Trigonometric Ratios

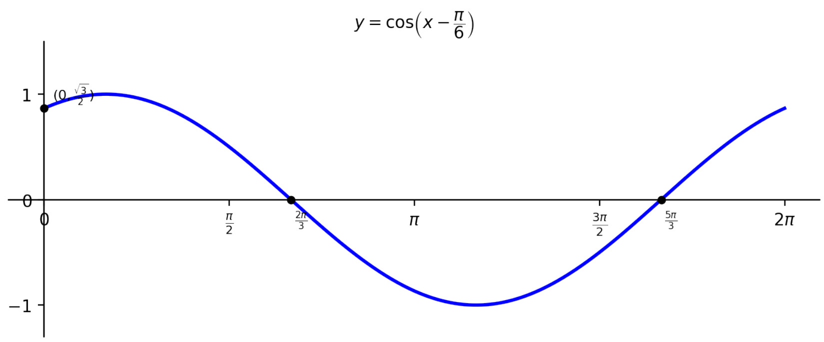
Worked Solutions

Suitable for:

Pearson Edexcel International A-Level (iAL) — Pure Mathematics 1

Question 1 — Worked Solution

(a) Sketch



(b) Axis intercepts

y-intercept (x = 0): y = cos(0 – π/6) = cos(–π/6) = √3/2

x-intercepts: cos(x – π/6) = 0 → x – π/6 = π/2 or 3π/2

$$x = \frac{\pi}{6} + \frac{\pi}{2} = \frac{2\pi}{3} \quad \text{and} \quad x = \frac{\pi}{6} + \frac{3\pi}{2} = \frac{5\pi}{3}$$

$$\therefore y\text{-intercept: } \left(0, \frac{\sqrt{3}}{2}\right); \quad x\text{-intercepts: } \left(\frac{2\pi}{3}, 0\right), \left(\frac{5\pi}{3}, 0\right)$$

(c) Solve cos(x – π/6) = 1/√2

1/√2 = cos(π/4), so:

$$x - \frac{\pi}{6} = \frac{\pi}{4} \quad \text{or} \quad x - \frac{\pi}{6} = -\frac{\pi}{4} + 2\pi = \frac{7\pi}{4}$$

$$x = \frac{\pi}{6} + \frac{\pi}{4} = \frac{2\pi + 3\pi}{12} = \frac{5\pi}{12}$$

$$x = \frac{\pi}{6} + \frac{7\pi}{4} = \frac{2\pi + 21\pi}{12} = \frac{23\pi}{12}$$

$$\therefore x = \frac{5\pi}{12} \quad \text{and} \quad x = \frac{23\pi}{12}$$

Question 2 — Worked Solution

(a) Axis intercepts of y = sin(x – 30°)

y-intercept (x = 0): y = sin(–30°) = –1/2. So (0, –1/2).

x-intercepts: sin(x – 30°) = 0 → x – 30° = 0°, ±180°, ±360°, ...

$$x = 30^\circ,\ 210^\circ,\ -150^\circ,\ -330^\circ$$

$\therefore$ y-intercept: $(0,\ -\frac{1}{2})$; x-intercepts: $(-330^\circ, 0)$, $(-150^\circ, 0)$, $(30^\circ, 0)$, $(210^\circ, 0)$

(b) Solve 4sin(x – 30°) = √6 – √2

Divide both sides by 4:

$$\sin(x - 30°) = \frac{\sqrt{6} - \sqrt{2}}{4} = \sin 15°$$

(using the exact value sin 15° = (√6 – √2)/4)

Principal solution: x – 30° = 15^{\circ} → x = 45°

Second solution in [–360°, 360°]: x – 30° = 180° – 15^{\circ} = 165^{\circ} → x = 195°

Subtract 360°: x = 45° – 360° = –315° and x = 195° – 360° = –165°

$$\therefore x = -315^\circ,\ -165^\circ,\ 45^\circ,\ 195^\circ$$

Question 3 — Worked Solution

(a) Length of BD

In triangle ABD, using the cosine rule:

$$BD^2 = AB^2 + AD^2 - 2 \cdot AB \cdot AD \cdot \cos(\angle DAB)$$

$$= 10^2 + 6^2 - 2(10)(6)\cos 75°$$

$$= 100 + 36 - 120\cos 75° = 136 - 120(0.2588\ldots) = 136 - 31.058\ldots$$

$$BD^2 = 104.942\ldots \Rightarrow BD = 10.244\ldots$$

$$\therefore BD = 10.24\ \text{m}$$

(b) Area of ABCD

Diagonal BD splits the quadrilateral into triangles ABD and BCD.

Area of triangle ABD:

$$\text{Area}_{ABD} = \tfrac{1}{2} \cdot AB \cdot AD \cdot \sin(\angle DAB) = \tfrac{1}{2}(10)(6)\sin 75^\circ = 28.98\ \text{m}^2$$

Area of triangle ABC (using AB=10, BC=8, ∠ABC=70°):

$$\text{Area}_{ABC} = \tfrac{1}{2} \cdot AB \cdot BC \cdot \sin(\angle ABC) = \tfrac{1}{2}(10)(8)\sin 70^\circ = 37.59\ \text{m}^2$$

Total area:

$$\text{Area} = 28.98 + 37.59 = 66.57 \approx 67\ \text{m}^2$$

$$\therefore \text{Area of } ABCD \approx 67\ \text{m}^2$$

Question 4 — Worked Solution

(a) Show sin ACB = 5/8

Using the sine rule in triangle ABC:

$$\frac{\sin ACB}{AB} = \frac{\sin BAC}{BC}$$

$$\frac{\sin ACB}{5x} = \frac{\sin 30^\circ}{4x}$$

$$\sin ACB = \frac{5x \cdot \sin 30^\circ}{4x} = \frac{5 \times \frac{1}{2}}{4} = \frac{5}{8} \quad \checkmark$$

(b) Angle ABC (ACB obtuse)

Since ACB is obtuse: ACB = 180° – arcsin(5/8) = 180° – 38.68° = 141.32°

$$\angle ABC = 180° - 30° - 141.32° = 8.68°$$

$$\therefore \angle ABC = 8.68°$$

(c) Value of x (area = 20)

$$\text{Area} = \tfrac{1}{2} \cdot AB \cdot BC \cdot \sin(\angle ABC) = \tfrac{1}{2}(5x)(4x)\sin 8.68^\circ = 10x^2 \sin 8.68^\circ = 20$$

$$x^2 = \frac{20}{10\sin 8.68^\circ} = \frac{2}{0.1508\ldots} = 13.26\ldots$$

$$\therefore x = 3.64$$

(d) Length AC

AB = 5 × 3.64 = 18.20. Using sine rule:

$$AC = \frac{AB \cdot \sin(\angle ABC)}{\sin(\angle ACB)} = \frac{18.20 \times \sin 8.68^\circ}{\sin 141.32^\circ} = \frac{18.20 \times 0.1508}{0.6250} = 4.39\ldots$$

$$\therefore AC = 4.40 \text{ (to 2 d.p.)}$$

Question 5 — Worked Solution

(a) Possible sizes of angle ABC

In triangle ABC, using sine rule:

$$\frac{\sin(\angle ACB)}{AB} = \frac{\sin(\angle CAB)}{BC}$$

$$\sin(\angle ACB) = \frac{7\sin 20^\circ}{5} = \frac{7 \times 0.3420}{5} = 0.4788$$

$$\angle ACB = 28.6° \quad \text{or} \quad \angle ACB = 151.4°$$

$$\angle ABC = 180° - 20° - 28.6° = 131.4° \quad \text{or} \quad \angle ABC = 180° - 20° - 151.4° = 8.6°$$

$$\therefore \angle ABC = 131.4° \text{ or } 8.6°$$

(b) Length of AC (angle ABC obtuse = 131.4°)

Using sine rule with ∠ABC = 131.4°, ∠ACB = 28.6°:

$$AC = \frac{BC \cdot \sin(\angle ABC)}{\sin(\angle CAB)} = \frac{5 \times \sin 131.4^\circ}{\sin 20^\circ} = \frac{5 \times 0.7501}{0.3420} = 10.97 \text{ cm}$$

$$\therefore AC = 10.97 \text{ cm}$$

(c) Area of parallelogram

Area of parallelogram = AB × BC × sin(∠ABC) = 7 × 5 × sin(131.4°)

$$= 35 \times 0.7501 = 26.25 \text{ cm}^2$$

$$\therefore \text{Area} = 26.3 \text{ cm}^2 \text{ (3 s.f.)}$$

Question 6 — Worked Solution

(a) Distance AC

From B, A is due north (bearing 000°) and C is on bearing 070°.

Angle ABC = 70°. AB = 12, BC = 9. Cosine rule:

$$AC^2 = AB^2 + BC^2 - 2 \cdot AB \cdot BC \cdot \cos(\angle ABC)$$

$$= 144 + 81 - 2(12)(9)\cos 70° = 225 - 216\cos 70°$$

$$= 225 - 216(0.3420) = 225 - 73.87 = 151.13$$

$$AC = \sqrt{151.13} = 12.2932\ldots$$

$$\therefore AC = 12.29 \text{ km (to nearest 10 m)}$$

(b) Bearing of C from A

Use sine rule to find angle BAC:

$$\frac{\sin(\angle BAC)}{BC} = \frac{\sin(\angle ABC)}{AC} \Rightarrow \sin(\angle BAC) = \frac{9\sin 70°}{12.2932} = 0.6882$$

$$\angle BAC = 43.5°$$

From A, B is due south (bearing 180°). C is to the east of AB, so:

$$\text{Bearing of } C \text{ from } A = 180° - 43.5° = 136.5°$$

$$\therefore \text{Bearing of } C \text{ from } A = 136.5°$$

Question 7 — Worked Solution

(a) Show angle APB = 88°

Bearing of A from P = 320°. Bearing of B from P = 048°.

Angle between the two bearings:

$$\angle APB = 360° - 320° + 48° = 88° \quad ✓$$

(b) Distance AB

PA = 9, PB = 4, ∠APB = 88°. Cosine rule:

$$AB^2 = 9^2 + 4^2 - 2(9)(4)\cos 88° = 81 + 16 - 72\cos 88°$$

$$= 97 - 72(0.03490) = 97 - 2.513 = 94.487$$

$$AB = \sqrt{94.487} = 9.7205\ldots$$

$$\therefore AB = 9.7 \text{ km (to 1 d.p.)}$$

(c) Bearing of B from A

Use sine rule to find angle PAB:

$$\frac{\sin(\angle PAB)}{PB} = \frac{\sin(\angle APB)}{AB} \Rightarrow \sin(\angle PAB) = \frac{4\sin 88^\circ}{9.7205} = 0.4114$$

$$\angle PAB = 24.3°$$

From A, the bearing of P is 320° – 180° = 140°.

B is to the right (east) of AP, so:

$$\text{Bearing of } B \text{ from } A = 140° + 24.3° = 164.3°$$

$$\therefore \text{Bearing of } B \text{ from } A = 164°$$