eClassroom

A-LEVEL MATHEMATICS • MECHANICS 2

# Impulses and Collisions

Worksheet

Pearson Edexcel International A Level (WME02) • Calculator allowed: take $g = 9.8\,\text{m s}^{-2}$ and give answers to 2 or 3 s.f.

Free Impulses and Collisions worksheet for A-Level and International A-Level Maths, with full worked solutions


A **Examples** & Your turn page 2

B **Fluency** core skills page 4

C **Reasoning** explain it page 6

D **Problem** solving page 7

E **Exam** practice page 9


Tap a section to jump straight to it

**By the end of this worksheet you can:** use impulse as a vector • apply conservation of momentum and Newton's law of restitution • find the rebound from a smooth wall • find the kinetic energy lost • *Stretch:* impulses at an angle, results in terms of $e$, and whether a further collision happens.

## Which questions are for me?

CORE The methods every A-level student must secure. Start here.

STRETCH Impulses at an angle, answers in terms of $e$, walls, second collisions and ranges of values.

Every section has both, clearly labelled. Questions get harder as you go, and every skill in Sections A to D is needed in Section E.

## Track your progress

| Section | B | C | D | E |
|---|---|---|---|---|
| Core marks | 23 | 10 | 13 | 20 |
| Stretch marks | 16 | 14 | 17 | 13 |
| My score | | | | |


Full worked solutions | Exam questions by topic | Mechanics 2 exam questions | Formula sheet builder


Name ..... Class ..... Date .....


★★★★★

**Join students already acing their exams.**

Book your FREE lesson at www.eclassroom.org.uk

# A Worked examples & Your turn

Read each example carefully. Cover it up, then answer the question beside it using the same method.

**CORE** *Secure the method*

### Worked example A1: impulse as a vector

A ball of mass 0.5 kg moving with velocity $(4\mathbf{i} - 2\mathbf{j})\,\text{m s}^{-1}$ receives an impulse of $(-3\mathbf{i} + 4\mathbf{j})\,\text{N s}$. Find its speed immediately afterwards.

$\mathbf{I} = m\mathbf{v} - m\mathbf{u}$

$-3\mathbf{i} + 4\mathbf{j} = 0.5\mathbf{v} - (2\mathbf{i} - \mathbf{j}),\ \mathbf{v} = -2\mathbf{i} + 6\mathbf{j}$

$\sqrt{40} = 6.32\,\text{m s}^{-1}$

### Your turn A1

A ball of mass 0.25 kg moving with velocity $(6\mathbf{i} + \mathbf{j})\,\text{m s}^{-1}$ receives an impulse of $(-2\mathbf{i} + 0.5\mathbf{j})\,\text{N s}$. Find its speed immediately afterwards.

### Worked example A2: a direct collision

A sphere $A$ of mass 3 kg moving at $4\,\text{m s}^{-1}$ hits a sphere $B$ of mass 1 kg at rest. The coefficient of restitution is $\frac{1}{2}$. Find their velocities after the impact.

*Momentum, then separation = $e\times$ approach*

$12 = 3v_A + v_B;\ v_B - v_A = 2$

$v_A = 2.5\,\text{m s}^{-1},\ v_B = 4.5\,\text{m s}^{-1}$

### Your turn A2

A sphere $A$ of mass 2 kg moving at $5\,\text{m s}^{-1}$ catches up with a sphere $B$ of mass 3 kg moving at $1\,\text{m s}^{-1}$ in the same direction. The coefficient of restitution is 0.4. Find their velocities after the impact.

### Worked example A3: a smooth wall

A ball of mass 0.4 kg hits a smooth vertical wall at right angles with speed $5\,\text{m s}^{-1}$. The coefficient of restitution is 0.6. Find the magnitude of the impulse on the ball.

*Rebound at $eu$; the direction changes*

$0.6(5) = 3;\ 0.4(3 - (-5))$

3.2 N s

### Your turn A3

A ball of mass 0.3 kg hits a smooth vertical wall at right angles with speed $8\,\text{m s}^{-1}$. The coefficient of restitution is 0.75. Find the magnitude of the impulse on the ball.

★★★★★
**Join students already acing their exams.**
Book your FREE lesson at www.eclassroom.org.uk

## STRETCH

### Worked example A4: an impulse at an angle

A particle of mass 0.2 kg moving at $10\,\text{m s}^{-1}$ receives an impulse of 3 N s at 60° to its direction of motion. Find its new speed.

*Triangle of momenta:* $|m\mathbf{v}|^2 = 2^2 + 3^2 + 2(2)(3)\cos 60°$

$$|m\mathbf{v}| = \sqrt{19},\ v = \frac{\sqrt{19}}{0.2}$$

$21.8\,\text{m s}^{-1}$

### Your turn A4

A particle of mass 0.5 kg moving at $6\,\text{m s}^{-1}$ receives an impulse of 4 N s at right angles to its direction of motion. Find its new speed and the angle its direction turns through.

### Worked example A5: kinetic energy lost

For the collision in Worked example A2, find the kinetic energy lost.

*Total KE before − total KE after*

$$\tfrac{1}{2}(3)(4^2) - \left[\tfrac{1}{2}(3)(2.5^2) + \tfrac{1}{2}(1)(4.5^2)\right]$$

4.5 J

### Your turn A5

For the collision in Your turn A2, find the kinetic energy lost.

### Worked example A6: finding $e$

$P$ (2 kg, $5\,\text{m s}^{-1}$) and $Q$ (1 kg, $4\,\text{m s}^{-1}$) move towards each other and collide. Afterwards $Q$ moves back at $5\,\text{m s}^{-1}$. Find $P$'s velocity and $e$

$$10 - 4 = 2v_P + 5,\ v_P = 0.5;\ e = \frac{5 - 0.5}{5 + 4}$$

$0.5\,\text{m s}^{-1}$, $e = \frac{1}{2}$

### Your turn A6

$P$ (3 kg, $4\,\text{m s}^{-1}$) and $Q$ (2 kg, $2\,\text{m s}^{-1}$) move towards each other and collide. Afterwards $Q$ moves back at $3.5\,\text{m s}^{-1}$. Find $P$'s velocity and $e$

### Worked example A7: a further collision?

After a collision, $A$ moves away from a wall at $0.5\,\text{m s}^{-1}$ and $B$ moves towards the wall at $3\,\text{m s}^{-1}$. $B$ rebounds with coefficient $f$. Find the values of $f$ for which $A$ and $B$ collide again.

*$B$ must catch $A$: rebound speed* $3f > 0.5$

$f > \frac{1}{6}$

### Your turn A7

After a collision, $A$ moves away from a wall at $0.8\,\text{m s}^{-1}$ and $B$ moves towards the wall at $2\,\text{m s}^{-1}$. $B$ rebounds with coefficient $f$. Find the values of $f$ for which they collide again.

★★★★★
Join students already acing their exams.
Book your FREE lesson at www.eclassroom.org.uk

## B Fluency

Each question is a designed sequence. From one part to the next, ask yourself: what changed, and what did it do to the answer?

**CORE** *Secure the method*

**B1** Find the impulse on the particle in each case.

**(a)** 2 kg: from $3\,\text{m s}^{-1}$ to $7\,\text{m s}^{-1}$ in the same direction (magnitude)

**(b)** 2 kg: from $3\,\text{m s}^{-1}$ to $7\,\text{m s}^{-1}$ in the opposite direction (magnitude)

**(c)** 0.5 kg: from $(2\mathbf{i}+3\mathbf{j})\,\text{m s}^{-1}$ to $(5\mathbf{i}-\mathbf{j})\,\text{m s}^{-1}$ (vector)

**(d)** The magnitude of the impulse in part (c)

**[4 marks]**

**B2** $A$ (2 kg) moving at $6\,\text{m s}^{-1}$ hits $B$ (1 kg) at rest. Find the velocities after impact when

**(a)** $e = 0$

**(b)** $e = 1$

**(c)** $e = \frac{1}{2}$

**(d)** Find the kinetic energy lost when $e = \frac{1}{2}$

**[4 marks]**

**B3** A ball of mass 0.5 kg hits a smooth wall at right angles at $6\,\text{m s}^{-1}$

**(a)** $e = 0.5$: find the rebound speed.

**(b)** Find the magnitude of the impulse.

**(c)** Find the kinetic energy lost.

**(d)** Find $e$ if half the kinetic energy is lost.

**[4 marks]**

**B4** In each collision, find the quantity asked for.

**(a)** $A$ at $4\,\text{m s}^{-1}$ hits $B$ at rest; then $A$ moves on at $1\,\text{m s}^{-1}$ and $B$ at $2.5\,\text{m s}^{-1}$: find $e$

**(b)** $A$ at $5\,\text{m s}^{-1}$ and $B$ at $3\,\text{m s}^{-1}$ approach; both reverse, $A$ at $1\,\text{m s}^{-1}$, $B$ at $4\,\text{m s}^{-1}$: find $e$

**(c)** For part (b), find $m_A : m_B$

**(d)** Equal masses, $A$ at $u$ hits $B$ at rest, $e = 1$: describe the motion.

**[4 marks]**

**B5** A particle of mass 0.2 kg moving at $5\mathbf{i}\,\text{m s}^{-1}$ is given an impulse, and its velocity becomes $(2\mathbf{i}+4\mathbf{j})\,\text{m s}^{-1}$

**(a)** Find the impulse as a vector.

**(b)** Find its magnitude.

**(c)** Find the angle between the impulse and the original direction of motion. **[3 marks]**

★★★★★
**Join students already acing their exams.**
Book your FREE lesson at www.eclassroom.org.uk

**B6** Find the kinetic energy lost in each collision.

**(a)** 1 kg at $6\,\text{m s}^{-1}$ catches 1 kg at $2\,\text{m s}^{-1}$ and they coalesce

**(b)** The same spheres with $e = 1$

**(c)** 2 kg at $4\,\text{m s}^{-1}$ hits 2 kg at rest, $e = \frac{1}{2}$

**(d)** Show that the loss in (c) is $\frac{3}{4}$ of the loss when $e = 0$

**[4 marks]**

## STRETCH

**B7** A particle of mass 0.4 kg moving at $5\,\text{m s}^{-1}$ receives an impulse of 3 N s at an angle $\theta$ to its direction of motion. Find its new speed when

**(a)** $\theta = 90^\circ$

**(b)** $\theta = 180^\circ$

**(c)** $\theta = 60^\circ$

**(d)** Find $\theta$ if its speed is unchanged.

**[4 marks]**

**B8** A sphere $A$ of mass $m$ moving with speed $u$ hits a sphere $B$ of mass $2m$ at rest. The coefficient of restitution is $e$

**(a)** Find the velocity of $A$ after impact.

**(b)** Find the velocity of $B$ after impact.

**(c)** Find the values of $e$ for which $A$ reverses.

**(d)** Show that the KE lost is $\frac{1}{3}(1 - e^2)mu^2$

**[4 marks]**

**B9** After a collision, $A$ (2 kg) moves away from a wall at $1\,\text{m s}^{-1}$ and $B$ (1 kg) moves towards it at $4\,\text{m s}^{-1}$. $B$ rebounds with coefficient $f$

**(a)** Find $B$'s speed after the wall.

**(b)** Find the magnitude of the impulse from the wall.

**(c)** Find the values of $f$ for a second collision.

**(d)** $f = \frac{1}{2}$ and $e = \frac{1}{2}$ between $A$ and $B$: find their speeds after the second collision.

**[4 marks]**

**B10** Three identical spheres $A$, $B$ and $C$ lie in a straight line. $A$ moves at $u$ towards $B$; $B$ and $C$ are at rest. For every impact $e = \frac{1}{2}$

**(a)** Find the velocities of $A$ and $B$ after the first impact.

**(b)** Find the velocities of $B$ and $C$ after $B$ hits $C$

**(c)** Show that $A$ and $B$ collide again.

**(d)** Find the fraction of the original KE remaining before that collision.

**[4 marks]**

# C Reasoning

Show your working and explain your thinking. A correct answer without a reason does not score full marks.

## CORE *Secure the method*

**C1** Asked for the magnitude of an impulse, Amir writes $(0.6\mathbf{i} + 2.1\mathbf{j})$ N s. Explain his error, and find the magnitude. **[2 marks]**

**C2** A ball of mass 0.4 kg hits a wall at $5\,\text{m s}^{-1}$ and rebounds at $2\,\text{m s}^{-1}$. Bea writes the impulse as $0.4(5-2) = 1.2$ N s. Explain her error, and find the impulse. **[2 marks]**

**C3** $A$ at $5\,\text{m s}^{-1}$ and $B$ at $2\,\text{m s}^{-1}$ move towards each other and collide. Afterwards both have reversed, $A$ at $1\,\text{m s}^{-1}$ and $B$ at $3\,\text{m s}^{-1}$. Cal writes $e = \dfrac{3-1}{5-2} = \dfrac{2}{3}$

Explain his error, and find $e$ **[3 marks]**

**C4** $A$ (2 kg) at $5\,\text{m s}^{-1}$ hits $B$ (2 kg) at rest. Afterwards $A$ moves at $1\,\text{m s}^{-1}$ and $B$ at $4\,\text{m s}^{-1}$. Dev says the kinetic energy lost is $\frac{1}{2}(2)(5^2) - \frac{1}{2}(2)(1^2) = 24$ J.

Explain his error, and find the kinetic energy lost. **[3 marks]**

## STRETCH

**C5** A particle of mass 0.5 kg moving at $4\,\text{m s}^{-1}$ receives an impulse of 3 N s at $60^\circ$ to its direction of motion. Eve resolves parallel to the motion as $2 + 3\sin 60^\circ$ and perpendicular as $3\cos 60^\circ$

Explain her error, and find the new speed. **[3 marks]**

**C6** After a collision, $A$ moves away from a wall at $1.5\,\text{m s}^{-1}$. $B$ hits the wall and rebounds at $1.5\,\text{m s}^{-1}$, following $A$. Fay says they must collide again because $B$ is moving towards $A$. Explain her error, and state the condition for a second collision.

**[3 marks]**

**C7** A sphere $Q$ of mass $2m$ moving at $3u$ hits a smooth wall at right angles and rebounds at $u$. Gus writes the impulse as $m(3u + u) = 4mu$

Explain his error, and find the impulse. **[3 marks]**

**C8** A sphere $A$ of mass $m$ moving with speed $u$ hits a sphere $B$ of mass $km$ at rest. The coefficient of restitution is $e$

**(a)** Show that the velocities of $A$ and $B$ after impact are $\dfrac{u(1-ke)}{1+k}$ and $\dfrac{u(1+e)}{1+k}$

**(3)**

**(b)** Show that $A$ is brought to rest only if $ke = 1$, and explain why this needs $k \geqslant 1$

**(2)**

**[5 marks]**

# D Problem solving

You decide the method. Read the whole question first, and show every step.

CORE *Secure the method*

**D1** [In this question **i** and **j** are horizontal perpendicular unit vectors.] A particle $P$ of mass 0.5 kg is moving on a smooth horizontal surface with velocity $(4\mathbf{i} - 2\mathbf{j})\,\text{m s}^{-1}$ when it receives an impulse of $(-\mathbf{i} + 3\mathbf{j})\,\text{N s}$.

**(a)** Find the speed of $P$ immediately after receiving the impulse.

**(2)**

**(b)** Find the angle through which the direction of motion of $P$ is turned.

**(1)**

**[3 marks]**

**D2** Two particles $P$ and $Q$, of masses $2m$ and $3m$, move towards each other along the same straight line on a smooth horizontal surface, with speeds $3u$ and $2u$, as shown in Figure 1. They collide directly, and the direction of motion of each particle is reversed. The coefficient of restitution between them is $\frac{1}{2}$

**(a)** Find the speeds of $P$ and $Q$ after the collision.

**(3)**

**(b)** Find the magnitude of the impulse received by $Q$

**(1)**

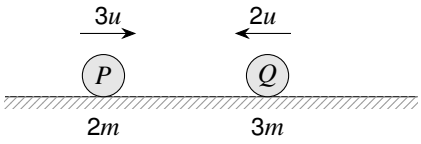



**Figure 1**

**[4 marks]**

**D3** [In this question **i** and **j** are horizontal perpendicular unit vectors.] A particle of mass 1 kg moving with velocity $4\mathbf{i}\,\text{m s}^{-1}$ receives a horizontal impulse $\mathbf{I}\,\text{N s}$ of magnitude $\sqrt{10}\,\text{N s}$. Immediately afterwards its velocity is $\lambda(\mathbf{i} + \mathbf{j})\,\text{m s}^{-1}$, where $\lambda$ is a constant, and its speed has increased.

**(a)** Find the value of $\lambda$

**(2)**

**(b)** Find $\mathbf{I}$ in terms of **i** and **j**

**(1)**

**[3 marks]**

**D4** [In this question **i** and **j** are horizontal perpendicular unit vectors.] A particle $A$ of mass 1 kg moving with velocity $(2\mathbf{i} + 3\mathbf{j})\,\text{m s}^{-1}$ collides with a particle $B$ of mass 2 kg moving with velocity $(-\mathbf{i} + \mathbf{j})\,\text{m s}^{-1}$. Immediately after the collision, the velocity of $A$ is $(-\mathbf{i} + 2\mathbf{j})\,\text{m s}^{-1}$

**(a)** Find the impulse received by $A$ in the collision.

**(1)**

**(b)** Find the velocity of $B$ immediately after the collision.

**(2)**

**[3 marks]**

## STRETCH

**D5** A particle $P$ of mass $m$ moving with speed $u$ collides directly with a particle $Q$ of mass $3m$ at rest on a smooth horizontal surface, and the direction of motion of $P$ is reversed. The coefficient of restitution is $e$. $Q$ then hits a smooth vertical wall at right angles, as shown in Figure 2, and the coefficient of restitution between $Q$ and the wall is $\frac{1}{2}$

**(a)** Find the speed of $P$ after the collision in terms of $u$ and $e$

**(2)**

**(b)** Given that there is a second collision between $P$ and $Q$, find the range of possible values of $e$

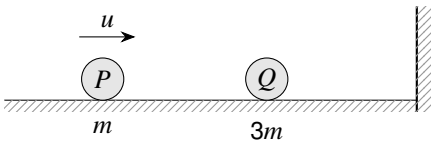



**Figure 2**

**(2)**

**[4 marks]**

**D6** A particle $A$ of mass $3m$ moving with speed $u$ collides directly with a particle $B$ of mass $m$ at rest on a smooth horizontal surface. The coefficient of restitution between $A$ and $B$ is $e$

**(a)** Show that the kinetic energy lost in the collision is $\frac{3}{8}(1-e^2)mu^2$

**(2)**

**(b)** The magnitude of the impulse received by $B$ is $\frac{6}{5}mu$. Find the value of $e$

**(2)**

**[4 marks]**

**D7** A particle of mass 0.5 kg is moving on a smooth horizontal surface at $6\,\text{m s}^{-1}$ when it receives a horizontal impulse of magnitude $I$ N s, acting at $60^\circ$ to its direction of motion. Immediately afterwards its speed is $14\,\text{m s}^{-1}$

**(a)** Find the value of $I$

**(3)**

**(b)** Find the angle through which the direction of motion of the particle is turned.

**(1)**

**[4 marks]**

**D8** A particle $A$ of mass $2m$ moving with speed $u$ collides directly with a particle $B$ of mass $m$ at rest on a smooth horizontal surface. The coefficient of restitution between $A$ and $B$ is $\frac{1}{2}$. After the collision $B$ hits a smooth vertical wall at right angles, as shown in Figure 3, and the coefficient of restitution between $B$ and the wall is also $\frac{1}{2}$

**(a)** Find the speeds of $A$ and $B$ after their first collision.

**(2)**

**(b)** Show that $A$ and $B$ collide again.

**(1)**

**(c)** Find the magnitude of the impulse received by $B$ in the second collision between $A$ and $B$

**(2)**

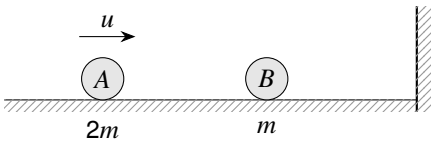



**Figure 3**

**[5 marks]**

# E Exam practice

Full-size exam questions in the style of WME02. Write your answers in the spaces, and show every step.

CORE *Exam questions for everyone*

**E1** A particle $P$ of mass $2m$ and a particle $Q$ of mass $3m$ are moving in opposite directions along the same straight line on a smooth horizontal surface, as shown in Figure 4. Particle $P$ collides directly with particle $Q$. Immediately before the collision, the speed of $P$ is $3u$ and the speed of $Q$ is $u$. Immediately after the collision, the speed of $P$ is $x$ and the speed of $Q$ is $y$. The direction of motion of each particle is reversed by the collision.

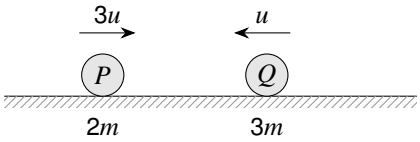



**Figure 4**

In the collision, $P$ receives an impulse of magnitude $\frac{36}{5}mu$

**(a)** Show that $x = \frac{3}{5}u$

**(3)**

**(b)** Find $y$ in terms of $u$

**(2)**

The coefficient of restitution between $P$ and $Q$ is $e$

**(c)** Find the value of $e$

**(2)**

**(d)** Find, in terms of $m$ and $u$, the total kinetic energy lost in the collision.

**(3)**

**(Total for Question E1 is 10 marks)**

**E2** A particle $A$ of mass $2m$ and a particle $B$ of mass $m$ are at rest on a smooth horizontal plane. Particle $A$ is projected with speed $u$ along the plane towards $B$ and the particles collide directly. The coefficient of restitution between the particles is $e$, where $e > 0$

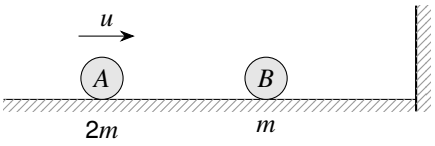



**Figure 5**

**(a)** Show that the speed of $B$ after the collision is $\frac{2}{3}(1+e)u$

**(3)**

**(b)** Explain why the direction of motion of $A$ is not reversed by the collision.

**(1)**

After the collision, $B$ goes on to hit a smooth vertical wall which is fixed at right angles to the direction of motion of $B$, as shown in Figure 5. The coefficient of restitution between $B$ and the wall is $f$. After $B$ has hit the wall, $B$ has the same speed as $A$.

**(c)** Show that $f = \dfrac{2-e}{2(1+e)}$

**(3)**

**(d)** Given that $e = \frac{1}{4}$, find, in terms of $m$ and $u$, the magnitude of the impulse received by $B$ from the wall.

**(2)**

**(e)** State, with a reason, whether $A$ and $B$ collide again.

**(1)**

**(Total for Question E2 is 10 marks)**

STRETCH

**E3** A particle $P$ of mass $m$ and a particle $Q$ of mass $km$, where $k$ is a constant, are moving towards each other along the same straight line on a smooth horizontal surface, as shown in Figure 6. Immediately before they collide, the speed of $P$ is $4u$ and the speed of $Q$ is $u$. As a result of the collision, the direction of motion of each particle is reversed, and immediately afterwards the speed of $P$ is $u$

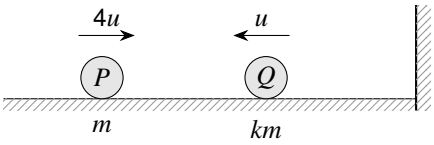



**Figure 6**

**(a)** Show that the speed of $Q$ immediately after the collision is $\frac{(5-k)u}{k}$, and hence state the restriction on $k$ given by the reversal of $Q$

**(3)**

**(b)** Show that the coefficient of restitution between $P$ and $Q$ is $\frac{1}{k}$

**(3)**

**(c)** Hence find the full range of possible values of $k$

**(2)**

In the rest of this question $k = 2$. After the collision, $Q$ hits a smooth vertical wall, fixed at right angles to its direction of motion. The coefficient of restitution between $Q$ and the wall is $f$

**(d)** Given that there is a second collision between $P$ and $Q$, find the range of possible values of $f$

**(3)**

**(e)** Find the fraction of the total kinetic energy before the first collision that is lost in the first collision.

**(2)**

**(Total for Question E3 is 13 marks)**

**Want more exam practice?** The **Exam Questions by Topic** booklets have 10 more full exam questions on Mechanics 2 Chapter 5, Impulses and Collisions, with a full mark scheme. Open the booklet

**Finished?** Mark your work with the **full worked solutions**, fill in your scores on the front page, then practise real exam questions on the **Mechanics 2 exam questions page**.