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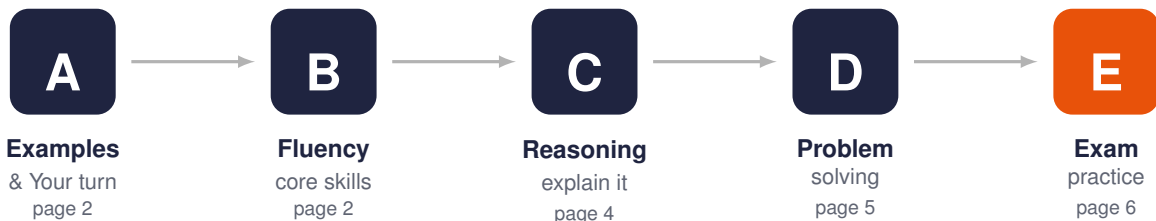
A-LEVEL MATHEMATICS • PURE 4

Vectors

Worked Solutions

Pearson Edexcel International A Level (WMA14) & A Level (9MA0) • Calculator allowed: show every step

Full worked solutions for the free Vectors worksheet, A-Level and International A-Level Maths



Tap a section to jump straight to it

Full worked solutions for every question on the Pure 4 Vectors worksheet, in the same order and with the same question numbers.

How to mark your work. Tick each step you got right, not just the final answer. At A-level most marks are for method, so a wrong final answer can still earn most of the marks. If you lost marks, find the *first* line where your working is different from ours. That is the thing to fix.

In Section E: **M** = method mark, **dM** = method mark that depends on the previous M mark, **A** = accuracy mark (needs the M mark before it), **B** = independent mark.

Question sheet

Exam questions by topic

Vectors revision engine

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A**Your turn answers**

Compare each line of your working with the solution.

CORE

A1 $\vec{PQ} = -2\mathbf{i} + 6\mathbf{j} + 3\mathbf{k}$, $|\vec{PQ}| = \sqrt{4+36+9} = 7$: $\frac{1}{7}(-2\mathbf{i} + 6\mathbf{j} + 3\mathbf{k})$

A2 $\mathbf{r} = (3\mathbf{j} + 2\mathbf{k}) + \mu(2\mathbf{i} - 2\mathbf{j} + 3\mathbf{k})$; x : $\mu = 3$ gives $y = -3$ but $z = 11 \neq 10$: **R is not on the line**

A3 $\cos \theta = \frac{4-8-14}{3 \times 9} = -\frac{2}{3}$: **131.8° ; acute angle 48.2°**

STRETCH

A4 $\mathbf{i}$: $2 + \lambda = 1 - 3\mu$; $\mathbf{j}$: $-1 + 2\lambda = 5 + 2\mu$; so $\lambda = 2$, $\mu = -1$

$\mathbf{k}$: $4 - 2 = 2$ and $-1 + 3 = 2$ ✓: **$(4, 3, 2)$**

A5 Directions $2\mathbf{i} + \mathbf{k}$ and $\mathbf{i} + \mathbf{j} - \mathbf{k}$ are not multiples, so not parallel.

$\mathbf{j}$: $1 = 2 + \mu$, $\mu = -1$; $\mathbf{i}$: $1 + 2\lambda = -1$, $\lambda = -1$; $\mathbf{k}$: $-1 \neq 4$: **no intersection, so skew**

A6 $X = (\mu, 2 - \mu, 1 + 2\mu)$; $\vec{PX} \cdot (\mathbf{i} - \mathbf{j} + 2\mathbf{k}) = (\mu - 5) - (-1 - \mu) + 2(2\mu - 4) = 6\mu - 12 = 0$

$\mu = 2$: **$X(2, 0, 5)$, $PX = |(-3, -3, 0)| = 3\sqrt{2}$**

A7 $\mathbf{r} = \mathbf{p} + \frac{1}{4}(\mathbf{q} - \mathbf{p}) = (-2, 4, 1) + \frac{1}{4}(8, -4, 8)$: **$R(0, 3, 3)$**

B**Fluency solutions**One mark for each correct part. The notes in *italics* explain why each sequence was chosen.**CORE**

B1 (a) **$4\mathbf{i} + 7\mathbf{j} - \mathbf{k}$** (b) **$\sqrt{14}$** (c) $|\mathbf{i} - 5\mathbf{j} + 5\mathbf{k}|$: **$\sqrt{51}$** (d) **$\frac{1}{\sqrt{21}}(\mathbf{i} + 4\mathbf{j} - 2\mathbf{k})$**

Combine first, then find the magnitude; a unit vector is the vector divided by its magnitude.

B2 (a) $-4\mathbf{i} + 4\mathbf{j} + 2\mathbf{k}$ (b) $\sqrt{16+16+4}$: 6 (c) $(1, 0, 6)$ (d) $\mathbf{a} + 3\overrightarrow{AB}$: $(-9, 10, 11)$

Every part starts from $\overrightarrow{AB} = \mathbf{b} - \mathbf{a}$

B3 (a) $\mathbf{r} = (2\mathbf{i} + \mathbf{j}) + \lambda(3\mathbf{i} - \mathbf{j} + 4\mathbf{k})$ (b) $\mathbf{r} = (\mathbf{i} + 2\mathbf{j} + 3\mathbf{k}) + \lambda(3\mathbf{i} - 2\mathbf{j} + 2\mathbf{k})$

(c) $\lambda = 3$ gives $(10, -4, 9)$: **on l** (d) $2 - 2\lambda = 6, \lambda = -2$: $p = -5, q = -1$

A point is on a line when one value of λ fits all three components.

B4 (a) $2 - 4 + 6$: 4 (b) $3 - 5 + 2 = 0$: **perpendicular** (c) $3t + 2t - 4 = 0$: $t = \frac{4}{5}$ (d) $\cos \theta = \frac{1}{2}$: 60°

A zero scalar product means perpendicular; otherwise divide by the magnitudes.

CORE

B5 (a) $1 + \lambda = 2, 1 = -1 + \mu$, so $\lambda = 1, \mu = 2$; $\mathbf{k}: 1 + 2 = 3$ ✓: **they meet**

(b) $(2, 1, 3)$ (c) $(\mathbf{i} + 2\mathbf{k}) \cdot \mathbf{j} = 0$: 90°

Two components give λ and μ ; the third is the check.

B6 (a) $\frac{16}{25}$: 50.2° (b) $\overrightarrow{BA} = -\mathbf{i} - \mathbf{j} + \mathbf{k}, \overrightarrow{BC} = 2\mathbf{i} - \mathbf{j} + 2\mathbf{k}, \cos = \frac{1}{3\sqrt{3}}$: 78.9°

(c) $\cos \theta = \frac{2}{3}$: 48.2° (d) $-\frac{1}{3}$

Both vectors must point away from the angle; an axis is just the direction $\mathbf{i}$

STRETCH

B7 (a) $2\mathbf{i} - 4\mathbf{j} + 6\mathbf{k} = -2(-\mathbf{i} + 2\mathbf{j} - 3\mathbf{k})$: **parallel**

(b) $(0, 1, 5)$ on l_1 needs $\lambda = -\frac{1}{2}$ from x , giving $y = 4 \neq 1$: **not the same line**

(c) $2 - 4\lambda = 0, \lambda = \frac{1}{2}$: $a = 2, b = 6$ (d) $\lambda = 3$: $(7, -10, 21)$

Parallel lines are the same line only if a point of one lies on the other.

B8 (a) $\lambda = 2, \mu = 1$: $(4, 4, 3)$ (b) $\lambda = 2, \mu = 3$, then $p + 2 = 4$: $p = 2$

(c) not parallel; $\mu = 2, \lambda = 1$, but $z: 1 \neq 3$: **skew** (d) directions are multiples, and $(1, 0, 0)$ is not on the first: **parallel, distinct**

Check parallel first; then meet or skew. State both facts for skew.



B9 (a) $(\lambda - 3, 2\lambda - 1, 2\lambda - 2) \cdot (1, 2, 2) = 9\lambda - 9 = 0$: $X(2, 2, 3)$ (b) $|(-2, 1, 0)|$: $\sqrt{5}$

(c) $2\mathbf{x} - \mathbf{p}$: $(0, 3, 3)$ (d) $\vec{OX} \cdot \mathbf{d} = 0$: $\lambda = -\frac{1}{3}$: $\left(\frac{2}{3}, -\frac{2}{3}, \frac{1}{3}\right)$, distance 1

The foot needs $\vec{PX} \cdot \mathbf{d} = 0$; the point nearest O is the foot from O

B10 (a) $\mathbf{d} = \mathbf{a} + \mathbf{c} - \mathbf{b}$: $(2, 2, 3)$ (b) $\vec{AB} = (3, 1, -2)$, $\vec{AD} = (1, 2, 1)$, $\cos = \frac{3}{\sqrt{84}}$: 70.9°

(c) $\sqrt{14}\sqrt{6}\sin A = \sqrt{84}\sqrt{\frac{75}{84}}$: $5\sqrt{3}$ (d) $(6, 3, -6) = 3(2, 1, -2)$, with a common point: collinear

Exact areas: $\sin = \sqrt{1 - \cos^2}$

C

Reasoning solutions

Your wording can be different. Check that your reason makes the same point.

CORE

C1 He used start minus end, which gives $\vec{BA}$: $\vec{AB} = \mathbf{b} - \mathbf{a} = 2\mathbf{i} - 3\mathbf{j} + 3\mathbf{k}$

C2 It must be " $\mathbf{r} =$ " (not " $l =$ "), and the direction needs a parameter: $\mathbf{r} = (\mathbf{i} + 2\mathbf{k}) + \lambda(\mathbf{i} + 3\mathbf{k})$

C3 Two lines make two angles, θ and $180^\circ - \theta$; the angle between lines is normally the acute one.

60°

C4 One component is not enough: $\lambda = 2$ gives $y = 3$ but $z = 1 - 2 = -1 \neq 1$: not on the line

STRETCH

C5 Lines that do not meet may be parallel. Skew lines are not parallel and do not meet: she must also show the directions are not multiples of each other.

C6 $\vec{OX}$ is the position vector, not the perpendicular. Use $\vec{PX} \cdot \mathbf{d} = 0$: $(\lambda - 3) + (\lambda - 4) = 0$

$\lambda = \frac{7}{2}$: $X\left(\frac{7}{2}, \frac{7}{2}, 0\right)$



C7 He used points, not directions. Use $\mathbf{i} - \mathbf{k}$ and $\mathbf{j} + 2\mathbf{k}$: $\cos \theta = \frac{-2}{\sqrt{2}\sqrt{5}}$, $\theta = 129.2^\circ$

acute angle 50.8°

C8 (a) $(\mathbf{a} + \mathbf{b}) \cdot (\mathbf{a} - \mathbf{b}) = \mathbf{a} \cdot \mathbf{a} - \mathbf{a} \cdot \mathbf{b} + \mathbf{b} \cdot \mathbf{a} - \mathbf{b} \cdot \mathbf{b}$: $|\mathbf{a}|^2 - |\mathbf{b}|^2$

(b) The diagonals are $\mathbf{a} + \mathbf{b}$ and $\mathbf{a} - \mathbf{b}$; their scalar product is $|\mathbf{a}|^2 - |\mathbf{b}|^2 = 0$: **perpendicular**

D

Problem solving solutions

There is often more than one correct method. Any complete, correct method scores full marks.

CORE

D1 $\vec{BC} = (a-1)\mathbf{i} - 2\mathbf{j}$; $\vec{OC} \cdot \vec{BC} = a(a-1) - 2 + 0 = 0$

$$a^2 - a - 2 = (a-2)(a+1) = 0: \quad a = 2 \text{ or } a = -1; \quad a = 2: \vec{BC} = \mathbf{i} - 2\mathbf{j}; \quad BC = \sqrt{5}$$

D2 (a) $\vec{AB} = 2\mathbf{i} + 4\mathbf{j} - 4\mathbf{k}$: $\mathbf{r} = (3\mathbf{i} - \mathbf{j} + 2\mathbf{k}) + \lambda(\mathbf{i} + 2\mathbf{j} - 2\mathbf{k})$

(b) $\vec{CP} = (\lambda - 1, 2\lambda + 2, -2\lambda - 3)$, $\cdot (1, 2, -2) = 9\lambda + 9 = 0$: $P(2, -3, 4)$

D3 $(3, -1, 2)$ is on l_1 with $\lambda = 1$ and on l_2 with $\mu = 0$

$\cos \theta = \frac{-4 + 0 - 6}{3 \times 5} = -\frac{2}{3}$ (obtuse), so the acute angle has $\cos \theta = \frac{2}{3}$, positive because the angle is acute.

D4 $|\vec{OA}|^2 = (1+2\lambda)^2 + \lambda^2 + (2\lambda-1)^2 = 9\lambda^2 + 2 = 11$, so $\lambda = \pm 1$

$3\mathbf{i} + \mathbf{j} + \mathbf{k}$ or $-\mathbf{i} - \mathbf{j} - 3\mathbf{k}$; distance $|(4, 2, 4)| = \sqrt{36} = 6$

STRETCH

D5 (a) $\mathbf{i}: 1 + 2\lambda = 4 + \mu$; $\mathbf{k}: 2 - \lambda = -3 + 3\mu$; so $\lambda = 2$, $\mu = 1$

$\mathbf{j}: p + 2 = 4$: $p = 2$, $X(5, 4, 0)$

(b) $\cos \theta = \frac{2+3-3}{\sqrt{6}\sqrt{19}} = \frac{2}{\sqrt{114}}$: 79.2°

D6 (a) $\vec{AC} = \mathbf{i} + 2\mathbf{j}$, $\vec{AC} \cdot (2\mathbf{i} - \mathbf{j} + 2\mathbf{k}) = 0$; area $\frac{1}{2} \times |\vec{AB}| \times |\vec{AC}| = \frac{1}{2} \times 3 \times \sqrt{5}$: $\frac{3\sqrt{5}}{2}$

(b) Twice the area needs $AD = 2AB = 6$, so $\lambda = \pm 2$: $D(5, -2, 6)$ or $D(-3, 2, -2)$



D7 (a) $\vec{PX} = (\lambda - 3, -\lambda - 3, 2\lambda - 3), \cdot (1, -1, 2) = 6\lambda - 6 = 0: \boxed{X(3, 0, 2)}$

(b) $\vec{PX} = (-2, -4, -1): \boxed{\sqrt{21}}; 2\mathbf{x} - \mathbf{p}: \boxed{(1, -4, 1)}$

D8 (a) $\mathbf{r} = (\mathbf{i} + 2\mathbf{j}) + \lambda(2\mathbf{i} + \mathbf{j} + 2\mathbf{k})$

(b) $\vec{AC} = (\alpha - 2)\mathbf{j} + 4\mathbf{k}; \cos 45^\circ = \frac{\alpha + 6}{3\sqrt{(\alpha - 2)^2 + 16}} = \frac{1}{\sqrt{2}}$

$2(\alpha + 6)^2 = 9((\alpha - 2)^2 + 16), \text{ so } 7\alpha^2 - 60\alpha + 108 = 0, (\alpha - 6)(7\alpha - 18) = 0: \boxed{\alpha = 6 \text{ or } \alpha = \frac{18}{7}}$

E

Exam practice solutions

Mark-scheme notes are shown on the right of each line.

CORE

Exam questions for everyone

E1 (a) $\mathbf{j}: 2\lambda = 4, \lambda = 2; \mathbf{i}: 1 + \lambda = -5 + 4\mu, \mu = 2$

M1 A1

$\mathbf{k}: 3 - 4 = -1 \text{ and } -7 + 6 = -1 \checkmark: \boxed{3\mathbf{i} + 4\mathbf{j} - \mathbf{k}}$

B1: check, A1

(b) $\cos \theta = \frac{4 + 0 - 6}{3 \times 5} = -\frac{2}{15}$

M1

$\boxed{\cos \theta = \frac{2}{15}}$, positive, so θ is acute

A1

E2 (a) $\mathbf{r} = (2\mathbf{i} + \mathbf{j}) + \lambda(2\mathbf{i} - 2\mathbf{j} + \mathbf{k})$

B1

(b) $\vec{CP} = (2\lambda - 6, 3 - 2\lambda, \lambda), \cdot (2, -2, 1) = 9\lambda - 18 = 0$

M1 dM1

$\boxed{P(6, -3, 2)}$

A1

(c) CP is perpendicular to l , so the angle at P is $90^\circ; AP = |(4, -4, 2)| = 6, CP = |(-2, -1, 2)| = 3$

M1

$\frac{1}{2} \times 6 \times 3: \boxed{9}$

A1



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STRETCH

E3 (a) $\mathbf{i} + \mathbf{j} - \mathbf{k}$ and $2\mathbf{i} - \mathbf{j} + \mathbf{k}$ are not multiples: not parallel

B1

$$\mathbf{i}, \mathbf{j}: 2 + \lambda = -1 + 2\mu, -1 + \lambda = 4 - \mu, \text{ so } \lambda = \frac{7}{3}, \mu = \frac{8}{3}$$

M1

$$\mathbf{k}: 3 - \frac{7}{3} = \frac{2}{3} \text{ but } 2 + \frac{8}{3} = \frac{14}{3}: \text{ no intersection, so skew}$$

A1

(b) $\overrightarrow{PQ} = (-3 + 2\mu - \lambda, 5 - \mu - \lambda, -1 + \mu + \lambda)$

M1

$$\cdot (1, 1, -1) = 3 - 3\lambda = 0 \text{ and } \cdot (2, -1, 1) = 6\mu - 12 = 0: \lambda = 1, \mu = 2$$

dM1

$$P(3, 0, 2), Q(3, 2, 4)$$

A1

(c) $\overrightarrow{PQ} = 2\mathbf{j} + 2\mathbf{k}$

M1

$$2\sqrt{2}$$

A1

