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INTERNATIONAL A LEVEL MATHEMATICS • FURTHER PURE MATHEMATICS 3

Further Pure 3

Chapter 5: Vectors

Exam Questions by Topic

Pearson Edexcel International Advanced Level (WFM03/01) • Calculator allowed

Multi-step exam questions at the difficulty of the real WFM03 paper, with a full mark scheme

Chapters (tap a chapter to jump to it)		Qs	Marks	Page	My score
1	**Hyperbolic Functions** Definitions, graphs, inverse functions, identities and equations	10	80	separate file	
2	**Further Coordinate Systems** Ellipses and hyperbolas, foci and directrices, tangents, normals and loci	10	80	separate file	
3	**Differentiation** Hyperbolic and inverse trigonometric and hyperbolic functions	10	80	separate file	
4	**Integration** Substitutions, reduction formulae, arc length and surface area	10	80	separate file	
5	**Vectors** Vector product, triple products, lines and planes, distances and angles	10	80	2	
6	**Further Matrix Algebra** 3×3 matrices, inverses, eigenvalues and eigenvectors, diagonalisation	10	80	separate file	

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CHAPTER 5 • KEY SKILLS

5 Vectors

Specification FP3: Vectors • 10 questions • 80 marks • about 96 minutes

1 Vector product

$\mathbf{a} \times \mathbf{b}$ by the determinant method; it is perpendicular to both $\mathbf{a}$ and $\mathbf{b}$, and $|\mathbf{a} \times \mathbf{b}| = |\mathbf{a}||\mathbf{b}| \sin\theta$

2 Areas

Triangle ABC: $\frac{1}{2}\left|\overrightarrow{AB} \times \overrightarrow{AC}\right|$;

parallelogram: $\left|\overrightarrow{AB} \times \overrightarrow{AD}\right|$

3 Volumes

Scalar triple product $\mathbf{a} \cdot (\mathbf{b} \times \mathbf{c})$:

parallelepiped $|\ldots|$, tetrahedron $\frac{1}{6}|\ldots|$

4 Lines

$\mathbf{r} = \mathbf{a} + \lambda\mathbf{b}$, or $(\mathbf{r} - \mathbf{a}) \times \mathbf{b} = \mathbf{0}$, i.e. $\mathbf{r} \times \mathbf{b} = \mathbf{a} \times \mathbf{b}$, or Cartesian $\dfrac{x-a_1}{b_1} = \dfrac{y-a_2}{b_2} = \dfrac{z-a_3}{b_3}$

5 Planes

$\mathbf{r} \cdot \mathbf{n} = \mathbf{a} \cdot \mathbf{n}$, i.e. $n_1x + n_2y + n_3z = d$; from $\mathbf{r} = \mathbf{a} + \lambda\mathbf{b} + \mu\mathbf{c}$, take $\mathbf{n} = \mathbf{b} \times \mathbf{c}$

6 Intersections

Line and plane: substitute the line into the plane equation. Two planes: direction $\mathbf{n}_1 \times \mathbf{n}_2$, plus one common point.

7 Angles

Between a line and a plane: $\sin\theta = \dfrac{|\mathbf{b} \cdot \mathbf{n}|}{|\mathbf{b}||\mathbf{n}|}$. Between planes: the angle between the normals.

8 Distances

Point to plane: $\dfrac{|n_1x_0 + n_2y_0 + n_3z_0 - d|}{|\mathbf{n}|}$. Skew lines: $\dfrac{|(\mathbf{a}_2 - \mathbf{a}_1) \cdot (\mathbf{b}_1 \times \mathbf{b}_2)|}{|\mathbf{b}_1 \times \mathbf{b}_2|}$

9 Feet and reflections

Move from the point along $\mathbf{n}$ to the plane (foot), then the same distance again (reflection). Answer lines as "$\mathbf{r} = \ldots$".

Skills map every skill above is tested at least once; interleaved skills come from GCSE or other chapters

Skill	Q1	Q2	Q3	Q4	Q5	Q6	Q7	Q8	Q9	Q10
Marks	6	6	6	6	6	10	10	10	10	10
1 Vector product	•	•	•				•	•	•	•
2 Areas	•								•	
3 Volumes		•							•	
4 Lines				•		•	•	•		•
5 Planes			•	•	•	•		•	•	•
6 Intersections				•		•	•	•		
7 Angles				•			•	•		
8 Distances					•	•	•		•	•
9 Feet and reflections					•	•				•
Interleaved: vectors and scalar product (P4)	•	•	•	•	•	•	•	•	•	•
Interleaved: simultaneous equations (P1)							•	•		
Interleaved: trigonometry and surds (P1)	•			•			•	•	•	

Exam difficulty. Questions 1 to 5 check each skill on its own. Questions 6 to 10 are multi-step questions built like the last questions on a real WFM03 paper, where one part leads into the next and skills from different topics are combined.

Must learn. Sketch the situation, choose the right vectors (direction and normal), and say which angle you have found before adjusting it. Write lines as "$\mathbf{r} =$" and planes with their constant.

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1. The vectors $\mathbf{a} = \mathbf{i} + 2\mathbf{j} - \mathbf{k}$ and $\mathbf{b} = 3\mathbf{i} + 2\mathbf{k}$ are the position vectors of the points A and B.

(a) Find $\mathbf{a} \times \mathbf{b}$.

(2)

(b) Verify that $\mathbf{a} \times \mathbf{b}$ is perpendicular to $\mathbf{a}$.

(1)

(c) Find the exact area of triangle OAB.

(2)

(d) Find a unit vector perpendicular to both $\mathbf{a}$ and $\mathbf{b}$.

(1)

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Question 1 continued

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(Total for Question 1 is 6 marks)

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2. The points A, B and C have position vectors $\mathbf{i}+2\mathbf{k}$, $2\mathbf{i}+\mathbf{j}$ and $3\mathbf{j}+\mathbf{k}$ respectively, and O is the origin.

(a) Find the volume of the tetrahedron $OABC$.

(4)

(b) Explain what it would mean if the scalar triple product $\overrightarrow{OA}\cdot\left(\overrightarrow{OB}\times\overrightarrow{OC}\right)$ were zero.

(2)

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Question 2 continued

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3. The plane Π passes through the points $A(1,0,2)$, $B(3,1,1)$ and $C(2,-1,4)$.

(a) Find a Cartesian equation of Π.

(4)

(b) Verify that C lies on your plane.

(1)

(c) Write the equation of Π in the form $\mathbf{r} \cdot \mathbf{n} = p$

(1)

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Question 3 continued

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4. The line l has equation $\mathbf{r} = \mathbf{i} + 2\mathbf{j} + \lambda(2\mathbf{i} - \mathbf{j} + 3\mathbf{k})$ and the plane Π has equation $x + 2y + 2z = 13$

(a) Find the coordinates of the point where l meets Π.

(3)

(b) Find the acute angle between l and Π, to the nearest 0.1°

(3)

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Question 4 continued

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5. The point P has coordinates $(3, -1, 2)$ and the plane Π has equation $2x - y + 2z = 1$

(a) Find the perpendicular distance from P to Π.

(3)

(b) Find the coordinates of the foot of the perpendicular from P to Π.

(3)

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Question 5 continued

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6. The line l passes through $A(2, 1, -1)$ in the direction $\mathbf{i} + 2\mathbf{j} + 2\mathbf{k}$. The plane Π has equation $2x - 2y + z = 9$

(a) Show that l is parallel to Π, and find the distance between them.

(5)

(b) Find the coordinates of the reflection A' of A in Π.

(3)

(c) Write down a vector equation of the reflection of l in Π.

(2)

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Question 6 continued

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Question 6 continued

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7. The lines l_1 and l_2 have equations

$$l_1 : \mathbf{r} = \mathbf{i} + 2\mathbf{k} + \lambda(\mathbf{i} + \mathbf{j}), \qquad l_2 : \mathbf{r} = 3\mathbf{j} + \mathbf{k} + \mu(2\mathbf{i} + \mathbf{k})$$

(a) Find the shortest distance between l_1 and l_2, giving an exact answer.

(5)

(b) Explain why your answer shows that the lines do not intersect.

(2)

(c) Find the acute angle between the directions of l_1 and l_2, to the nearest 0.1°

(3)

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Question 7 continued

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Question 7 continued

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8. The planes Π_1 and Π_2 have equations $x+y+z=6$ and $2x-y+z=3$ respectively.

(a) Find the acute angle between Π_1 and Π_2, to the nearest 0.1°

(3)

(b) Find a vector equation of the line of intersection of Π_1 and Π_2.

(4)

(c) Write this line in Cartesian form.

(1)

(d) Write this line in the form $\mathbf{r}\times\mathbf{b}=\mathbf{c}$

(2)

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Question 8 continued

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Question 8 continued

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9. A tetrahedron has vertices $A(1,1,0)$, $B(3,0,1)$, $C(2,3,1)$ and $D(1,2,4)$.

(a) Find the exact area of the face ABC.

(3)

(b) Find the volume of the tetrahedron.

(3)

(c) Hence find the exact perpendicular distance from D to the plane ABC.

(2)

(d) Find a Cartesian equation of the plane ABC, and use it to check your answer to part (c).

(2)

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Question 9 continued

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Question 9 continued

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10. The plane Π has vector equation $\mathbf{r} = \mathbf{i} + s(\mathbf{i} + \mathbf{j}) + t(\mathbf{j} + \mathbf{k})$

(a) Find a Cartesian equation of Π.

(3)

(b) The line l has equation $\mathbf{r} = 2\mathbf{i} + 3\mathbf{j} + 4\mathbf{k} + \lambda(\mathbf{i} - \mathbf{k})$. Show that l is parallel to Π and find the exact distance between them.

(3)

(c) Find the coordinates of the point of Π nearest to the origin.

(2)

(d) Find a Cartesian equation of the plane that contains l and is perpendicular to Π.

(2)

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Question 10 continued

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Question 10 continued

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TOTAL FOR CHAPTER 5 IS 80 MARKS

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