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INTERNATIONAL A LEVEL MATHEMATICS • FURTHER PURE 2 (WFM02)

Further Pure 2 Practice Paper 3

Edexcel IAL Further Pure Mathematics 2

AIMING FOR A/A*

Written in the style of the Pearson Edexcel International Advanced Level paper (WFM02/01) • Calculator allowed

A full 75-mark paper set above the hardest recent series: long questions, few printed results and a trap in every question

Time	Total	You must have
1 hour 30 minutes	**75 marks**	**Mathematical Formulae and Statistical Tables, calculator**

Instructions

- Use black ink or ball-point pen. Pencil may be used for diagrams and sketches.
- Answer **all** questions, in the spaces provided.
- Show enough working to make your method clear. Answers without working may not gain full credit.
- Give inexact answers to **three significant figures** unless the question says otherwise.

Information

- There are 7 questions in this paper. The total mark is 75.
- The marks for each question are shown in brackets.

Advice

- Sit the paper in one go, in 1 hour 30 minutes, with the mark scheme closed.
- Read each question carefully and try every question. Check your answers if you have time.
- Then mark it line by line with the mark scheme and fill in your score.

Track your score

Q	Topic	Marks	Score
1	A modulus against a rational function	10	
2	A sum of logarithms and the least n	10	
3	de Moivre's theorem and exact roots	11	
4	The image of a quarter disc	11	
5	A second-order equation by substitution	11	
6	A product of series and a limit	11	
7	Tangents and the area of a limaçon	11	
	Total	**75**	

Rough grade guide (out of 75): A* **58** • A **52** • B **45** • C **38**

Based on recent WFM02 grade boundaries. Real boundaries change every series, so treat this as a guide, not a grade.

Mark scheme | Exam questions by topic | More practice papers

Name ..... Date .....

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1. In this question you must show all stages of your working.

Solutions relying on calculator technology are not acceptable.

(a) On the same axes, sketch the curve with equation $y = \dfrac{x+3}{x-1}$ and the graph of $y = |x-3|$, stating the equations of any asymptotes.

(3)

(b) Use algebra to find the set of values of x for which

$$\frac{x+3}{x-1} \geqslant |x-3|$$

(7)

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Question 1 continued

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(Total for Question 1 is 10 marks)

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2. **(a)** Show that, for $r \geqslant 1$,

$$\ln \frac{(r+1)^2}{r(r+2)} = \ln \frac{r+1}{r+2} - \ln \frac{r}{r+1}$$

(2)

(b) Hence, using the method of differences, find $\sum_{r=1}^{n} \ln \frac{(r+1)^2}{r(r+2)}$, giving your answer as a single logarithm in terms of n

(4)

(c) Find the least value of n for which

$$\sum_{r=1}^{n} \ln \frac{(r+1)^2}{r(r+2)} > \ln 1.99$$

(4)

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Question 2 continued

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(Total for Question 2 is 10 marks)

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3. In this question you must show all stages of your working.

Solutions relying on calculator technology are not acceptable.

(a) Use de Moivre's theorem to show that

$$\sin 5\theta = 16\sin^5\theta - 20\sin^3\theta + 5\sin\theta$$

(5)

(b) Hence find the four roots of the equation

$$16x^4 - 20x^2 + 5 = 0$$

giving each root in the form $\sin k\pi$, where $-\frac{1}{2} < k < \frac{1}{2}$

(4)

(c) Hence find the exact value of $\sin\frac{\pi}{5}\sin\frac{2\pi}{5}$

(2)

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Question 3 continued

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(Total for Question 3 is 11 marks)

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4. The transformation T from the z-plane to the w-plane, where $w = u + iv$, is given by

$$w = \frac{z-1}{z+1}, \qquad z \neq -1$$

(a) Show that T maps the circle $|z| = 1$, $z \neq -1$, onto the imaginary axis in the w-plane.

(3)

The region R of the z-plane is given by $|z| < 1$ and $0 < \arg z < \frac{\pi}{2}$

(b) Find the image of R under T, and shade it on an Argand diagram.

(5)

(c) Find the set of possible values of $|w-1|$ for points w in the image of R

(3)

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Question 4 continued

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(Total for Question 4 is 11 marks)

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5.

$$x^2\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} + 2y = x^3, \qquad x > 0$$

(a) Show that the substitution $x = e^u$ transforms this differential equation into

$$\frac{d^2y}{du^2} - 3\frac{dy}{du} + 2y = e^{3u}$$

(5)

(b) Hence find the general solution of the original differential equation, giving y in terms of x

(4)

(c) Find the particular solution for which $y = 0$ and $\frac{dy}{dx} = 1$ when $x = 1$

(2)

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Question 5 continued

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(Total for Question 5 is 11 marks)

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6. **(a)** Using the Maclaurin series for $\ln(1+x)$, find the series expansion of $\ln(1+2x)$ in ascending powers of x, up to and including the term in x^3, and state the range of values of x for which the expansion is valid.

(4)

(b) Hence find the series expansion of $e^x \ln(1+2x)$ in ascending powers of x, up to and including the term in x^3

(4)

(c) Hence find

$$\lim_{x \to 0} \frac{e^x \ln(1+2x) - 2x}{x^3}$$

(2)

(d) Explain why the series in part (b) should not be used to estimate the value of $e \ln 3$

(1)

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Question 6 continued

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(Total for Question 6 is 11 marks)

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7.

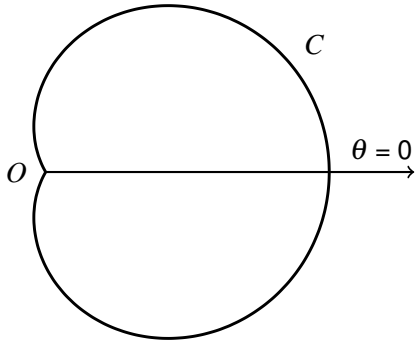

Figure 1

Figure 1 shows the curve C with polar equation

$$r = 1 + 2\cos\theta, \qquad -\frac{2\pi}{3} \leqslant \theta \leqslant \frac{2\pi}{3}$$

(a) Find the polar coordinates of each point on C at which the tangent to C is perpendicular to the initial line, giving any non-exact values of θ to 3 significant figures.

(6)

(b) Use calculus to find the exact area of the region enclosed by C

(5)

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Question 7 continued

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(Total for Question 7 is 11 marks)

TOTAL FOR PAPER IS 75 MARKS

END

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