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A-LEVEL MATHEMATICS • PURE 4

Partial Fractions

Worked Solutions

Pearson Edexcel International A Level (WMA14) & A Level (9MA0) • Calculator allowed: show every step

Full worked solutions for the free Partial Fractions worksheet, A-Level and International A-Level Maths



Tap a section to jump straight to it

Full worked solutions for every question on the Pure 4 Partial Fractions worksheet, in the same order and with the same question numbers.

How to mark your work. Tick each step you got right, not just the final answer. At A-level most marks are for method, so a wrong final answer can still earn most of the marks. If you lost marks, find the *first* line where your working is different from ours. That is the thing to fix.

In Section E: **M** = method mark, **dM** = method mark that depends on the previous M mark, **A** = accuracy mark (needs the M mark before it), **B** = independent mark.

Question sheet

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A**Your turn answers**

Compare each line of your working with the solution.

CORE

A1 $x + 11 \equiv A(x+1) + B(2x-3)$: $x = \frac{3}{2}$: $\frac{25}{2} = \frac{5}{2}A$, $A = 5$; $x = -1$: $10 = -5B$, $B = -2$

$$\frac{5}{2x-3} - \frac{2}{x+1}$$

A2 $13x - 7 \equiv A(x+1) + B(3x-2)$: $A + 3B = 13$ and $A - 2B = -7$, so $5B = 20$

$B = 4$, $A = 1$: $\frac{1}{3x-2} + \frac{4}{x+1}$

A3 $2x^2 - 7x + 28 \equiv A(x-3)^2 + B(2x-1)(x-3) + C(2x-1)$

$x = 3$: $25 = 5C$, $C = 5$; $x = \frac{1}{2}$: $25 = \frac{25}{4}A$, $A = 4$; x^2 : $2 = A + 2B$, $B = -1$

$$\frac{4}{2x-1} - \frac{1}{x-3} + \frac{5}{(x-3)^2}$$

STRETCH

A4 $A = 3$; $3x^2 - 3x + 2 \equiv 3(x+1)(x-3) + B(x-3) + C(x+1)$

$x = -1$: $8 = -4B$, $B = -2$; $x = 3$: $20 = 4C$, $C = 5$: $3 - \frac{2}{x+1} + \frac{5}{x-3}$

A5 Dividing by $x^2 + 2x - 3$: quotient $x + 3$, remainder $x - 5$

$x - 5 \equiv C(x-1) + D(x+3)$: $x = 1$: $-4 = 4D$, $D = -1$; $x = -3$: $-8 = -4C$, $C = 2$

$$x + 3 + \frac{2}{x+3} - \frac{1}{x-1}$$

A6 $2x^3 + x^2 = x^2(2x+1)$: $x^2 + 3x + 1 \equiv Ax(2x+1) + B(2x+1) + Cx^2$

$x = 0$: $B = 1$; $x = -\frac{1}{2}$: $-\frac{1}{4} = \frac{1}{4}C$, $C = -1$; x^2 : $1 = 2A + C$, $A = 1$

$$\frac{1}{x} + \frac{1}{x^2} - \frac{1}{2x+1}$$



A7 $(x-2)(x+1)^2 = x^3 - 3x - 2$, so the fraction is $2 + \frac{5x-1}{(x-2)(x+1)^2}$

$5x-1 \equiv A(x+1)^2 + B(x-2)(x+1) + C(x-2): x=2: A=1; x=-1: C=2; x^2: B=-1$

$$2 + \frac{1}{x-2} - \frac{1}{x+1} + \frac{2}{(x+1)^2}$$

B**Fluency solutions**

One mark for each correct part. The notes in italics explain why each sequence was chosen.

CORE

B1 (a) $\frac{A}{x+2} + \frac{B}{x-5}$ (b) $\frac{A}{x-1} + \frac{B}{x+3} + \frac{C}{2x+1}$

(c) $\frac{A}{x+1} + \frac{B}{(x+1)^2} + \frac{C}{x-2}$ (d) $A + \frac{B}{x-1} + \frac{C}{x+4}$

One term per factor, two for a squared factor, and a polynomial part when the top degree is not lower.

B2 (a) $\frac{3}{x-1} + \frac{2}{x+1}$ (b) $\frac{2}{x-3} - \frac{1}{x+2}$ (c) $\frac{2}{x-3} - \frac{4}{2x+1}$ (d) $\frac{2}{x} - \frac{1}{x-1} + \frac{3}{x+2}$

For $2x+1$, substitute $x = -\frac{1}{2}$; with three factors, three substitutions.

B3 (a) $A+B=5, 2A+B=7: \frac{2}{x+1} + \frac{3}{x+2}$ (b) $A+2B=1, -3A+B=-10: \frac{3}{2x+1} - \frac{1}{x-3}$

(c) $2(x+4)+3(x-1)=5x+5: a=5, b=5$ (d) $A=3, B=-4, C=6$

Compare x terms and constants; in (d), start with x^2 and work down.

B4 (a) $\frac{1}{x+1} + \frac{4}{(x+1)^2}$ (b) $\frac{1}{x+2} + \frac{2}{x-1} - \frac{3}{(x-1)^2}$

(c) $\frac{1}{x} - \frac{2}{2x-1} + \frac{4}{(2x-1)^2}$ (d) $-\frac{1}{2x} + \frac{2}{x^2} + \frac{1}{2(x+2)}$

Two substitutions give the outer constants; the x^2 coefficient gives the middle one.

B5 (a) $1 - \frac{2}{x+1} + \frac{2}{x+2}$ (b) $2 - \frac{3}{x} + \frac{4}{x-1}$ (c) $x-1 - \frac{2}{x} + \frac{3}{x+1}$

Divide first: the quotient is the polynomial part, and only the remainder is split.



B6 (a) $(x-2)(x+3): \frac{3}{x-2} - \frac{2}{x+3}$ (b) $(x-1)(x+1): \frac{2}{x-1} + \frac{4}{x+1}$

(c) $x(x-1)(x+1): -\frac{1}{x} + \frac{5}{2(x-1)} - \frac{3}{2(x+1)}$ (d) $(x-1)^2$, improper: $2 + \frac{4}{x-1} + \frac{7}{(x-1)^2}$

The form depends on the factorised denominator, so always factorise first.

STRETCH

B7 (a) $x-2 + \frac{3}{x+1} - \frac{1}{(x+1)^2}$ (b) $x-1 - \frac{1}{x} + \frac{1}{x^2} + \frac{2}{x+1}$

(c) $6x^2 + x - 1 = (2x+1)(3x-1): \frac{3x-1}{x-1} = 3 + \frac{2}{x-1}$ (d) $B = \frac{4+2k+8}{(1)(-1)} = 0: k = -6$

In (c) and (d) a factor cancels, so a term disappears: always look for a common factor first.

B8 (a) $\frac{1}{2a} \left(\frac{1}{x-a} - \frac{1}{x+a} \right)$ (b) $\frac{1}{k} \left(\frac{1}{x} - \frac{1}{x+k} \right)$

(c) $-\frac{1}{y} + \frac{1}{y-1} + \frac{1}{y+1}$ (d) $(2x-1)(x+3): \frac{1}{2x-1} - \frac{1}{x+3}$

The method does not change when the variable or the constants are letters.

B9 (a) constants: $8+2-C=5, C=5; x: 8-1+C=a: a=12, C=5$

(b) $A=2, B=-1, C=1$ (c) $2 - \frac{1}{x-1} + \frac{1}{(x-1)^2}$

(d) $1 \equiv A(x+2)(x+3) + B(x+1)(x+3) + C(x+1)(x+2);$ the x^2 terms give $A+B+C=0: A=\frac{1}{2}, B=-1, C=\frac{1}{2}$

Comparing coefficients gives facts that substitution does not show directly.

B10 (a) $2 + \frac{1}{x-1} - \frac{3}{x+2}$ (b) $x=1, x=-2, y=2$

(c) $\frac{1}{x-1} - \frac{3}{x+2} = \frac{5-2x}{(x-1)(x+2)} < 0$ for large $x: \text{below } 2$ (d) $\frac{1}{x-1} = \frac{3}{x+2}: x = \frac{5}{2}$

The constant part is the horizontal asymptote; the other terms say how the curve approaches it.



C

Reasoning solutions

Your wording can be different. Check that your reason makes the same point.

CORE

- C1** $3x + 1 \equiv A(x + 1)^2 + B(x - 1)$: the x^2 terms give $A = 0$, but $x = 1$ gives $A = 1$, a contradiction.

$$\frac{A}{x-1} + \frac{B}{x+1} + \frac{C}{(x+1)^2}$$

- C2** The fraction is improper (degree 2 over degree 2), so it needs a constant term first.

$$A + \frac{B}{x-1} + \frac{C}{x+2}$$

- C3** She multiplied $\frac{A}{x-2}$ by $(x+1)$ only; it must be multiplied by $(x+1)^2$

$$4x + 1 \equiv A(x + 1)^2 + B(x - 2)(x + 1) + C(x - 2); x = 2: 9 = 9A: \boxed{A = 1}$$

- C4** After multiplying out, both sides are polynomials that are equal for every x except 1 and -2 , so they are identical and equal for all x , including $x = 1$

$$x = 1: 9 = 3A: \boxed{A = 3}$$

STRETCH

- C5** $x^2 + 2x = x(x + 2)$, so the fraction is $\frac{x}{x-3}$: there is no factor $x + 2$ left.

$$\frac{x}{x-3} = 1 + \frac{3}{x-3}, \text{ so } B = 0 \text{ is correct.}$$

- C6** $3x + 2 = 3(x - 1) + 5: \boxed{A = 3, B = 5}$; and $\boxed{P = 3, Q = 2}$

Both forms are correct, but $\frac{A}{x-1} + \frac{B}{(x-1)^2}$ has constant numerators, so each term can be used directly, for example differentiated.

- C7** One value gives one equation, but there are two unknowns, so many wrong answers pass one check.

$$x = 2: \text{ the left side is } \frac{9}{5} \text{ but his right side is } \frac{17}{5}: \boxed{\frac{2}{x-1} - \frac{1}{x+3}}$$



C8 (a) $\frac{1}{x+a} - \frac{1}{x+b} = \frac{(x+b) - (x+a)}{(x+a)(x+b)} = \frac{b-a}{(x+a)(x+b)}$; dividing by $b-a \neq 0$ gives the result.

(b) $a=2, b=3$: $\frac{1}{x+2} - \frac{1}{x+3}$; $\frac{1}{4} \times \frac{1}{\left(x-\frac{1}{2}\right)\left(x+\frac{1}{2}\right)}$: $\frac{1}{2(2x-1)} - \frac{1}{2(2x+1)}$

D**Problem solving solutions**

There is often more than one correct method. Any complete, correct method scores full marks.

CORE

D1 $x^2 - 7x + 32 \equiv A(x-3)^2 + B(x+2)(x-3) + C(x+2)$

$x=3$: $20 = 5C$; $x=-2$: $50 = 25A$; x^2 : $1 = A+B$: $A=2, B=-1, C=4$

Check $x=0$: $\frac{32}{18} = \frac{16}{9}$ and $\frac{2}{2} + \frac{1}{3} + \frac{4}{9} = \frac{16}{9}$ ✓

D2 (a) $A=1$: $x^2 + 2x + 3 \equiv (x-1)(x+2) + B(x+2) + C(x-1)$; $x=1$: $6 = 3B$; $x=-2$: $3 = -3C$

$y = 1 + \frac{2}{x-1} - \frac{1}{x+2}$

(b) $y=1$ when $\frac{2}{x-1} = \frac{1}{x+2}$, so $2x+4 = x-1$: $P(-5, 1)$

D3 $kx+4 \equiv A(x-2) + A(x+1) = A(2x-1)$

constants: $-A=4$, so $A=-4$; x : $k=2A$: $k=-8, A=-4$: $\frac{-8x+4}{(x+1)(x-2)} \equiv -\frac{4}{x+1} - \frac{4}{x-2}$

D4 $x^3 - x = x(x-1)(x+1)$: $x^2 + 10x + 1 \equiv A(x-1)(x+1) + Bx(x+1) + Cx(x-1)$

$x=0$: $1 = -A$; $x=1$: $12 = 2B$; $x=-1$: $-8 = 2C$: $-\frac{1}{x} + \frac{6}{x-1} - \frac{4}{x+1}$

Check $x=2$: $\frac{25}{6}$ and $-\frac{1}{2} + 6 - \frac{4}{3} = \frac{25}{6}$ ✓

STRETCH

D5 $f(0) = 3 + 2 - \frac{B}{2} = 1$: $B=8$

$ax^2 + bx + c \equiv 3(x+1)(x-2) + 2(x-2) + 8(x+1) = 3x^2 + 7x - 2$: $B=8, a=3, b=7, c=-2$



D6 (a) $5x^2 + 6x + 2 \equiv A(x+1)^2 + Bx(x+1) + Cx$; $x = 0$: $A = 2$; $x = -1$: $C = -1$; x^2 : $B = 3$

$$\frac{2}{x} + \frac{3}{x+1} - \frac{1}{(x+1)^2}$$

(b) $\frac{3}{x+1} - \frac{1}{(x+1)^2} = \frac{3x+2}{(x+1)^2} > 0$ for $x > 0$, so $f(x) > \frac{2}{x}$

D7 (a) $x(x-1)^2 = x^3 - 2x^2 + x$, so $y = 1 + \frac{2x^2 - 4x + 4}{x(x-1)^2}$

$x = 0$: $B = 4$; $x = 1$: $E = 2$; x^2 : $B + D = 2$, $D = -2$: $1 + \frac{4}{x} - \frac{2}{x-1} + \frac{2}{(x-1)^2}$

(b) $y - 1 = \frac{2(x^2 - 2x + 2)}{x(x-1)^2}$ and $x^2 - 2x + 2 = (x-1)^2 + 1 > 0$, so $y \neq 1$ for every x

D8 (a) $3x + 7 = 3(x+2) + 1$: $3 + \frac{1}{x+2}$

(b) $x > -2$, so $\frac{1}{x+2}$ takes every positive value: $f(x) > 3$

(c) $y - 3 = \frac{1}{x+2}$, so $x = -2 + \frac{1}{y-3}$: $f^{-1}(x) = -2 + \frac{1}{x-3}$, $x > 3$

E

Exam practice solutions

Mark-scheme notes are shown on the right of each line.

CORE

Exam questions for everyone

E1 (a) $3x^2 - 6x + 9 \equiv A(x-2)^2 + B(x+1)(x-2) + C(x+1)$

$x = 2$: $9 = 3C$, $C = 3$; $x = -1$: $18 = 9A$, $A = 2$

x^2 : $3 = A + B$: $A = 2, B = 1, C = 3$

M1: correct identity

A1: two constants

A1: all three

(b) $f'(x) = -\frac{2}{(x+1)^2} - \frac{1}{(x-2)^2} - \frac{6}{(x-2)^3}$

M1: differentiates the terms

$f'(3) = -\frac{2}{16} - 1 - 6$: $-\frac{57}{8}$

A1: cao



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E2 (a) $A = 2$

B1

$$4x^2 + 3x + 2 \equiv 2(2x - 1)(x + 1) + B(x + 1) + C(2x - 1)$$

M1

$$x = \frac{1}{2}: \frac{9}{2} = \frac{3}{2}B, B = 3; \quad x = -1: 3 = -3C, C = -1$$

A1 A1

$$g(x) = 2 + \frac{3}{2x-1} - \frac{1}{x+1}$$

$$(b) \quad g(x) - 2 = \frac{3}{2x-1} - \frac{1}{x+1} = \frac{x+4}{(2x-1)(x+1)}$$

M1: combines

$$\text{for } x > \frac{1}{2}, \text{ this is positive, so } g(x) > 2 \text{ and never equals } 2$$

A1: reason

STRETCH**E3 (a)** $15t \equiv A(t+4) + B(t+1)$

M1

$$t = -1: -15 = 3A; \quad t = -4: -60 = -3B: \quad \frac{20}{t+4} - \frac{5}{t+1}$$

A1 A1

$$(b) \quad \frac{dC}{dt} = -\frac{20}{(t+4)^2} + \frac{5}{(t+1)^2}$$

M1

$$= 0 \text{ when } (t+4)^2 = 4(t+1)^2, \text{ so } t+4 = 2(t+1) \text{ (both positive)}$$

dM1

$$t = 2 \text{ hours}$$

A1

$$C = \frac{20}{6} - \frac{5}{3}: \quad \frac{5}{3} \text{ mg per litre}$$

A1

$$(c) \text{ both terms tend to } 0 \text{ as } t \rightarrow \infty: \quad \text{the concentration tends to } 0$$

B1

