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INTERNATIONAL A LEVEL MATHEMATICS • FURTHER PURE MATHEMATICS 1

Further Pure 1

Chapter 4: Coordinate Systems

Exam Questions by Topic

Pearson Edexcel International Advanced Level (WFM01/01) • Calculator allowed

Multi-step exam questions at the difficulty of the real WFM01 paper, with a full mark scheme

Chapters (tap a chapter to jump to it)		Qs	Marks	Page	My score
1	**Complex Numbers** Arithmetic, conjugates, Argand diagrams, modulus and argument, roots of polynomial equations	10	80	separate file	
2	**Roots of Quadratic Equations** Sums and products of roots, symmetric functions, equations with transformed roots	10	80	separate file	
3	**Numerical Solutions of Equations** Interval bisection, linear interpolation, Newton–Raphson	10	80	separate file	
4	**Coordinate Systems** Parametric equations, parabolas, rectangular hyperbolas, tangents and normals	10	80	2	
5	**Matrices** Matrix arithmetic, determinants, inverses of 2×2 matrices	10	80	separate file	
6	**Transformations Using Matrices** Linear transformations, combined transformations, area scale factor, inverses	10	80	separate file	
7	**Series** Standard sums of r, r^2 and r^3, sigma notation	10	80	separate file	
8	**Proof** Mathematical induction for sums, divisibility, matrices and sequences	10	80	separate file	

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CHAPTER 4 • KEY SKILLS

4 Coordinate Systems

Specification FP1: Coordinate systems • 10 questions • 80 marks • about 96 minutes

1 Parametric to Cartesian

Eliminate t: $x = at^2$, $y = 2at$ gives $y^2 = 4ax$; $x = ct$, $y = \frac{c}{t}$ gives $xy = c^2$

2 The parabola

$y^2 = 4ax$: focus $S(a, 0)$, directrix $x = -a$, general point $(at^2, 2at)$

3 Focus and directrix

Every point P on the parabola is as far from S as from the directrix: $PS = x_P + a$

4 Rectangular hyperbola

$xy = c^2$: general point $\left(ct, \frac{c}{t}\right)$; the axes are asymptotes.

5 Gradients

$\frac{dy}{dx} = \frac{dy/dt}{dx/dt}$: $\frac{1}{t}$ on $y^2 = 4ax$ and $-\frac{1}{t^2}$ on $xy = c^2$. Differentiate; do not use t as the gradient.

6 Tangents and normals

Use $y - y_1 = m(x - x_1)$; the normal has gradient $-\frac{1}{m}$. Rearrange into the form asked for.

7 Meeting the curve again

Substitute the line into the curve (or the parameters into the line) and solve the quadratic with working; one root is already known.

8 Lengths and areas

Exact lengths by Pythagoras (surds). Triangle area $\frac{1}{2}|x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)|$, or use a right angle.

9 General results

Work with parameters p, q, t throughout, and simplify fully to reach a given answer.

Skills map every skill above is tested at least once; interleaved skills come from GCSE or other chapters

Skill	Q1	Q2	Q3	Q4	Q5	Q6	Q7	Q8	Q9	Q10
Marks	6	6	6	6	6	10	10	10	10	10
1 Parametric to Cartesian	•	•					•			
2 The parabola	•	•	•		•		•	•		•
3 Focus and directrix	•				•			•		
4 Rectangular hyperbola		•		•		•	•		•	
5 Gradients			•	•		•	•	•	•	•
6 Tangents and normals			•	•		•	•	•	•	•
7 Meeting the curve again		•				•	•			•
8 Lengths and areas				•		•	•		•	•
9 General results			•	•	•	•		•	•	•
Interleaved: differentiation and straight lines (P1)			•	•		•	•	•	•	•
Interleaved: quadratics and simultaneous equations (P1)		•			•	•	•			•
Interleaved: surds and algebraic manipulation (P1)						•		•	•	

Exam difficulty. Questions 1 to 5 check each skill on its own. Questions 6 to 10 are multi-step questions built like the last questions on a real WFM01 paper, where one part leads into the next and skills from different topics are combined.

Must learn. Sketch the curve and the lines first. Find gradients by differentiation, show the quadratic when a line meets a curve again, and give lengths exactly.

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1. The parabola C has equation $y^2 = 12x$

(a) Write down the coordinates of the focus S of C and the equation of its directrix.

(2)

(b) The point P on C has y-coordinate 6. Find the distance PS.

(2)

(c) Write down parametric equations for C in the form $x = at^2$, $y = 2at$

(1)

(d) Find the coordinates of the points on C whose distance from the directrix is 15

(1)

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Question 1 continued

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(Total for Question 1 is 6 marks)

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2. The curve C_1 has parametric equations $x = 5t^2$, $y = 10t$, and the curve C_2 has parametric equations $x = 5s$, $y = \frac{10}{s}$, $s \neq 0$

(a) Find a Cartesian equation for each curve.

(3)

(b) Find, using algebra, the coordinates of the point where C_1 and C_2 meet.

(3)

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Question 2 continued

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(Total for Question 2 is 6 marks)

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3. The point $P\left(at^2, 2at\right)$, where $t \neq 0$, lies on the parabola with equation $y^2 = 4ax$, where a is a positive constant.

(a) Show that the tangent to the parabola at P has equation $ty = x + at^2$

(4)

(b) Find the equation of the tangent to the parabola $y^2 = 12x$ at the point where $t = 2$

(2)

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Question 3 continued

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4. The point $P\left(3t, \frac{3}{t}\right)$, $t \neq 0$, lies on the rectangular hyperbola H with equation $xy = 9$

(a) Show that the tangent to H at P has equation $x + t^2y = 6t$

(4)

(b) The tangent at the point where $t = 3$ meets the x-axis at A and the y-axis at B. Find the area of triangle OAB.

(2)

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Question 4 continued

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5. The point $P\left(2t^2, 4t\right)$ lies on the parabola $y^2 = 8x$, which has focus S.

(a) Show that $PS = 2\left(t^2 + 1\right)$

(3)

(b) Given that $PS = 10$, find the possible coordinates of P.

(3)

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Question 5 continued

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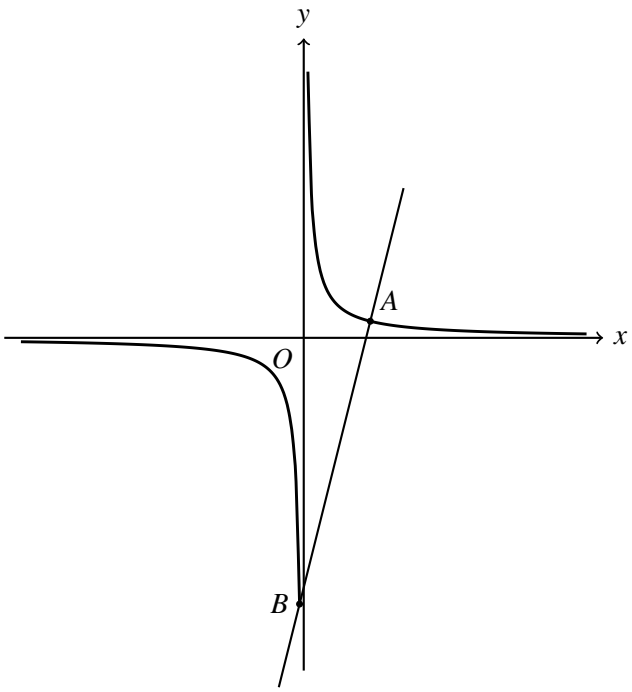

Figure 1

The rectangular hyperbola H has equation $xy = 16$. The point $P\left(4t, \frac{4}{t}\right)$, $t \neq 0$, lies on H.

(a) Show that the normal to H at P has equation $t^3x - ty = 4t^4 - 4$

(5)

The normal to H at the point A, where $t = 2$, meets H again at the point B, as shown in Figure 1.

(b) Find the coordinates of B.

(3)

(c) Find the exact length of AB, giving your answer in the form $k\sqrt{17}$

(2)

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Question 6 continued

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Question 6 continued

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7. The parabola C has equation $y^2 = 16x$ and the rectangular hyperbola H has equation $xy = 32$

(a) Using algebra, show that C and H meet at exactly one point, P, and find its coordinates.

(3)

(b) Find an equation of the normal to C at P, in the form $x + y = k$

(3)

(c) The normal meets C again at the point Q. Find the coordinates of Q.

(3)

(d) Find the exact length of PQ.

(1)

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Question 7 continued

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Question 7 continued

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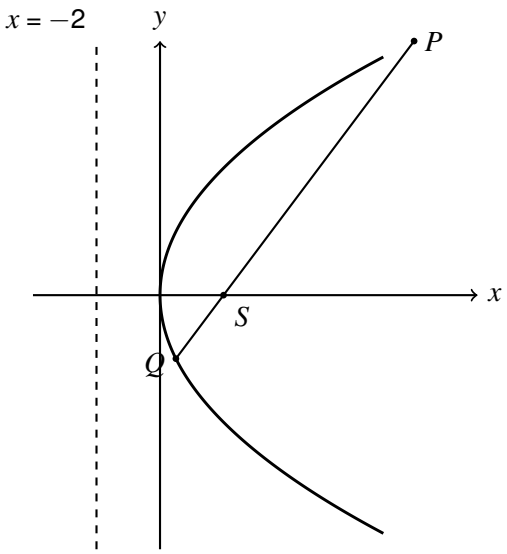

Figure 2

The parabola C has equation $y^2 = 8x$ and focus S. The points $P\left(2p^2, 4p\right)$ and $Q\left(2q^2, 4q\right)$, where $p \neq q$, lie on C, and the chord PQ passes through S, as shown in Figure 2.

(a) Show that $pq = -1$

(4)

(b) Show that the tangents to C at P and Q are perpendicular.

(2)

(c) Show that the tangents to C at P and Q meet on the directrix of C.

(4)

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Question 8 continued

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Question 8 continued

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9. The points $P\left(4p, \frac{4}{p}\right)$ and $Q\left(4q, \frac{4}{q}\right)$, where p, q are non-zero and $p \neq q$, lie on the rectangular hyperbola $xy = 16$

(a) Show that the gradient of the chord PQ is $-\frac{1}{pq}$

(3)

(b) The tangents to the hyperbola at P and Q meet at R. Given that the tangent at P has equation $x + p^2y = 8p$, find the coordinates of R in terms of p and q.

(4)

(c) For $p = 1$ and $q = 3$, find the area of triangle PQR.

(3)

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Question 9 continued

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Question 9 continued

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10. The point $P\left(at^2, 2at\right)$, $t \neq 0$, lies on the parabola $y^2 = 4ax$

(a) Show that the normal to the parabola at P has equation $y + tx = 2at + at^3$

(3)

(b) The normal meets the parabola again at the point with parameter s. Show that $s = -t - \frac{2}{t}$

(4)

(c) For $a = 1$ and $t = 1$, find the coordinates of the point Q where the normal meets the parabola again, and the area of triangle OPQ, where O is the origin.

(3)

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Question 10 continued

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TOTAL FOR CHAPTER 4 IS 80 MARKS

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