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INTERNATIONAL A LEVEL MATHEMATICS • PURE 4

Pure Mathematics 4

Exam Questions by Topic: Mark Scheme

Pearson Edexcel International Advanced Level (WMA14/01) • Calculator allowed

Every question in the booklet, marked line by line the way an examiner marks it

Chapters (tap a chapter to jump to it)	Qs	Marks	Page	My score
1 Proof Proof by contradiction: irrational numbers, primes, integers, inequalities	10	73	3	
2 Partial Fractions Linear and repeated factors, improper fractions	10	78	own PDF	
3 Coordinate Geometry in the (x, y) Plane Parametric equations and conversion to Cartesian form	10	76	own PDF	
4 Binomial Expansion $(a + bx)^n$ for rational n , validity, using partial fractions	10	81	own PDF	
5 Differentiation Parametric and implicit differentiation, rates of change	10	81	own PDF	
6 Integration Volumes of revolution, substitution, parts, partial fractions, differential equations	10	86	own PDF	
7 Vectors 3D vectors, vector equations of lines, intersections, scalar product	10	82	own PDF	

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How to use this mark scheme

The same marking rules Pearson examiners use for WMA14.

Mark every line, not just the answer. Method marks are given for a correct method even when the final answer is wrong. If you lost marks, find the *first* line where your working differs from the scheme. That is the thing to fix.

Code	Meaning
M1	Method mark: for knowing a correct method and attempting to apply it. Small slips in the working do not lose this mark.
dM1	Dependent method mark: can only be given if the method mark before it has been given.
A1	Accuracy mark: for a correct answer or line of working. It can only be given if the method mark it depends on has been given.
B1	Independent mark: for a correct result or statement, whatever method was used.
*	The answer is given in the question. Every step must be shown, and the answer must follow from correct working (cso).
cao	Correct answer only.
cso	Correct solution only: no errors anywhere in the working.
ft	Follow through: the mark can be given for correct work using your own earlier (wrong) answer.
oe	Or equivalent: any correct equivalent form is accepted.

“Solutions relying on calculator technology are not acceptable.” When a question says this, a correct final answer with no algebraic working scores **no marks**. The notes under each scheme line say when this applies.

Marking a proof. A proof by contradiction earns its final mark only when it has all three parts: a clear assumption at the start (“Assume there is ...”), correct deductions in the right order, and a statement of the contradiction with a conclusion. A sketch or calculator values are not a proof.



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Chapter 1: Proof

Mark scheme. Marks are shown on the right of each line.

Question 1

5 marks

Part	Scheme	Marks
(a)	Line 1 must say that a and b have <i>no common factors</i> (the fraction is in its lowest terms) Without this, finding that a and b are both even contradicts nothing: e.g. $\frac{2}{4} = \frac{1}{2}$ <i>This is the omission that cost most candidates the final mark in the Oct 2021 and June 2023 series.</i>	B1 B1 (2)
(b)	If a were odd, $a = 2k + 1$ and $a^2 = 4k^2 + 4k + 1 = 2(2k^2 + 2k) + 1$ would be odd; so if a^2 is even, a is even *	B1* (1)
(c)	$3b^2 = 2m^2$, so $3b^2$ is even 3 is odd, so b^2 must be even (odd $\times$ odd is odd), and therefore b is even (by part (b)) * <i>Writing "b is even" straight from $2m^2 = 3b^2$ skips the step that uses 3 being odd.</i>	M1 A1* (2)
Total for Question 1: 5 marks		

Question 2

5 marks

Part	Scheme	Marks
	Assume C has a stationary point, so $\frac{dy}{dx} = 0$ for some value of x Differentiates to the form $pe^{3x} + 6 \pm q \cos 3x$ $\frac{dy}{dx} = 3e^{3x} + 6 - 6 \cos 3x$ At this x : $3e^{3x} = 6(\cos 3x - 1)$, and $\cos 3x \leq 1$ so the right-hand side is ≤ 0 But $e^{3x} > 0$ for all x , so the left-hand side is positive: a contradiction. So C has no stationary points * <i>Assuming $\frac{dy}{dx} = 0$ "for all x" is B0. Both facts are needed: $e^{3x} > 0$ and $\cos 3x \leq 1$. A sketch of the curve is not a proof. Alternatively, $3e^{3x} > 0$ and $6 - 6 \cos 3x \geq 0$, so $\frac{dy}{dx} > 0$, contradicting $\frac{dy}{dx} = 0$.</i>	B1 M1 A1 dM1 A1* (5)
Total for Question 2: 5 marks		



Question 3

5 marks

Part	Scheme	Marks
(a)	$(x + y) + (x - y) = 2x$, which is even Two integers with an even sum are both even or both odd (an even and an odd number have an odd sum) * <i>Considering the four cases (both even, both odd, one of each) and showing $x + y$ and $x - y$ match in each is equally valid.</i>	M1 A1* (2)
(b)	Assume there are positive integers x and y with $x^2 - y^2 = 18$, so $(x - y)(x + y) = 18$ Uses (a): if both factors are odd, the product is odd; if both are even, the product is $2p \times 2q = 4pq$, a multiple of 4 18 is even but not a multiple of 4, so neither case is possible: a contradiction. So there are no such positive integers * <i>Alternative: the factor pairs of 18 are $1 \times 18, 2 \times 9, 3 \times 6$; each has one odd and one even factor, which contradicts (a).</i>	B1 M1 A1* (3)
Total for Question 3: 5 marks		

Question 4

5 marks

Part	Scheme	Marks
(a)	Assume there is a real number $x > 1$ for which $\frac{x^2}{x-1} < 4$ $x - 1 > 0$, so multiplying by $x - 1$ keeps the inequality: $x^2 < 4x - 4$ $x^2 - 4x + 4 < 0 \Rightarrow (x - 2)^2 < 0$ A square cannot be negative: a contradiction. So $\frac{x^2}{x-1} \geq 4$ for all $x > 1$ * <i>The reason $x - 1 > 0$ must be given: multiplying by a negative number would reverse the inequality.</i>	B1 M1 dM1 A1* (4)
(b)	A correct counter-example, e.g. $x = 0: \frac{0^2}{0-1} = 0 < 4$ <i>Any $x < 1$ works, since then $\frac{x^2}{x-1} \leq 0$. The value of the expression must be shown.</i>	B1 (1)
Total for Question 4: 5 marks		



Question 5

6 marks

Part	Scheme	Marks
(a)	Assume $\log_2 3$ is rational: $\log_2 3 = \frac{a}{b}$, where a and b are positive integers (positive since $3 > 1$) $2^{a/b} = 3$, and raising both sides to the power b: $2^a = 3^b$ 2^a is even (as $a \geq 1$) and 3^b is odd (a product of odd numbers) An even number cannot equal an odd number: a contradiction. So $\log_2 3$ is irrational *	B1 M1 M1 A1* (4)
(b)	$\log_8 9 = \frac{\log_2 9}{\log_2 8} = \frac{2 \log_2 3}{3}$ If $\log_8 9 = r$ were rational, then $\log_2 3 = \frac{3}{2}r$ would be rational, contradicting (a). So $\log_8 9$ is irrational *	M1 A1* (2)
Total for Question 5: 6 marks		

Question 6

10 marks

Part	Scheme	Marks
(a)	Every integer that is not a multiple of 3 is $3k+1$ or $3k+2$; cubes one of them $(3k+1)^3 = 27k^3 + 27k^2 + 9k + 1 = 3(9k^3 + 9k^2 + 3k) + 1$ $(3k+2)^3 = 27k^3 + 54k^2 + 36k + 8 = 3(9k^3 + 18k^2 + 12k + 2) + 2$: neither is a multiple of 3 *	M1 A1 A1* (3)
(b)	Assume $\sqrt[3]{3} = \frac{a}{b}$, where a and b are integers with no common factors $a^3 = 3b^3$, so a^3 is a multiple of 3 So a is a multiple of 3 (by (a): if a were not, a^3 would not be) $a = 3m$: $27m^3 = 3b^3 \Rightarrow b^3 = 9m^3 = 3(3m^3)$ b^3 is a multiple of 3, so b is too. Then 3 is a common factor of a and b: a contradiction, so $\sqrt[3]{3}$ is irrational * <i>"a^3 is a multiple of 3 so a is a multiple of 3" must be justified by part (a): this is the step most often missed (Oct 2021 report).</i>	B1 M1 A1 M1 A1* (5)
(c)	$a^3 = 8b^3$ and $a = 2m$ give $8m^3 = 8b^3$, so $b^3 = m^3$, i.e. $b = m$ Nothing forces b to be even, so there is no common factor and no contradiction (indeed $\sqrt[3]{8} = 2 = \frac{2}{1}$ is rational)	B1 B1 (2)
Total for Question 6: 10 marks		



Question 7

10 marks

Part	Scheme	Marks
(a)	$\frac{1}{\cos^2 x} + \frac{1}{\sin^2 x} = \frac{\sin^2 x + \cos^2 x}{\sin^2 x \cos^2 x}$ $= \frac{1}{\sin^2 x \cos^2 x}$ $\sin x \cos x = \frac{1}{2} \sin 2x, \text{ so } \frac{1}{\frac{1}{4} \sin^2 2x} = 4 \operatorname{cosec}^2 2x \quad *$	M1 A1 A1* (3)
(b)	Assume there is a value of x (not a multiple of $\frac{\pi}{2}$) for which $\sec^2 x + \operatorname{cosec}^2 x < 4$ Then $4 \operatorname{cosec}^2 2x < 4$, so $\operatorname{cosec}^2 2x < 1$, i.e. $\sin^2 2x > 1$ But $\sin^2 2x \leq 1$ for all x: a contradiction. So $\sec^2 x + \operatorname{cosec}^2 x \geq 4 \quad *$	B1 M1 A1* (3)
(c)	$4 \operatorname{cosec}^2 2x = \frac{16}{3} \Rightarrow \sin^2 2x = \frac{3}{4}$ $\sin 2x = \pm \frac{\sqrt{3}}{2}$ $0 < 2x < 2\pi: 2x = \frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}$ $x = \frac{\pi}{6}, \frac{\pi}{3}, \frac{2\pi}{3}, \frac{5\pi}{6}$ <i>Taking only $\sin 2x = +\frac{\sqrt{3}}{2}$ loses the last two solutions. Decimal answers score A0.</i>	M1 A1 M1 A1 (4)
Total for Question 7: 10 marks		



Question 8

8 marks

Part	Scheme	Marks
(a)	Each integer k with $2 \leq k \leq n$ is a factor of $n! = 1 \times 2 \times \dots \times n$ So $n! + 1$ leaves remainder 1 when divided by k : k is not a factor of $n! + 1$ *	M1 A1* (2)
(b)	Assume there are finitely many primes, and let p be the largest Consider $N = p! + 1$, which is greater than p By (a), none of $2, 3, \dots, p$ is a factor of N ; $N > 1$, so N is prime or has a prime factor q , and then $q > p$ Either way there is a prime greater than p : a contradiction. So there are infinitely many primes * <i>N itself need not be prime (see part (c)): the argument only needs a prime factor of N that is bigger than p.</i>	B1 M1 M1 A1* (4)
(c)	Tries values of n , e.g. $2! + 1 = 3$, $3! + 1 = 7$ (both prime), then $n = 4$ $4! + 1 = 25 = 5 \times 5$, which is not prime, so the student is wrong	M1 A1 (2)
Total for Question 8: 8 marks		



Question 9

9 marks

Part	Scheme	Marks
(a)	Assume the curves meet: $2x^2 = 6 - \frac{8}{x^2}$ for some real $x \neq 0$ Multiplies by x^2: $2x^4 - 6x^2 + 8 = 0 \Rightarrow x^4 - 3x^2 + 4 = 0$, a quadratic in x^2 Discriminant $(-3)^2 - 4 \times 1 \times 4 = -7 < 0$ (or $(x^2 - \frac{3}{2})^2 + \frac{7}{4} > 0$) No real x^2, so no real x: a contradiction. So C_1 and C_2 do not meet * <i>The assumption must be written (Oct 2024 report: many lost the first mark by going straight to the algebra). Values of x^2 from a calculator, or reading the figure, score no marks.</i>	B1 M1 dM1 A1* (4)
(b)	$2x^4 - kx^2 + 8 = 0$, i.e. $2u^2 - ku + 8 = 0$ with $u = x^2$; real roots need $k^2 - 64 \geq 0$ Product of roots $4 > 0$, so the roots have the same sign; $u = x^2 > 0$ needs their sum $\frac{k}{2} > 0$ $k \geq 8$ <i>$k \leq -8$ gives two negative values of x^2, so it must be rejected. At $k = 8$ the curves touch at $x = \pm\sqrt{2}$.</i>	M1 M1 A1 (3)
(c)	$2u^2 - 10u + 8 = 0 \Rightarrow u = 1$ or $u = 4$, so $x = \pm 1, \pm 2$ $(-2, 8), (-1, 2), (1, 2), (2, 8)$	M1 A1 (2)
Total for Question 9: 9 marks		



Question 10

10 marks

Part	Scheme	Marks
(a)	Let $a = \frac{m}{n}$ be rational and b irrational. Assume $a + b = \frac{p}{q}$ is rational (m, n, p, q integers, $n, q \neq 0$) $b = \frac{p}{q} - \frac{m}{n} = \frac{pn - mq}{qn}$ $pn - mq$ and qn are integers with $qn \neq 0$, so b is rational: a contradiction. So $a + b$ is irrational *	B1 M1 A1* (3)
(b)	Assume $\sqrt{2} + \sqrt{3} = r$, where r is rational Squares: $2 + 2\sqrt{6} + 3 = r^2$ $\sqrt{6} = \frac{r^2 - 5}{2}$ $\frac{r^2 - 5}{2}$ is rational, but $\sqrt{6}$ is irrational: a contradiction. So $\sqrt{2} + \sqrt{3}$ is irrational *	B1 M1 dM1 A1* (4)
(c)	e.g. $\sqrt{2} + (-\sqrt{2}) = 0$, or $(1 + \sqrt{2}) + (1 - \sqrt{2}) = 2$	B1 (1)
(d)	$(\sqrt{3} - \sqrt{2})(\sqrt{3} + \sqrt{2}) = 3 - 2 = 1$, so $\sqrt{3} - \sqrt{2} = \frac{1}{\sqrt{3} + \sqrt{2}}$ If $\sqrt{3} - \sqrt{2} = s$ were rational ($s \neq 0$), then $\sqrt{3} + \sqrt{2} = \frac{1}{s}$ would be rational, contradicting (b). So $\sqrt{3} - \sqrt{2}$ is irrational *	M1 A1* (2)
Total for Question 10: 10 marks		

