



eClassroom

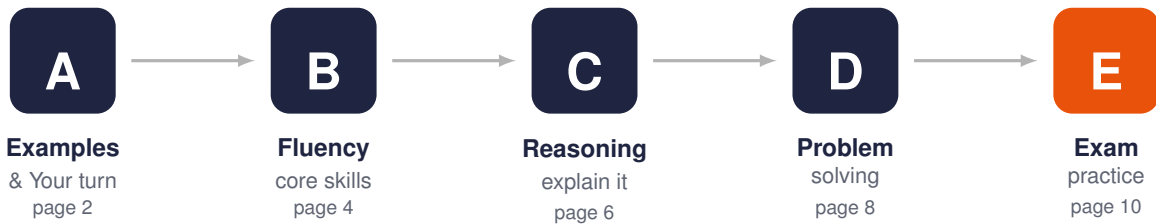
GCSE MATHEMATICS • PROBABILITY

Mixed Sample Space

Worksheet

Pearson Edexcel GCSE (1MA1) & International GCSE (4MA1) • No calculator

Free Mixed Sample Space worksheet for GCSE and iGCSE Maths, with full worked solutions



Tap a section to jump straight to it

By the end of this worksheet you can: list all the outcomes of two events • draw and use sample space diagrams for totals, products and differences • find probabilities and expected numbers from a sample space • *Higher:* probabilities given that something has happened, unknown numbers of outcomes, and three events.

Which questions are for me?

FOUNDATION Everyone starts here. These skills appear on both tiers.

HIGHER The same skills where Higher papers use them: probabilities given information, unknown numbers of sides, three events and algebra. Grades 6 to 9.

Every section has both, clearly labelled. Questions get harder as you go, and every skill in Sections A to D is needed in Section E.

Track your progress

Section	B	C	D	E
Foundation marks	23	10	13	17
Higher marks	16	14	17	15
My score				

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A**Worked examples & Your turn**

Read each example carefully. Cover it up, then answer the question beside it using the same method.

FOUNDATION

Everyone starts here

Worked example A1

Two fair spinners are each numbered 1, 2, 3. The scores are added. Find $P(\text{total} = 4)$.

Draw the grid: $3 \times 3 = 9$ equally likely outcomes.

Totals of 4: $1 + 3, 2 + 2, 3 + 1$

$$\frac{3}{9} = \frac{1}{3}$$

Your turn A1

A fair coin is thrown and a fair spinner numbered 1 to 4 is spun. Find $P(\text{tails and a number greater than 2})$.

Worked example A2

Two fair dice are rolled and the scores are added. Find $P(\text{total} = 9)$.

36 outcomes. Totals of 9: (3,6), (4,5), (5,4), (6,3)

$$\frac{4}{36} = \frac{1}{9}$$

Your turn A2

Two fair dice are rolled and the scores are added. Find $P(\text{total} = 4)$.

Worked example A3

Spinner A: 1, 2, 3, 4. Spinner B: 1, 2, 3. The scores are multiplied. Find $P(\text{product is even})$.

12 outcomes. Even if A is even (6 outcomes), or A is odd and B is 2 (2 more).

$$\frac{8}{12} = \frac{2}{3}$$

Your turn A3

Spinner A: 1, 2, 3. Spinner B: 1, 2, 3, 4, 5. The scores are added. Find $P(\text{total is a prime number})$.



HIGHER*Higher strategies: these are new ways of thinking, not longer sums***Worked example A4: the difference grid**

Two fair dice are rolled. The score is the difference between the numbers. Find $P(\text{difference} = 2)$.

Differences of 2: (1,3), (2,4), (3,5), (4,6) and the reverse

8 of the 36 outcomes

$$\frac{8}{36} = \frac{2}{9}$$

Your turn A4

Two fair dice are rolled. Find $P(\text{the difference is 4 or more})$.

Worked example A5: an unknown number of outcomes

A fair coin is thrown and a fair spinner numbered 1 to n is spun. $P(\text{heads and a 3}) = \frac{1}{16}$. Find n .

There are $2n$ equally likely outcomes, and only one is (H, 3).

$$\frac{1}{2n} = \frac{1}{16}$$

$$n = 8$$

Your turn A5

A fair dice is rolled and a fair spinner numbered 1 to n is spun. $P(\text{both show 1}) = \frac{1}{30}$. Find n .

Worked example A6: given that...

Two fair dice are rolled. The total is 8. Find the probability that both dice show the same number.

Only look at totals of 8: (2,6), (3,5), (4,4), (5,3), (6,2)

1 of these 5 is a double.

$$\frac{1}{5}$$

Your turn A6

Two fair dice are rolled. At least one dice shows a 5. Find the probability that the total is 10

Worked example A7: three events

A family has three children. Each child is a boy or a girl, equally likely. Find $P(\text{at least two girls})$.

8 outcomes: BBB, BBG, BGB, GBB, BGG, GBG, GGB, GGG

At least two girls: 4 of them

$$\frac{4}{8} = \frac{1}{2}$$

Your turn A7

Three fair coins are thrown. Find the probability that all three land the same way up.



B**Fluency**

Each question is a designed sequence. From one part to the next, ask yourself: what changed, and what did it do to the answer?

FOUNDATION*Everyone starts here*

B1 Two fair dice are rolled and the scores are added. Find

- (a) $P(\text{total} = 7)$ (b) $P(\text{total} = 12)$ (c) $P(\text{total} = 1)$ (d) $P(\text{total less than } 4)$
[4 marks]

B2 Two fair spinners are each numbered 1 to 4. The scores are multiplied. Find

- (a) $P(\text{product} = 4)$ (b) $P(\text{product is odd})$ (c) $P(\text{product} = 12)$ (d) $P(\text{product is square})$
[4 marks]

B3 A fair coin is thrown and a fair dice is rolled.

- (a) How many outcomes are there? (b) Find $P(\text{heads and a } 6)$
(c) Find $P(\text{tails and an even number})$ (d) Find $P(\text{heads, or a } 6, \text{ or both})$
[4 marks]

B4 Two fair dice are rolled. The score is the difference between the numbers. Find

- (a) $P(\text{difference} = 0)$ (b) $P(\text{difference} = 5)$ (c) $P(\text{difference} = 1)$ (d) $P(\text{difference} \leq 2)$
[4 marks]

B5 Two fair dice are rolled 360 times. How many times would you expect

- (a) a total of 7 (b) a total of 12 (c) a total of 2 or 12
[3 marks]

B6 Spinner A has four equal sections, 1, 2, 2, 3. Spinner B has two equal sections, 1 and 2. The scores are added. Find

- (a) $P(\text{total} = 3)$ (b) $P(\text{total} = 4)$ (c) $P(\text{total} = 5)$ (d) $P(\text{total is even})$
[4 marks]



HIGHER*Higher tier only***B7** Two fair dice are rolled. Find

- (a) $P(\text{double, given that the total is 6})$
- (b) $P(\text{total more than 6, given that it is a double})$
- (c) $P(\text{double, given that at least one dice shows 6})$
- (d) $P(\text{at least one 6, given that the total is 10 or more})$

[4 marks]**B8** (a) A coin and a spinner numbered 1 to n : $P(\text{heads and 1}) = \frac{1}{20}$. Find n (b) A dice and a spinner numbered 1 to n : $P(\text{both 6}) = \frac{1}{48}$. Find n (c) Two spinners, each numbered 1 to n : $P(\text{both show the same number}) = \frac{1}{5}$. Find n (d) A dice and a spinner numbered 1 to n : $P(\text{total} = 2) = \frac{1}{42}$. Find n **[4 marks]****B9** (a) Three coins are thrown. How many outcomes are there? (b) Find $P(\text{three heads})$ (c) Find $P(\text{at least one head})$

(d) Two coins are thrown and a dice is rolled. How many outcomes are there?

[4 marks]**B10** Two fair dice, each numbered 1 to n , are rolled. Find(a) $P(\text{double})$ when $n = 6$ (b) $P(\text{double})$ when $n = 10$ (c) $P(\text{total} = 2)$ when $n = 10$ (d) $P(\text{first number is bigger than the second})$ when $n = 10$ **[4 marks]**

C**Reasoning**

Show your working and explain your thinking. A correct answer without a reason does not score full marks.

FOUNDATION

Everyone starts here

- C1** Kai says, “Two dice can give 11 different totals, from 2 to 12, so $P(\text{total} = 7) = \frac{1}{11}$ ”
Explain Kai’s mistake and find the correct probability. [2 marks]
- C2** Two fair coins are thrown. Mia says, “Either there is a head or there is not, so $P(\text{at least one head}) = \frac{1}{2}$ ”
Explain why Mia is wrong and find the correct probability. [2 marks]
- C3** Two fair dice are rolled and the scores are added.
Show that 7 is the most likely total. Use a sample space diagram. [3 marks]
- C4** A fair spinner numbered 1 to 3 is spun and a fair coin is thrown. Ali lists the outcomes: 1H, 2H, 3H, 1T, 2T
(a) What has Ali missed? (b) Find $P(\text{an odd number and tails})$. [3 marks]



HIGHER*Higher tier only*

C5 Two fair dice are rolled. The total is 8

Jo says, "The probability that the dice show a double is $\frac{1}{36}$ "

Explain Jo's mistake and find the correct probability.

[3 marks]

C6 Two fair dice are rolled. Which is more likely: at least one 6, or a total of 7? You must show your working.

[3 marks]

C7 Two fair dice are rolled and the scores are multiplied.

Show that the probability that the product is even is $\frac{3}{4}$

[3 marks]

C8 Two fair dice, each numbered 1 to n , are rolled.

(a) Show that the probability that the total is greater than n is $\frac{n+1}{2n}$

(b) This probability is $\frac{9}{16}$. Find n .

[5 marks]



D**Problem solving**

You decide the method. Read the whole question first, and show every step.

FOUNDATION

Everyone starts here

- D1** A fair coin is thrown and a fair dice is rolled. If the coin shows heads, the score is double the dice number. If it shows tails, the score is the dice number plus 1. Find the probability that the score is 8 or more. [3 marks]
- D2** Two fair dice are rolled. The score is the larger of the two numbers (if they are the same, the score is that number).

	1	2	3	4	5	6
1	1	2				
2	2					
3						
4						
5						
6						

- Complete the sample space. Find $P(\text{score} = 6)$ and $P(\text{score} = 3)$. [4 marks]
- D3** Two fair spinners each have four equal sections coloured red, blue, green and yellow. Both are spun. Find the probability that they land on different colours. [3 marks]
- D4** A game costs 50p to play. Two fair dice are rolled, and a total of 10 or more wins £2. The game is played 180 times. Work out the expected profit for the person running the game. [3 marks]



HIGHER*Higher tier only*

- D5** Dice A has faces 1, 1, 2, 2, 3, 3. Dice B has faces 0, 0, 0, 2, 2, 2. Both are fair and are rolled together, and the scores are added.
(a) Find $P(\text{total} = 3)$ (b) Find $P(\text{total is even})$ [4 marks]
- D6** Bag A holds cards numbered 1, 2, 3, 4. Bag B holds cards numbered 1, 2, 3. One card is taken at random from each bag.
Find $P(\text{the card from A is bigger})$, $P(\text{the cards are equal})$ and $P(\text{the card from B is bigger})$. [4 marks]
- D7** Two fair dice are rolled. Jo is told that the total is 10. She bets that the dice show a double.
(a) Find the probability that Jo wins her bet.
(b) Instead, Jo is told that at least one dice shows a 5. Find the probability that the dice show a double. [4 marks]
- D8** A fair dice numbered 1 to 6 is rolled and a fair spinner numbered 1 to k is spun, where k is a whole number. The score is the difference between the two numbers.
The probability that the score is 0 is $\frac{1}{8}$
(a) Find k . (b) Find the probability that the score is 5 or more. [5 marks]



E**Exam practice**

Exam-style questions. Write your answers in the spaces. Aim for about one minute per mark.

FOUNDATION

Foundation and Higher crossover

- E1** Spinner A is numbered 1, 2, 3. Spinner B is numbered 1, 2, 3, 4. Both are fair. The scores are added.

	1	2	3	4
1	2	3		
2	3			
3				7

Rows: spinner A. Columns: spinner B.

- (a) Complete the sample space diagram.

(2)

- (b) Find the probability that the total is 5

.....

(1)

(Total for Question E1 is 3 marks)

- E2** A fair coin is thrown and a fair dice is rolled.

- (a) Find the probability of heads and a square number.

.....

(1)

- (b) Find the probability of tails, or an odd number, or both.

.....

(2)

(Total for Question E2 is 3 marks)



E3 Two fair dice are rolled and the scores are multiplied.

(a) Find the probability that the product is 12

.....
(2)

(b) Find the probability that the product is a square number.

.....
(1)

(Total for Question E3 is 3 marks)

E4 Sam and Jo play a game with two fair dice. Sam wins if the total is 7. Jo wins if the total is 6 or 8.
Is the game fair? You **must** show your working.

(Total for Question E4 is 3 marks)

E5 Two fair coins are thrown 200 times. How many times would you expect them both to land on heads?

.....
(Total for Question E5 is 2 marks)

E6 Spinner A is numbered 1 to 5 and spinner B is numbered 1 to 3. Both are fair. The score is the difference between the two numbers.

Which score is most likely? Find its probability.

.....
(Total for Question E6 is 3 marks)



HIGHER*Higher tier only*

- E7** Two fair dice are rolled. Find the probability that the difference between the numbers is 1 or less.

.....

(Total for Question E7 is 3 marks)

- E8** Spinner A is numbered 1, 2, 3. Spinner B is numbered 2, 4, x , where x is a whole number greater than 4. Both are fair and the scores are added.

The probability that the total is more than 7 is $\frac{2}{9}$. Find x .

.....

(Total for Question E8 is 4 marks)

- E9** Three fair coins are thrown.

(a) Find the probability of exactly two heads.

.....

(2)

(b) Find the probability of at least one tail.

.....

(1)**(Total for Question E9 is 3 marks)**

E10 Spinner A is numbered 1 to 5 and spinner B is numbered 1 to 4. Both are fair. The scores are multiplied.

(a) Draw a sample space diagram.

(2)

(b) Find the probability that the product is odd.

.....

(1)

(c) The product is even. Find the probability that it is more than 10

.....

(2)

(Total for Question E10 is 5 marks)

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