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INTERNATIONAL A LEVEL MATHEMATICS • PURE 4 (WMA14)

Pure 4 Practice Paper 2

Mark Scheme

AIMING FOR A

Written in the style of the Pearson Edexcel International Advanced Level paper (WMA14/01) • 75 marks

Every question marked line by line the way an examiner marks it, with notes on the common errors

Mark every line, not just the answer. Method marks are given for a correct method even when the final answer is wrong. If you lost marks, find the *first* line where your working differs from the scheme. That is the thing to fix.

Code	Meaning
M1	Method mark: for knowing a correct method and attempting to apply it. Small slips in the working do not lose this mark.
dM1	Dependent method mark: can only be given if the method mark before it has been given.
A1	Accuracy mark: for a correct answer or line of working. It depends on the method mark before it.
B1	Independent mark: for a correct result or statement, whatever method was used.
*	The answer is given in the question. Every step must be shown, and the answer must follow from correct working (cso).
cao / cso	Correct answer only / correct solution only, with no errors anywhere in the working.
ft / oe	Follow through from your own earlier answer / or any correct equivalent form.

"Solutions relying on calculator technology are not acceptable." Questions 3, 5, 6 and 9 say this. There, a correct final answer with no algebraic working scores **no marks**.

Rough grade guide (out of 75): A* **61** • A **54** • B **46** • C **38**

Based on recent WMA14 grade boundaries. Real boundaries change every series, so treat this as a guide, not a grade.

Question paper **Exam questions by topic** **More practice papers**

What to revise next

Write in your score for each question. Where you dropped marks, tap the chapter to practise ten more exam questions on it.

Q	What it tested	Marks	Practise it	Score
1	An improper fraction in partial fractions; series and validity; an unknown multiplier	9	Ch 2 Partial Fractions; Ch 4 Binomial Expansion	
2	A binomial approximation to a surd; proving a square root irrational	8	Ch 4 Binomial Expansion; Ch 1 Proof	
3	A maximum on a parametric curve; area with parametric limits	8	Ch 3 Coordinate Geometry; Ch 5 Differentiation; Ch 6 Integration	
4	Forming and solving a differential equation in context	8	Ch 5 Differentiation; Ch 6 Integration	
5	A stationary point; volume by integrating by parts twice	9	Ch 5 Differentiation; Ch 6 Integration	
6	Partial fractions with a repeated factor; an exact area	8	Ch 2 Partial Fractions; Ch 6 Integration	
7	Implicit differentiation; a tangent; a proof about gradients	8	Ch 5 Differentiation; Ch 1 Proof	
8	Distance from a point to a line; the foot of the perpendicular; a reflection	9	Ch 7 Vectors	
9	A series approximation to an integral; substitution; percentage error	8	Ch 4 Binomial Expansion; Ch 6 Integration	
	Total	**75**		

Where did the marks go? If you lost the first marks of a question, the method needs work: go back to that chapter's key skills page. If you lost the final marks, the method is fine and the gap is accuracy, a limit in the wrong order, or a range of validity. Paper 3 gives almost no help between the parts and adds a trap to most questions, so check every answer against the conditions in the question.

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Question 1

9 marks

Part	Scheme	Marks
(a)	$A = -2$ (by division, or comparing x^2 terms: $4 = -2A$)	**B1**
	$4x^2 + x + 4 \equiv A(1+2x)(1-x) + B(1-x) + C(1+2x)$, with $x = 1$ and $x = -\frac{1}{2}$	**M1**
	$\boxed{B = 3,\ C = 3}$	**A1**
	Writing the form without A (as if the fraction were proper) scores B0 M0 A0.	**(3)**
(b)	$3(1+2x)^{-1} = 3 - 6x + 12x^2$	**M1**
	$3(1-x)^{-1} = 3 + 3x + 3x^2$, and adds the constant -2	**M1**
	$\boxed{4 - 3x + 15x^2}$	**A1**
	$\boxed{\lvert x\rvert < \frac{1}{2}}$	**B1**
	The constant $A = -2$ is part of the series. Leaving it out gives $6 - 3x + 15x^2$. The range is set by $(1+2x)^{-1}$.	**(4)**
(c)	Coefficient of x^2 in $(1+kx)\left(4 - 3x + 15x^2\right)$: $15 - 3k = 0$	**M1**
	$\boxed{k = 5}$	**A1**
		(2)
		Total for Question 1: 9 marks

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Question 2

8 marks

Part	Scheme	Marks
(a)	$(1-8x)^{\frac{1}{2}} = 1 + \frac{1}{2}(-8x) + \frac{\frac{1}{2}\left(-\frac{1}{2}\right)}{2!}(-8x)^2 + \frac{\frac{1}{2}\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)}{3!}(-8x)^3$	M1
	Two of the terms in x, x^2 and x^3 correct	A1
	$1 - 4x - 8x^2 - 32x^3$	A1
		(3)
(b)	$\sqrt{1-0.08} = \sqrt{\frac{92}{100}} = \frac{2\sqrt{23}}{10} = \frac{\sqrt{23}}{5}$, so $\sqrt{23} \approx 5(1 - 0.04 - 0.0008 - 0.000032)$	M1
	$\frac{14987}{3125}$ $(= 4.79584)$	A1
	The factor 5 is the link: $\sqrt{0.92}$ is $\frac{\sqrt{23}}{5}$, not $\sqrt{23}$ itself.	
		(2)
(c)	Assume $\sqrt{23} = \frac{p}{q}$, where p and q are integers with no common factor	B1
	$p^2 = 23q^2$, so 23 divides p^2, and as 23 is prime, 23 divides p: $p = 23m$, so $q^2 = 23m^2$ and 23 divides q	M1
	p and q share the factor 23, contradicting the assumption, so $\sqrt{23}$ is irrational	A1
	The step "23 is prime" is needed: it is why 23 dividing p^2 means 23 divides p.	
		(3)
	Total for Question 2: 8 marks	

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Question 3

8 marks

Part	Scheme	Marks
(a)	$\frac{dy}{dx} = \frac{2\cos 2t}{-4\sin t}$, or uses $\frac{dy}{dt} = 0$	M1
	$\cos 2t = 0$, so $t = \frac{\pi}{4}$	M1
	$\boxed{M\left(2\sqrt{2}, 1\right)}$	A1
		(3)
(b)	Area $= \int y\frac{dx}{dt}\,dt = \int \sin 2t\,(-4\sin t)\,dt$	M1
	$= \int -8\sin^2 t\cos t\,dt$ (uses $\sin 2t = 2\sin t\cos t$)	M1
	Limits $t = \frac{\pi}{2}$ (at $x = 0$) to $t = 0$ (at $x = 4$), or reverses them and the sign	A1
	$\left[-\frac{8}{3}\sin^3 t\right]$ evaluated: $0 - \left(-\frac{8}{3}\right)$	M1
	$\boxed{\frac{8}{3}}$	A1
	Using limits 0 to $\frac{\pi}{2}$ without changing the sign gives $-\frac{8}{3}$. A negative area is the clue that the limits are the wrong way round.	
		(5)
		Total for Question 3: 8 marks

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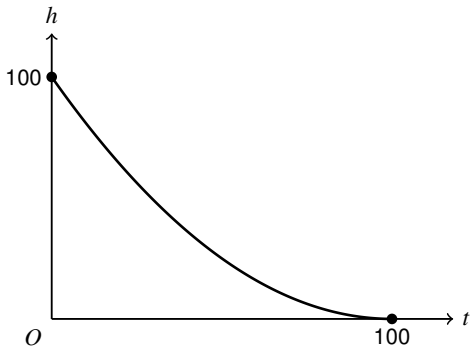
Question 4

8 marks

Part	Scheme	Marks
(a)	$V = 400\pi h$, so $\frac{dV}{dt} = 400\pi\frac{dh}{dt}$	M1
	$400\pi\frac{dh}{dt} = -80\pi\sqrt{h}$, so $\frac{dh}{dt} = -0.2\sqrt{h}$	A1
	Separates and integrates: $\int h^{-\frac{1}{2}}\,dh = \int -0.2\,dt \Rightarrow 2\sqrt{h} = -0.2t + c$	M1
	$t = 0$, $h = 100$: $c = 20$	M1
	$\boxed{h = (10 - 0.1t)^2}$ oe	A1
	The minus sign is needed because the depth is falling. Without πr^2 the rate is wrong by a factor of 400π.	(5)
(b)		B1
	A decreasing curve from (0, 100), flattening as it reaches (100, 0)	(1)
(c)	$h = 50$: $\sqrt{50} = 10 - 0.1t$, so $t = 100 - 10\sqrt{50}$	M1
	$\boxed{t = 29.3}$ minutes	A1
	The tank empties at $t = 100$, but half the depth is reached long before half that time, because water leaves fastest when the tank is full.	(2)
		Total for Question 4: 8 marks

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Question 5

9 marks

Part	Scheme	Marks
(a)	Product rule: $\frac{dy}{dx} = \frac{\ln x}{2\sqrt{x}} + \frac{\sqrt{x}}{x} = \frac{\ln x + 2}{2\sqrt{x}}$	M1
	$\ln x = -2$, so $x = e^{-2}$	M1
	$\boxed{P\left(e^{-2}, -2e^{-1}\right)}$	A1
		(3)
(b)	$V = \pi \int_1^e x(\ln x)^2 \, dx$ (lower limit 1, where C meets the x-axis)	B1
	By parts, $u = (\ln x)^2$: $\frac{1}{2}x^2(\ln x)^2 - \int x \ln x \, dx$	M1
	By parts again: $\int x \ln x \, dx = \frac{1}{2}x^2 \ln x - \frac{1}{4}x^2$	M1
	$\frac{1}{2}x^2(\ln x)^2 - \frac{1}{2}x^2 \ln x + \frac{1}{4}x^2$	A1
	Uses the limits: $\left(\frac{1}{2}e^2 - \frac{1}{2}e^2 + \frac{1}{4}e^2\right) - \frac{1}{4}$	dM1
	$\boxed{\frac{\pi}{4}\left(e^2 - 1\right)}$	A1
	Squaring $y = \sqrt{x}\ln x$ gives $x(\ln x)^2$. Choosing $u = x$ cannot work, as $(\ln x)^2$ has no simple integral.	
		(6)
		Total for Question 5: 9 marks

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Question 6

8 marks

Part	Scheme	Marks
(a)	$\frac{A}{x}+\frac{B}{x^2}+\frac{C}{2x+1}$; $x=0$: $B=1$	B1
	$2x^2+1\equiv Ax(2x+1)+B(2x+1)+Cx^2$, using $x=-\frac{1}{2}$ and one other value or comparison	M1
	$\boxed{-\frac{2}{x}+\frac{1}{x^2}+\frac{6}{2x+1}}$	A1
		(3)
(b)	Integrates to the form $p\ln x+\frac{q}{x}+r\ln(2x+1)$	M1
	$-2\ln x-\frac{1}{x}+3\ln(2x+1)$	A1
	Uses the limits 1 and 2: $\left(-2\ln 2-\frac{1}{2}+3\ln 5\right)-(-1+3\ln 3)$	M1
	Combines the logarithms correctly	M1
	$\boxed{\frac{1}{2}+\ln\frac{125}{108}}$	A1
	$\int \frac{6}{2x+1}\,dx=3\ln(2x+1)$, *not* $6\ln(2x+1)$, *and* $\int x^{-2}\,dx=-x^{-1}$.	
		(5)
		Total for Question 6: 8 marks

Question 7

8 marks

Part	Scheme	Marks
(a)	Differentiates y^3+y to $\left(3y^2+1\right)\dfrac{\mathrm{d}y}{\mathrm{d}x}$	M1
	$\left(3y^2+1\right)\dfrac{\mathrm{d}y}{\mathrm{d}x}=3x^2-1$	M1
	$\boxed{\dfrac{\mathrm{d}y}{\mathrm{d}x}=\dfrac{3x^2-1}{3y^2+1}}$	A1
		(3)
(b)	$x=1$: $y^3+y=0\Rightarrow y\left(y^2+1\right)=0$, so $y=0$ only; gradient $=\dfrac{2}{1}=2$	M1
	$\boxed{y=2x-2}$	A1
		(2)
(c)	Assume there is a point with $\dfrac{3x^2-1}{3y^2+1}<-1$	B1
	$3y^2+1>0$, so $3x^2-1<-3y^2-1$, giving $3x^2+3y^2<0$	M1
	Impossible, as $3x^2+3y^2\geqslant 0$: a contradiction, so no gradient is less than -1	A1
	Multiplying through by $3y^2+1$ keeps the inequality the same way only because it is positive, and that reason is part of the M1.	
		(3)
		Total for Question 7: 8 marks

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Question 8

9 marks

Part	Scheme	Marks
(a)	$\overrightarrow{AP} = \begin{pmatrix} \lambda - 4 \\ 2\lambda - 5 \\ 2 - 2\lambda \end{pmatrix}$	M1
	$\boxed{AP^2 = 9\lambda^2 - 36\lambda + 45}$	A1
		(2)
(b)	$\frac{\mathrm{d}}{\mathrm{d}\lambda}\left(AP^2\right) = 18\lambda - 36 = 0$	M1
	$\lambda = 2$	A1
	$\boxed{F(4, 3, -1)}$	A1
	Shortest distance $= \sqrt{36 - 72 + 45} = \boxed{3}$	A1
	Minimising AP^2 is easier than minimising AP and gives the same λ.	
		(4)
(c)	$\overrightarrow{AF} \cdot \mathbf{d} = \begin{pmatrix} -2 \\ -1 \\ -2 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 2 \\ -2 \end{pmatrix} = -2 - 2 + 4 = 0$, so AF is perpendicular to l	B1
		(1)
(d)	F is the midpoint of A and its image A': $\overrightarrow{OA'} = 2\overrightarrow{OF} - \overrightarrow{OA}$	M1
	$\boxed{(2, 2, -3)}$	A1
		(2)
		Total for Question 8: 9 marks

Question 9

8 marks

Part	Scheme	Marks
(a)	$(1+x)^{-\frac{1}{2}} = 1 - \frac{1}{2}x + \frac{\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)}{2}x^2$	M1
	$1 - \frac{1}{2}x + \frac{3}{8}x^2$	A1
		(2)
(b)	$\int_0^{0.2} \left(x - \frac{1}{2}x^2 + \frac{3}{8}x^3\right) \mathrm{d}x = \left[\frac{1}{2}x^2 - \frac{1}{6}x^3 + \frac{3}{32}x^4\right]_0^{0.2}$	M1
	0.018817	A1
		(2)
(c)	$u = 1 + x$: $\int_1^{1.2} \frac{u-1}{\sqrt{u}} \mathrm{d}u = \int_1^{1.2} \left(u^{\frac{1}{2}} - u^{-\frac{1}{2}}\right) \mathrm{d}u$, with the limits changed	M1
	$\left[\frac{2}{3}u^{\frac{3}{2}} - 2u^{\frac{1}{2}}\right]_1^{1.2}$	A1
	$\sqrt{1.2} = \frac{\sqrt{30}}{5}$: $\frac{4}{3} - \frac{6\sqrt{30}}{25}$	A1
		(3)
(d)	$\frac{0.018817 - 0.018799}{0.018799} \times 100 =$ 0.093% (accept 0.09% to 0.1%)	B1
	The percentage error is measured against the exact value, not the approximation.	(1)
		Total for Question 9: 8 marks

TOTAL FOR PAPER: 75 MARKS