigonometric functions	10	72	own PDF	
4 Trigonometric Addition Formulae Addition and double-angle formulae, $R\cos(\theta \pm \alpha)$	10	75	own PDF	
5 Exponentials and Logarithms e^x , $\ln x$, modelling, logarithmic graphs	10	78	2	
6 Differentiation Chain, product and quotient rules, trigonometric and exponential functions	10	76	own PDF	
7 Integration Standard integrals, $f(ax+b)$, trigonometric identities, reverse chain rule	10	77	own PDF	
8 Numerical Methods Locating roots, iteration	10	77	own PDF	

[Full mark scheme](#)

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CHAPTER 5 • KEY SKILLS

5

Exponentials and Logarithms

Specification P3: Exponentials and logarithms • 10 questions • 78 marks • about 95 minutes

1 Graphs

$y = e^x$: asymptote $y = 0$, through $(0, 1)$.
 $y = \ln x$: asymptote $x = 0$, through $(1, 0)$. Transform asymptotes too.

2 Inverse functions

$\ln(e^x) = x$ and $e^{\ln x} = x$. The domain of $\ln$ is $x > 0$.

3 Laws of logarithms

$\ln a + \ln b = \ln ab$, $\ln a - \ln b = \ln \frac{a}{b}$,
 $k \ln a = \ln a^k$

4 Exact solutions

Take $\ln$ of both sides:
 $3e^{2x-1} = 12 \Rightarrow 2x - 1 = \ln 4$. Leave answers in terms of $\ln$ or e .

5 Hidden quadratics

Multiply through by e^x and let $u = e^x$.
 Reject $u \leq 0$, since $e^x > 0$

6 Exponential models

Initial value: $t = 0$. Use a known point to find k exactly, e.g. $k = \frac{1}{5} \ln 2$

7 Rates of change

$\frac{d}{dt}(Ae^{kt}) = kAe^{kt}$. A negative rate means decreasing: say so in context.

8 Long-term behaviour

As $t \rightarrow \infty$, $e^{-kt} \rightarrow 0$ (for $k > 0$): the model tends to a limit. State limitations in context.

9 Logarithmic graphs

$y = ax^n$: $\log y = \log a + n \log x$. $y = kb^x$: $\log y = \log k + x \log b$. Match the base used on the axis.

Skills map every skill above is tested at least once; interleaved skills come from GCSE or other chapters

Skill	Q1	Q2	Q3	Q4	Q5	Q6	Q7	Q8	Q9	Q10
Marks	5	5	5	5	6	10	10	10	10	12
1 Graphs			•						•	
2 Inverse functions									•	
3 Laws of logarithms	•			•	•			•	•	•
4 Exact solutions	•	•			•	•	•		•	•
5 Hidden quadratics		•								•
6 Exponential models						•	•	•		•
7 Rates of change						•				•
8 Long-term behaviour			•				•	•		
9 Logarithmic graphs				•	•			•		•
Interleaved: functions, inverse and composite (Chapter 2)			•						•	
Interleaved: algebraic fractions (Chapter 1)	•						•			
Interleaved: quadratics, straight lines and logarithms (P1, P2)		•		•	•			•		•

Exam difficulty. Questions 1 to 5 check each skill on its own. Questions 6 to 10 are multi-step questions built like the last questions on a real WMA13 paper, where one part leads into the next and skills from different topics are combined.

Must learn. Give exact answers when asked (in terms of $\ln$ or e). Find rates by differentiating, not by averaging, and finish log-graph questions by writing the equation in the form asked.



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1. In this question you must show all stages of your working.

Solutions relying on calculator technology are not acceptable.

Find the exact solutions of

(a) $3e^{2x-1} = 12$, giving your answer in the form $a + \ln b$, where a and b are rational numbers, (2)

(b) $\ln(2x+3) - \ln x = 1$, $x > 0$ (3)

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Question 1 continued

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(Total for Question 1 is 5 marks)**Join students already acing their exams.**Book your FREE lesson at www.eclassroom.org.uk

2. In this question you must show all stages of your working.

Solutions relying on calculator technology are not acceptable.

Solve the equation

$$2e^x + 3e^{-x} = 7$$

giving your answers as exact values.

(5)

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Question 2 continued

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3. In this question you must show all stages of your working.

Solutions relying on calculator technology are not acceptable.

$$f(x) = 3 - 2e^{-x} \quad x \in \mathbb{R}$$

(a) Sketch the graph of $y = f(x)$, showing the coordinates of the points where it crosses the coordinate axes and the equation of its asymptote.

(3)

(b) Find the range of f when the domain is restricted to $x \geq 0$

(2)

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Question 3 continued

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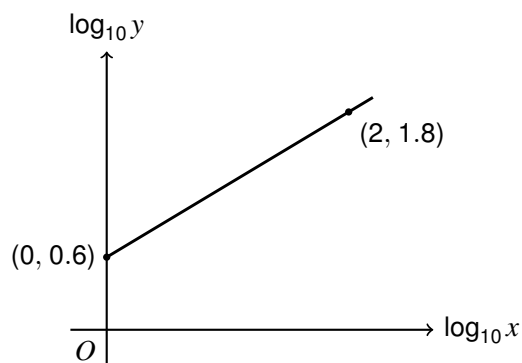
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(Total for Question 3 is 5 marks)**Join students already acing their exams.**Book your FREE lesson at www.eclassroom.org.uk

4.

**Figure 1**

The straight line in Figure 1 shows the relationship between $\log_{10} y$ and $\log_{10} x$. The line passes through the points $(0, 0.6)$ and $(2, 1.8)$.

(a) Write down an equation for the line.

(2)

(b) Hence express y in the form ax^n , giving the value of a to 3 significant figures.

(3)

Question 4 continued

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Solutions relying on calculator technology are not acceptable.

- Sketch the graph of $\log_{10} y$ against $\log_{10} x$, showing the coordinates of the points where it meets the coordinate axes.

(3)

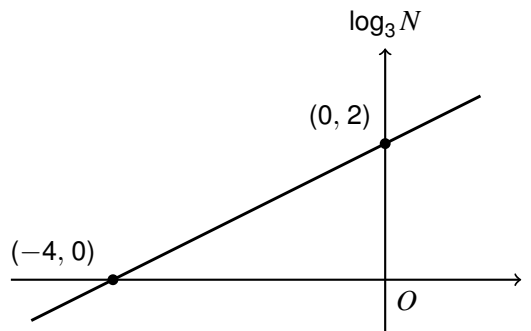


Figure 2

(b) Show that $N = ab^t$, where a and b are exact constants to be found.

(3)



Question 5 continued

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(Total for Question 5 is 6 marks)**Join students already acing their exams.**Book your FREE lesson at www.eclassroom.org.uk

6. The temperature, θ °C, of a cup of coffee t minutes after it is made is modelled by

$$\theta = 20 + Ae^{-kt}$$

where A and k are positive constants. The coffee is made at 90 °C and has cooled to 55 °C after 5 minutes.

- (a) Find the value of A . (1)
- (b) Show that $k = \frac{1}{5} \ln 2$ (3)
- (c) Find, to the nearest tenth of a minute, the time at which the temperature of the coffee is 30 °C. (3)
- (d) Find the rate at which the temperature is decreasing when $t = 5$, giving your answer in °C per minute to 3 significant figures. (3)



Question 6 continued

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Question 6 continued

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(Total for Question 6 is 10 marks)**Join students already acing their exams.**Book your FREE lesson at www.eclassroom.org.uk

7. In this question you must show all stages of your working.**Solutions relying on calculator technology are not acceptable.**The number of birds, P , on an island t years after the start of a study is modelled by

$$P = \frac{500e^{kt}}{4 + e^{kt}}$$

where k is a positive constant.

- (a) Find the number of birds at the start of the study.

(1)

Given that there are 250 birds on the island 10 years after the start of the study,

- (b) show that $k = \frac{1}{5} \ln 2$

(3)

- (c) State the number of birds that the model predicts in the long term, giving a reason.

(2)

- (d) Find, according to the model, the exact number of years after the start of the study at which there are 400 birds.

(3)

- (e) Give one reason why the model may not be appropriate in the long term.

(1)

Question 7 continued

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Question 7 continued

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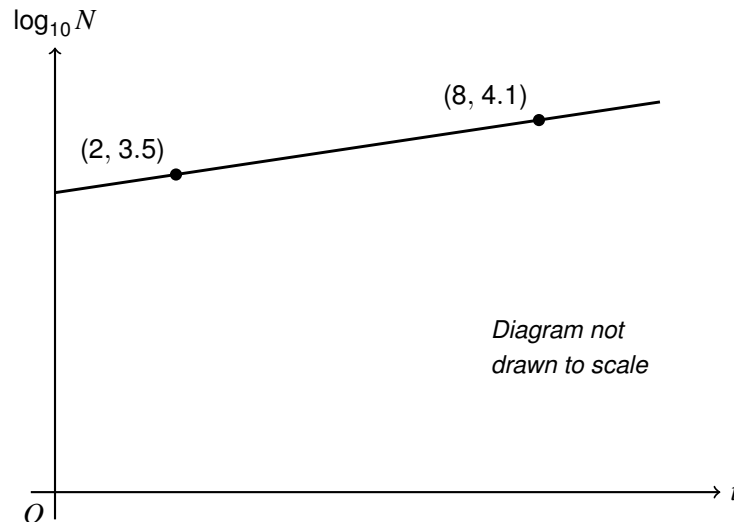
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8.

**Figure 3**

The number of subscribers, N , to an online service t years after it was launched is modelled by

$$N = ab^t$$

where a and b are constants. The line in Figure 3 shows the linear relationship between $\log_{10} N$ and t . The line passes through the points $(2, 3.5)$ and $(8, 4.1)$.

- (a) Find an equation for the line. (2)
- (b) Find the value of a and the value of b , giving the value of b to 3 significant figures. (3)
- (c) Give a practical interpretation of the constant b in this model. (1)
- (d) Find the number of subscribers predicted by the model 10 years after launch, to the nearest thousand. (1)
- (e) Use the model to find the value of t at which the number of subscribers first exceeds 50 000 (2)
- (f) State one reason why the model may not be reliable for large values of t . (1)



Question 8 continued

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Question 8 continued

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Solutions relying on calculator technology are not acceptable.

The functions f and g are defined by

$$f(x) = \ln(2x - 4) + 1 \quad x > 2 \qquad g(x) = e^{2x} \quad x \in \mathbb{R}$$

- (a) Sketch the graph of $y = f(x)$, showing the equation of its asymptote and the exact coordinates of the point where it crosses the x -axis. (3)
- (b) Find $f^{-1}(x)$ and state its domain and range. (3)
- (c) Solve the equation $fg(x) = 1 + \ln 4$, giving your answer in exact form. (4)

Question 9 continued

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Question 9 continued

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(Total for Question 9 is 10 marks)



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10. In this question you must show all stages of your working.**Solutions relying on calculator technology are not acceptable.**

The numbers of two species of insect, A and B , in a garden t days after the start of a study are modelled by

$$A = 40e^{0.1t} \qquad B = 200 - 80e^{-0.1t} \qquad t \geq 0$$

- (a) Find the numbers of each species at the start of the study. (1)
- (b) Show that, when $A = B$, the value $u = e^{0.1t}$ satisfies $u^2 - 5u + 2 = 0$ (3)
- (c) Hence find the exact value of t when $A = B$, giving your answer in the form $10 \ln \left(\frac{p + \sqrt{q}}{2} \right)$.
Explain why only one value of t is possible. (4)
- (d) Find the rate at which the number of species A is increasing at this time, to 3 significant figures. (2)
- (e) Show that the graph of $\ln A$ against t is a straight line, stating its gradient and the exact value of its intercept on the $\ln A$ axis. (2)



Question 10 continued

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Question 10 continued

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TOTAL FOR CHAPTER 5 IS 78 MARKS

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