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INTERNATIONAL A LEVEL MATHEMATICS • PURE 2 (WMA12)

Pure 2 Practice Paper 3

Mark Scheme

AIMING FOR A/A*

Written in the style of the Pearson Edexcel International Advanced Level paper (WMA12/01) • 75 marks

Every question marked line by line the way an examiner marks it, with notes on the common errors

Mark every line, not just the answer. Method marks are given for a correct method even when the final answer is wrong. If you lost marks, find the *first* line where your working differs from the scheme. That is the thing to fix.

Code	Meaning
M1	Method mark: for knowing a correct method and attempting to apply it. Small slips in the working do not lose this mark.
dM1	Dependent method mark: can only be given if the method mark before it has been given.
A1	Accuracy mark: for a correct answer or line of working. It depends on the method mark before it.
B1	Independent mark: for a correct result or statement, whatever method was used.
*	The answer is given in the question. Every step must be shown, and the answer must follow from correct working (cso).
cao / cso	Correct answer only / correct solution only, with no errors anywhere in the working.
ft / oe	Follow through from your own earlier answer / or any correct equivalent form.

"Solutions relying on calculator technology are not acceptable." Every question in this paper says this. There, a correct final answer with no algebraic working scores **no marks**.

Rough grade guide (out of 75): A **45** • B **38** • C **31**

Based on recent WMA12 grade boundaries. Real boundaries change every series, so treat this as a guide, not a grade.

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What to revise next

Write in your score for each question. Where you dropped marks, tap the chapter to practise ten more exam questions on it.

Q	What it tested	Marks	Practise it	Score
1	Binomial coefficients in a product; a cubic in n by the factor theorem	8	Ch 4 The Binomial Expansion Ch 1 Algebraic Methods	
2	Sum to infinity and its condition; a cubic; trigonometric equations	9	Ch 5 Sequences and Series Ch 1 Algebraic Methods Ch 6 Trig Identities and Equations	
3	Definite integrals of negative powers forming a geometric series	10	Ch 8 Integration Ch 5 Sequences and Series Ch 3 Exponentials and Logarithms	
4	Circle properties; two circles; segment areas in radians	10	Ch 2 Coordinate Geometry	
5	Optimisation in a circle without the chain rule	10	Ch 7 Differentiation Ch 2 Coordinate Geometry Ch 1 Algebraic Methods	
6	Proof by exhaustion and disproof by counter-example	9	Ch 1 Algebraic Methods	
7	Laws of logarithms; a quadratic in $\sin^2 x$; domain of log	10	Ch 3 Exponentials and Logarithms Ch 6 Trig Identities and Equations	
8	The trapezium rule and exact integration used backwards	9	Ch 8 Integration	
	Total	**75**		

Where did the marks go? If you lost the first marks of a question, the method needs work: go back to that chapter's key skills page. If you lost the final marks, the method is fine and the gap is accuracy or linking the parts together. In this paper the lost marks are usually the trap: an extra root, a condition such as $|r| < 1$, or a region counted once instead of twice. Before you move on, check every answer against the conditions in the question.

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Question 1

8 marks

Part	Scheme	Marks
(a)	An x^3 term comes from $3\times$(the x^3 term of $(1+2x)^n$) and $x\times$(the x^2 term)	M1
	$3\binom{n}{3}2^3+\binom{n}{2}2^2=540$	A1
	Expands: $4n(n-1)(n-2)+2n(n-1)=540$, i.e. $2n(n-1)(2n-3)=540$	M1
	$2n^3-5n^2+3n-270=0$	A1
	Factor theorem: $n=6$ is a root, and divides: $(n-6)\left(2n^2+7n+45\right)=0$	M1
	$2n^2+7n+45$ has discriminant $49-360<0$, so no other real roots: $\boxed{n=6}$ only	A1
	Finding $n=6$ by trial earns the M marks for the cubic but not the final A1: the question asks you to show that 6 is the only value.	
		(6)
(b)	$3\binom{6}{4}2^4+\binom{6}{3}2^3=3(15)(16)+20(8)$	M1
	$\boxed{880}$	A1
		(2)
		Total for Question 1: 8 marks

Question 2

9 marks

Part	Scheme	Marks
(a)	$S_\infty=\dfrac{5}{1-r}$ and second + third terms $=5r+5r^2$, where $r=2\cos\theta$: $\dfrac{5}{1-r}=\dfrac{8}{3}\left(5r+5r^2\right)$	M1
	$3=8r(1+r)(1-r)$, so $8r^3-8r+3=0$	A1
	Factor theorem: $r=\frac{1}{2}$ is a root (as $1-4+3=0$), so $(2r-1)\left(4r^2+2r-3\right)=0$	M1
	Solves the quadratic: $r=\dfrac{-2\pm\sqrt{52}}{8}=\dfrac{-1\pm\sqrt{13}}{4}$	M1
	Three roots: $\frac{1}{2}$, $\frac{-1+\sqrt{13}}{4}=0.651\ldots$, $\frac{-1-\sqrt{13}}{4}=-1.151\ldots$	A1
	A sum to infinity needs $\lvert r\rvert<1$, so reject $-1.151\ldots$: $\boxed{r=\frac{1}{2}\text{ or }r=\frac{-1+\sqrt{13}}{4}}$	B1
	This is the trap: $r=-1.151\ldots$ would give $\cos\theta=-0.576$, a perfectly good angle. The only reason to reject it is that the series would have no sum to infinity.	
		(6)
(b)	$\cos\theta=\frac{r}{2}$: $\cos\theta=\frac{1}{4}$ or $\cos\theta=\frac{-1+\sqrt{13}}{8}=0.3257\ldots$	M1
	One pair correct, e.g. 75.5°, 284.5°	A1
	$\boxed{\theta=71.0^\circ,\ 75.5^\circ,\ 284.5^\circ,\ 289.0^\circ}$ and no others	A1
	Answers 125.2° and 234.8° come from the rejected ratio and lose the final A1.	
		(3)
		Total for Question 2: 9 marks

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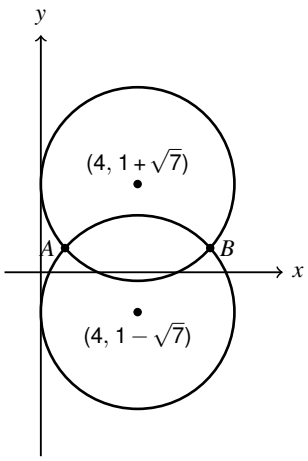
Question 3

10 marks

Part	Scheme	Marks
(a)	$\int x^{-\frac{3}{2}}\,\mathrm{d}x = -2x^{-\frac{1}{2}}$	M1
	Area of $R_n = \left[-2x^{-\frac{1}{2}}\right]_{k^{n-1}}^{k^n} = 2k^{-\frac{n-1}{2}} - 2k^{-\frac{n}{2}}$	M1
	$= 2k^{-\frac{n}{2}}\left(k^{\frac{1}{2}} - 1\right)$ (oe)	A1
	Considers $\dfrac{\text{Area } R_{n+1}}{\text{Area } R_n}$	M1
	$= \dfrac{2k^{-\frac{n+1}{2}}\left(k^{\frac{1}{2}} - 1\right)}{2k^{-\frac{n}{2}}\left(k^{\frac{1}{2}} - 1\right)} = k^{-\frac{1}{2}}$, which does not depend on n	A1
	So the areas form a geometric sequence with common ratio $\boxed{\dfrac{1}{\sqrt{k}}}$	A1
	Showing that the first three areas have a common ratio is not enough: the ratio must be shown to be constant for every n.	(6)
(b)	First term $2\left(1 - k^{-\frac{1}{2}}\right)$, and $0 < k^{-\frac{1}{2}} < 1$ because $k > 1$, so S_∞ exists: $S_\infty = \dfrac{2\left(1 - k^{-\frac{1}{2}}\right)}{1 - k^{-\frac{1}{2}}}$	M1
	$= 2$, which does not depend on k	A1
		(2)
(c)	$k = 4$: first term 1, ratio $\frac{1}{2}$, so $2\left(1 - \left(\frac{1}{2}\right)^n\right) > 1.99$, giving $\left(\frac{1}{2}\right)^n < 0.005$, i.e. $n > \dfrac{\log 0.005}{\log 0.5} = 7.64\ldots$	M1
	$\boxed{n = 8}$	A1
	Dividing by $\log 0.5$*, which is negative, reverses the inequality.* $S_7 = 1.984$ *and* $S_8 = 1.992$.	(2)
		Total for Question 3: 10 marks

Question 4

10 marks

Part	Scheme	Marks
(a)	The centre is on the perpendicular bisector of the chord AB, the line $x = 4$, so the centre is $(4, c)$	M1
	Distance to A is 4: $(4-1)^2 + (c-1)^2 = 16$, so $(c-1)^2 = 7$	M1
	$c = 1 \pm \sqrt{7}$: two values, so exactly two circles	A1
	$\boxed{(x-4)^2 + \left(y-1-\sqrt{7}\right)^2 = 16}$ and $\boxed{(x-4)^2 + \left(y-1+\sqrt{7}\right)^2 = 16}$	A1
		(4)
(b)		B1
	Two circles of the same size, one above and one below AB, both passing through A and B	
	Centres $\left(4, 1+\sqrt{7}\right)$ and $\left(4, 1-\sqrt{7}\right)$ shown, with each circle touching the y-axis	B1
		(2)
(c)	Angle subtended by AB at a centre: $\sin\frac{\alpha}{2} = \frac{3}{4}$, so $\alpha = 2\sin^{-1}\frac{3}{4} = 1.696\ldots$ radians	M1
	Segment area $= \frac{1}{2}r^2(\alpha - \sin\alpha) = 8(1.696\ldots - 0.992\ldots) = 5.63\ldots$	M1
	The region inside both circles is two such segments, one on each side of AB	M1
	$\boxed{11.3}$ $(11.26\ldots)$	A1
	The common error is to stop at one segment, 5.63. Using degrees in $\frac{1}{2}r^2(\alpha - \sin\alpha)$ scores M0 for the segment.	
		(4)
		Total for Question 4: 10 marks

Question 5

10 marks

Part	Scheme	Marks
(a)	P and Q: $x^2+h^2=9$, so $x=\pm\sqrt{9-h^2}$ and $PQ=2\sqrt{9-h^2}$	**M1**
	Height from $A(0,3)$ to the line $y=-h$ is $3+h$	**M1**
	Area $\boxed{A=(3+h)\sqrt{9-h^2}}$	**A1**
		(3)
(b)	Maximises A^2 instead (the same h gives the maximum, as $A>0$): $A^2=(3+h)^2\left(9-h^2\right)$	**M1**
	$A^2=81+54h-6h^3-h^4$	**A1**
	$\dfrac{\mathrm{d}\left(A^2\right)}{\mathrm{d}h}=54-18h^2-4h^3$	**M1**
	Sets to 0 and factorises: $-2\left(2h^3+9h^2-27\right)=-2(2h-3)(h+3)^2=0$	**M1**
	$h=\frac{3}{2}$ ($h=-3$ is outside $0<h<3$)	**A1**
	$\dfrac{\mathrm{d}^2\left(A^2\right)}{\mathrm{d}h^2}=-36h-12h^2=-81<0$ at $h=\frac{3}{2}$, so a maximum	**M1**
	$A=\frac{9}{2}\sqrt{9-\frac{9}{4}}=\frac{9}{2}\times\frac{3\sqrt{3}}{2}=\boxed{\frac{27\sqrt{3}}{4}}$	**A1**
	WMA12 has no chain rule, so A itself cannot be differentiated: squaring first is the key idea. A correct method using the chain rule (from WMA13) may be marked the same way. The maximum triangle is equilateral.	
		(7)
		Total for Question 5: 10 marks

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Question 6

9 marks

Part	Scheme	Marks
(a)	$n^5 - n = n\left(n^4 - 1\right) = n\left(n^2 - 1\right)\left(n^2 + 1\right)$	M1
	$\boxed{n(n-1)(n+1)\left(n^2+1\right)}$	A1
		(2)
(b)	$n-1$, n, $n+1$ are three consecutive integers, so one is even and one is a multiple of 3: $n^5 - n$ is divisible by 2 and by 3	B1
	For 5, considers every integer as $5k$, $5k \pm 1$ or $5k \pm 2$	M1
	$n = 5k$: the factor n is a multiple of 5; $n = 5k \pm 1$: the factor $n \mp 1$ is a multiple of 5	A1
	$n = 5k \pm 2$: $n^2 + 1 = 25k^2 \pm 20k + 4 + 1$	M1
	$= 5\left(5k^2 \pm 4k + 1\right)$, a multiple of 5	A1
	So $n^5 - n$ is divisible by 2, 3 and 5, which have no common factor, so it is divisible by 30 for every integer n	A1
	Testing a few values is not a proof. The $5k \pm 2$ case is where most attempts fail: neither $n-1$, n nor $n+1$ helps there.	
		(6)
(c)	e.g. $n = 2$: $2^5 - 2 = 30$, which is not divisible by 60	B1
		(1)
		Total for Question 6: 9 marks

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Question 7

10 marks

Part	Scheme	Marks
(a)	$\log_2(\sin x \cos x) = -2$	**M1**
	$\sin x \cos x = \frac{1}{4}$	**A1**
	Squares and uses $\cos^2 x = 1 - \sin^2 x$: $\sin^2 x\left(1 - \sin^2 x\right) = \frac{1}{16}$	**M1**
	$16\sin^4 x - 16\sin^2 x + 1 = 0$	**A1**
	Solves as a quadratic in $\sin^2 x$: $\sin^2 x = \dfrac{2 \pm \sqrt{3}}{4}$	**M1**
	Logs need $\sin x > 0$ and $\cos x > 0$, so x is in the first quadrant only; this also removes the extra roots made by squaring	**B1**
	$\sin x = 0.2588\ldots$ or $0.9659\ldots$: $\boxed{x = 0.262,\ 1.31}$	**A1**
	Squaring creates solutions with $\sin x \cos x = -\frac{1}{4}$ (second and fourth quadrants), and the logs rule out the third quadrant. Any answer outside $0 < x < \frac{\pi}{2}$ loses the final A1.	**(7)**
(b)	$\sin x \cos x = 2^k$, so $\sin^2 x\left(1 - \sin^2 x\right) = 4^k$	**M1**
	$t(1-t)$ with $t = \sin^2 x$ has greatest value $\frac{1}{4}$ (completing the square, or at $t = \frac{1}{2}$)	**M1**
	$4^k \leqslant \frac{1}{4}$, so the greatest value is $\boxed{k = -1}$	**A1**
		(3)
		Total for Question 7: 10 marks

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Question 8

9 marks

Part	Scheme	Marks
(a)	$\int_1^4 (ax^2+b)\,\mathrm{d}x = \left[\frac{a}{3}x^3+bx\right]_1^4$	M1
	$21a+3b=45$	A1
	Three strips, $h=1$: $\frac{1}{2}[(a+b)+(16a+b)+2(4a+b)+2(9a+b)]$	M1
	$\frac{1}{2}(43a+6b)=46$, i.e. $43a+6b=92$	A1
	Solves: $\boxed{a=2,\ b=1}$	A1
	Using $h=\frac{3}{4}$ (four ordinates, not three strips) is the common error.	(5)
(b)	$h=0.5$; y values $3,\ 5.5,\ 9,\ 13.5,\ 19,\ 25.5,\ 33$	B1
	$\frac{1}{2}\times 0.5\times[3+33+2(5.5+9+13.5+19+25.5)]$	M1
	$=45.25$; error $0.25=\frac{1}{4}\times 1$ *	A1*
		(3)
(c)	Error $\frac{1}{4}\times 0.25=0.0625$, so the estimate is $\boxed{45.0625}$	B1
	For any quadratic, the trapezium-rule error is exactly proportional to h^2, which is why doubling the strips divides the error by 4.	(1)
		Total for Question 8: 9 marks

TOTAL FOR PAPER: 75 MARKS

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