



eClassroom

A-LEVEL MATHEMATICS • PURE 3

Integration

Worksheet

Pearson Edexcel International A Level (WMA13) & A Level (9MA0) • Calculator allowed: show every step

Free Integration worksheet for A-Level and International A-Level Maths, with full worked solutions



Tap a section to jump straight to it

By the end of this worksheet you can: standard integrals and $f(ax + b)$ • exact definite integrals • *Stretch:* the reverse chain rule, identities and areas.

Which questions are for me?

CORE The methods every A-level student must secure. Start here.

STRETCH Standard integrals, the reverse chain rule, identities and exact areas.

Every section has both, clearly labelled. Questions get harder as you go, and every skill in Sections A to D is needed in Section E.

Track your progress

Section	B	C	D	E
Core marks	23	10	13	11
Stretch marks	16	14	17	8
My score				

Full worked solutions

Exam questions by topic

Integration revision engine

Formula sheet builder

Name

Class

Date



Join students already acing their exams.

Book your FREE lesson at www.eclassroom.org.uk

A

Worked examples & Your turn

Read each example carefully. Cover it up, then answer the question beside it using the same method.

CORE

Secure the method

Worked example A1

Find $\int \left(e^x + \frac{3}{x} - 2 \sin x \right) dx$

Integrate term by term; $\int \sin x dx = -\cos x$

$$e^x + 3 \ln |x| + 2 \cos x + c$$

Your turn A1

Find $\int \left(4 \cos x - \frac{1}{x} + \sec^2 x \right) dx$

Worked example A2

Find $\int e^{4x-1} dx$ and $\int \frac{1}{2x+5} dx$

Integrate as usual, then divide by the coefficient of x

$$\frac{1}{4} e^{4x-1} + c; \quad \frac{1}{2} \ln |2x+5| + c$$

Your turn A2

Find $\int \cos(3x+1) dx$ and $\int (5-2x)^4 dx$

Worked example A3

Find the exact value of $\int_0^1 e^{2x} dx$

$$\left[\frac{1}{2} e^{2x} \right]_0^1 = \frac{1}{2} e^2 - \frac{1}{2} e^0$$

$$\frac{1}{2} (e^2 - 1)$$

Your turn A3

Find the exact value of $\int_1^3 \frac{2}{2x-1} dx$



STRETCH

Worked example A4: $\frac{f'(x)}{f(x)}$

Find $\int \frac{2x}{x^2+3} dx$

The numerator is the derivative of the denominator.

$$\ln(x^2+3) + c$$

Your turn A4

Find $\int \frac{\cos x}{2 + \sin x} dx$

Worked example A5: the reverse chain rule

Find $\int x(x^2+1)^3 dx$

Try $(x^2+1)^4$: its derivative is $8x(x^2+1)^3$, so divide by 8

$$\frac{1}{8}(x^2+1)^4 + c$$

Your turn A5

Find $\int x^2 e^{x^3} dx$

Worked example A6: an identity first

Find $\int \sin^2 x dx$

$\sin^2 x = \frac{1}{2}(1 - \cos 2x)$, then integrate each term.

$$\frac{1}{2}x - \frac{1}{4}\sin 2x + c$$

Your turn A6

Find $\int \tan^2 x dx$

Worked example A7: an area

Find the area under $y = \frac{1}{x}$ from $x = 1$ to $x = e^2$

$$[\ln x]_1^{e^2} = 2 - 0$$

$$2$$

Your turn A7

Find the exact area under $y = e^{\frac{x}{2}}$ from $x = 0$ to $x = 2$



B**Fluency**

Each question is a designed sequence. From one part to the next, ask yourself: what changed, and what did it do to the answer?

CORE*Secure the method***B1** Find

(a) $\int e^{3x} dx$

(b) $\int \frac{6}{x} dx$

(c) $\int \sin 2x dx$

(d) $\int \sec^2 5x dx$

[4 marks]

B2 Find

(a) $\int (3x+2)^5 dx$

(b) $\int \frac{1}{4x-3} dx$

(c) $\int e^{2-x} dx$

(d) $\int \frac{3}{1-2x} dx$

[4 marks]

B3 Find the exact value of

(a) $\int_0^{\frac{\pi}{2}} \cos x dx$

(b) $\int_0^{\frac{\pi}{6}} \sin 3x dx$

(c) $\int_1^e \frac{2}{x} dx$

(d) $\int_0^{\ln 2} e^{2x} dx$

[4 marks]

B4 Find

(a) $\int \frac{3x^2}{x^3+1} dx$

(b) $\int \frac{x}{x^2+4} dx$

(c) $\int \tan x dx$

(d) $\int \frac{e^x}{e^x+1} dx$

[4 marks]



CORE

continued: identities and division

B5 Find

(a) $\int \cos^2 x \, dx$

(b) $\int \sin x \cos x \, dx$

(c) $\int (1 + \tan^2 x) \, dx$

[3 marks]

B6

(a) Find $\int \frac{x+3}{x+1} \, dx$, by writing the fraction as $1 + \frac{2}{x+1}$

(b) Find $\int \frac{2x^2+1}{x} \, dx$

(c) Find the exact value of $\int_0^1 \frac{x+3}{x+1} \, dx$

(d) Find $\int \frac{x^2+x+1}{x+1} \, dx$

[4 marks]

STRETCH

B7 Find

(a) $\int x(2x^2-1)^4 \, dx$

(b) $\int \sin x \cos^3 x \, dx$

(c) $\int e^{\sin x} \cos x \, dx$

(d) $\int \frac{x}{\sqrt{x^2+9}} \, dx$

[4 marks]

B8 Find

(a) $\int \sin^2 3x \, dx$

(b) $\int (\cos x - \sin x)^2 \, dx$

(c) $\int \cot^2 x \, dx$

(d) $\int \frac{\cos 2x}{\cos x + \sin x} \, dx$

[4 marks]

B9 Find the exact value of the constant in each case.

(a) $\int_1^k \frac{1}{x} \, dx = 2$

(b) $\int_0^k e^{2x} \, dx = 4$

(c) $\int_0^a \cos x \, dx = \frac{1}{2}, 0 < a < \frac{\pi}{2}$

(d) $\int_2^b \frac{1}{x-1} \, dx = \ln 3$

[4 marks]

B10

(a) Find where the curve $y = 4 - e^x$ meets the x -axis.(b) Find the exact area between $y = 4 - e^x$ and the axes, for $x \geq 0$ (c) Find the area under $y = \sin 2x$ from $x = 0$ to $x = \frac{\pi}{2}$ (d) Find the exact area under $y = \frac{1}{x+2}$ from $x = 0$ to $x = 2$

[4 marks]



C**Reasoning**

Show your working and explain your thinking. A correct answer without a reason does not score full marks.

CORE

Secure the method

C1 Sam writes $\int \frac{1}{2x+3} dx = \ln|2x+3| + c$

Explain Sam's mistake, and give the correct integral.

[2 marks]

C2 Lily writes $\int \sin^2 x dx = \frac{1}{3} \sin^3 x + c$

Explain Lily's mistake, and say what she should do first.

[2 marks]

C3 Kai writes $\int \sin 4x dx = \frac{1}{4} \cos 4x$

Explain Kai's two mistakes, and give the correct integral.

[3 marks]

C4 Jo finds $\int_{\pi}^{2\pi} \sin x dx = -2$ and says the area between $y = \sin x$ and the x -axis from π to 2π is -2

Explain Jo's mistake, and give the area.

[3 marks]



STRETCH

C5 Max writes $\int_0^1 e^{2x} dx = \left[\frac{1}{2} e^{2x} \right]_0^1 = \frac{1}{2} e^2 - 0$

Explain Max's mistake, and give the correct value.

[3 marks]

C6 Ali writes $\int x(x^2 + 1)^4 dx = \frac{x^2}{2} \times \frac{(x^2 + 1)^5}{5} + c$

Explain Ali's mistake, and give the correct integral.

[3 marks]

C7 Ali says $\int_{-3}^{-1} \frac{1}{x} dx$ cannot be found, because $\ln(-1)$ and $\ln(-3)$ are undefined.

Explain why Ali is wrong, and find the exact value of the integral.

[3 marks]

C8 (a) Show that $\frac{d}{dx}(x \ln x - x) = \ln x$

(2)

(b) Hence find the exact value of $\int_1^e \ln x dx$

(2)

(c) Explain what your answer to (b) tells you about the region under $y = \ln x$ between $x = 1$ and $x = e$

(1)

[5 marks]



D**Problem solving**

You decide the method. Read the whole question first, and show every step.

CORE

Secure the method

- D1** *In this question you must show all stages of your working. Solutions relying on calculator technology are not acceptable.*

By first differentiating $(1 - x^2)^4$ with respect to x , and using your result to integrate by recognition, find the exact value of

$$\int_0^1 x(1 - x^2)^3 dx$$

showing clearly the substitution of both limits, including the lower limit.

[3 marks]

- D2** The function f is defined by $f(x) = \frac{2x^2 + 3x - 1}{x + 2}$, $x > -2$. The numerator has a higher degree than the denominator.

(a) Write $f(x)$ in the form $Ax + B + \frac{C}{x + 2}$, where A , B and C are integers to be found.

(2)

(b) Hence, using algebraic integration, show that $\int_0^2 f(x) dx = p + \ln q$, where p and q are integers to be found.

(2)

[4 marks]

- D3** *In this question you must show all stages of your working. Solutions relying on calculator technology are not acceptable.*

Using the identity $\cos 2x \equiv 2\cos^2 x - 1$, find the exact value of

$$\int_0^{\frac{\pi}{3}} 2\cos^2 x dx$$

giving your answer in the form $a\pi + b\sqrt{3}$, where a and b are rational numbers to be found.

[3 marks]



- D4** Figure 1 shows part of the curve with equation $y = \frac{3}{2x+1}$, $x \geq 0$. The region R , shown shaded, is bounded by the curve, the coordinate axes and the line $x = 4$

Use algebraic integration to find the exact area of R , giving your answer in the form $k \ln 3$, where k is a rational number to be found.

(3)

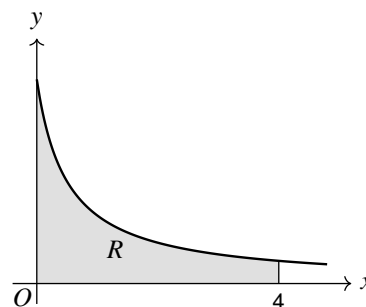


Figure 1

[3 marks]

STRETCH

- D5** In this question you must show all stages of your working. Solutions relying on calculator technology are not acceptable.

(a) Prove that $\cos^4 x - \sin^4 x \equiv \cos 2x$

(2)

- (b) Hence find the exact value of $\int_0^{\frac{\pi}{4}} (\cos^4 x - \sin^4 x) dx$, explaining why the identity makes this possible without expanding any powers.

(2)

[4 marks]

- D6** In this question you must use algebraic integration, and show all stages of your working. Given that k is a positive constant and that

$$\int_1^2 \frac{4}{2x+k} dx = 2 \ln \frac{5}{3}$$

(a) show that the integral is equal to $2 \ln \left(\frac{4+k}{2+k} \right)$

(2)

- (b) hence find the value of k

(2)

[4 marks]



- D7** Figure 2 shows the curve with equation $y = e^{2x}$ and the line with equation $y = e^2$. The curve and the line meet at the point A . The region R , shown shaded, is bounded by the curve, the line and the y -axis.

(a) Show that the x coordinate of A is 1

(1)

(b) Use algebraic integration to find the exact area of R , giving your answer in terms of e

(3)

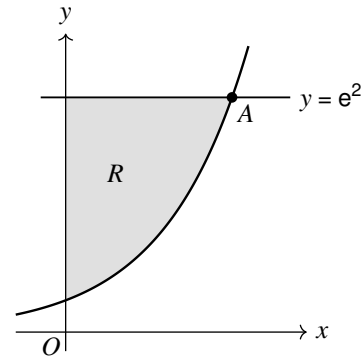


Figure 2

[4 marks]

- D8** In this question you must show all stages of your working. Solutions relying on calculator technology are not acceptable.

(a) Show that $\tan x + \tan^3 x \equiv \tan x \sec^2 x$

(1)

(b) Hence, by recognising $\tan x \sec^2 x$ as the derivative of a function, find the exact value of

$$\int_0^{\frac{\pi}{4}} (\tan x + \tan^3 x) \, dx$$

(2)

(c) Using an identity connecting $\tan^2 x$ and $\sec^2 x$, find the exact value of $\int_0^{\frac{\pi}{4}} \tan^2 x \, dx$, giving your answer in terms of π

(2)

[5 marks]



E**Exam practice**

Exam-style questions in the style of WMA13. Write your answers in the spaces, and show every step.

CORE

Exam questions for everyone

E1 *In this question you must show all stages of your working. Solutions relying on calculator technology are not acceptable.*

Find each of the following integrals, showing each step of your working. Remember to include a constant of integration.

(a) $\int \cos^2 3x \, dx$, using a suitable double angle identity,

.....
(2)

(b) $\int x^2 (2x^3 + 5)^4 \, dx$, by considering the derivative of $(2x^3 + 5)^5$

.....
(3)

(Total for Question E1 is 5 marks)



E2 Given that

$$\frac{x^3 + 2x^2 + 3x + 2}{x^2 + 1} \equiv Ax + B + \frac{Cx}{x^2 + 1}$$

(a) find the values of the constants A , B and C , showing your method clearly (by long division or by comparing coefficients),

.....
(3)

(b) hence, using algebraic integration, find the exact value of $\int_0^1 \frac{x^3 + 2x^2 + 3x + 2}{x^2 + 1} dx$, giving your answer in the form $p + \ln q$, where p and q are rational.

.....
(3)

(Total for Question E2 is 6 marks)



STRETCH

- E3** In this question you must show all stages of your working. Solutions relying on calculator technology are not acceptable.

Figure 3 shows the curves with equations

$$y = e^x \quad y = 4 - 3e^{-x}$$

The curves meet at the points A and B , where A lies on the y -axis. The finite region R , shown shaded, is bounded by the two curves.

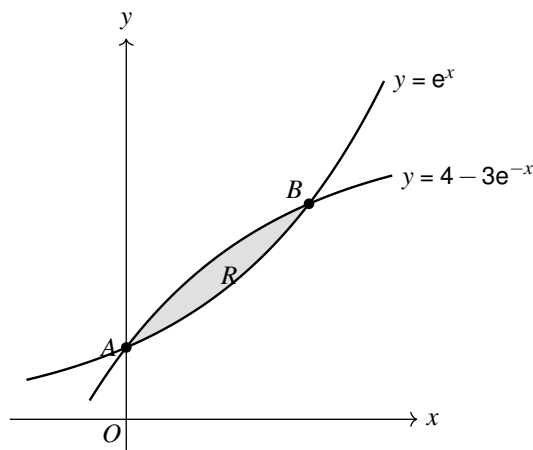


Figure 3

- (a) Show that the x coordinates of A and B satisfy $e^{2x} - 4e^x + 3 = 0$

(2)

- (b) Hence find the exact x coordinates of A and B

(2)

- (c) Use algebraic integration to find the exact area of R , giving your answer in the form $a \ln 3 + b$, where a and b are integers.

(4)

(Total for Question E3 is 8 marks)

Want more exam practice? The **Exam Questions by Topic** booklets have 10 more full exam questions on Pure 3 Chapter 7, with a full mark scheme.

[Open the booklet](#)

Finished? Mark your work with the **full worked solutions**, fill in your scores on the front page, then keep the skill sharp with unlimited questions on the **Integration revision engine**.



Join students already acing their exams.

Book your FREE lesson at www.eclassroom.org.uk