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A-LEVEL MATHEMATICS • STATISTICS 2

Continuous Random Variables

Worked Solutions

Pearson Edexcel International A Level (WST02) • Calculator and statistical tables allowed: show every step

Full worked solutions for the free Continuous Random Variables worksheet, A-Level and International A-Level Maths

A **Examples** & Your turn page 2

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Tap a section to jump straight to it

Full worked solutions for every question on the Statistics 2 Continuous Random Variables worksheet, in the same order and with the same question numbers.

How to mark your work. Tick each step you got right, not just the final answer. At A-level most marks are for method, so a wrong final answer can still earn most of the marks. If you lost marks, find the *first* line where your working is different from ours. That is the thing to fix.

In Section E: **M** = method mark, **dM** = method mark that depends on the previous M mark, **A** = accuracy mark (needs the M mark before it), **B** = independent mark.

Question sheet | Exam questions by topic | Statistics 2 exam questions | More worksheets

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A Your turn answers

Compare each line of your working with the solution.

CORE

A1 $k\left[\frac{1}{3}x^3+x^2\right]_0^3 = 18k = 1$: $\boxed{k=\frac{1}{18}}$; $\frac{1}{18}\left[(9+9)-\left(\frac{8}{3}+4\right)\right] = \frac{17}{27}$: $\boxed{0.630}$

A2 $F(x) = \frac{x^2+x}{12}$ on $[0,3]$ (0 below, 1 above); $m^2+m-6=0$, $m=2$ (not -3): $\boxed{m=2}$

A3 $E(X) = \frac{2}{9}\left[\frac{3}{2}x^2-\frac{1}{3}x^3\right]_0^3 = 1$; $E(X^2) = \frac{2}{9}\left[x^3-\frac{1}{4}x^4\right]_0^3 = \frac{3}{2}$: $\boxed{1,\ \frac{1}{2}}$

STRETCH

A4 $f(x) = \frac{3x^2+1}{10}$; $F(1.5)-F(1) = 0.4875-0.2$: $\boxed{0.2875}$

A5 $\frac{1}{64}(16-3x^2)=0$, $x^2=\frac{16}{3}$: $\boxed{\frac{4}{\sqrt{3}} = 2.31}$

A6 $F(1) = 0.2 < 0.5$, so the second piece: $1-\frac{(5-m)^2}{20} = 0.5$, $(5-m)^2 = 10$: $\boxed{m = 5-\sqrt{10} \approx 1.84}$

A7 $E(X) = \frac{3}{2}$, $E(X^2) = \frac{12}{5}$; $4\left(\frac{3}{2}\right)-\frac{12}{5}$: $\boxed{3.6}$

B Fluency solutions

One mark for each correct part. The notes in italics explain why each sequence was chosen.

CORE

B1 **(a)** $4k=1$: $\boxed{\frac{1}{4}}$ **(b)** $12k=1$: $\boxed{\frac{1}{12}}$ **(c)** $6k=1$: $\boxed{\frac{1}{6}}$ **(d)** $\frac{2}{15}\left(a+\frac{a^2}{2}\right)=1$, $a^2+2a-15=0$: $\boxed{a=3}$

The constant always comes from a total area of 1.

B2 **(a)** $\frac{8}{125}$: **0.064** **(b)** $1-\frac{64}{125}$: **0.488** **(c)** $\frac{56}{125}$: **0.448** **(d)** **0**

Probabilities are areas; a single value has no area.

B3 **(a)** $\frac{x}{4}$ **(b)** $\frac{1}{2}+\int_2^x \frac{4-t}{4}\,\mathrm{d}x$: $-\frac{x^2}{8}+x-1$

(c) **0 for $x<0$, 1 for $x>4$** **(d)** $\mathrm{F}(2)=\frac{1}{2}$: **median 2**

Each new piece starts from the value of F *at the join.*

B4 **(a)** $\frac{3}{16}\int_0^2 (4x-x^3)\,\mathrm{d}x$: $\frac{3}{4}$ **(b)** $\frac{3}{16}\int_0^2 (4x^2-x^4)\,\mathrm{d}x$: $\frac{4}{5}$ **(c)** $\frac{4}{5}-\frac{9}{16}$: $\frac{19}{80}$ **(d)** $16\times\frac{19}{80}$: **3.8**

$\mathrm{Var}(aX+b)=a^2\,\mathrm{Var}(X)$

B5 **(a)** $\frac{x^2}{9}$ **(b)** $\frac{8-1}{27}$: **0.259** **(c)** $m^3=13.5$: **2.38**

Differentiate F for f; use F itself for probabilities and the median.

B6 **(a)** $\sqrt{12.5}$: **3.54** **(b)** $\sqrt{6.25}$: **2.5** **(c)** $\sqrt{18.75}$: **4.33** **(d)** $\sqrt{22.5}$: **4.74**

Every percentile solves $\mathrm{F}(x)=$ *the required proportion.*

STRETCH

B7 **(a)** $3-2x=0$: **1.5** **(b)** $6x-3x^2=0$: **2** **(c)** $(3-x)(3-3x)=0$: **1** **(d)** increasing, so the end point: **3**

The mode is where f is greatest: a turning point or an end point.

B8 **(a)** Area: $2a+2b=1$, so $a+b=\frac{1}{2}$, as required **(b)** $1.2a+0.72b=0.5$: $a=\frac{7}{24}, b=\frac{5}{24}$

(c) $\int_0^2 \frac{7x+5x^2}{24}\,\mathrm{d}x$: $\frac{41}{36}\approx 1.14$ **(d)** f is increasing: **mode 2**

Two unknowns need two conditions: the area and the median.

B9 **(a)** $(4-x)(4-3x)=0$: $\frac{4}{3}$ **(b)** **1.6** **(c)** $\mathrm{F}(1.535)<0.5<\mathrm{F}(1.545)$, so $m=1.54$

(d) mode $<$ median $<$ mean: **positive skew**

Compare all three averages before deciding.

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B10 **(a)** 2 **(b)** $\frac{9}{2} - 4$: $\frac{1}{2}$ **(c)** $\int_0^3 \frac{2x^4}{9}\,dx$: **10.8** **(d)** $\int_0^3 \frac{2}{9}\,dx$: $\frac{2}{3}$

$E(g(X)) = \int g(x)f(x)\,dx$ *for any function g*

C Reasoning solutions

Your wording can be different. Check that your reason makes the same point.

CORE

C1 For a continuous variable f(2) is a density, not a probability; a single value has zero area: $P(X = 2) = 0$

C2 F must be given for all values of x: $F(x) = 0$ **for** $x < 0$, $\frac{x^2}{9}$ **for** $0 \leqslant x \leqslant 3$, **1 for** $x > 3$

C3 f is the derivative of F: $f(x) = \frac{2x+2}{15}$

C4 $E(X^2) \neq [E(X)]^2$: $E(X^2) = \int_0^2 \frac{3x^4}{8}\,dx = 2.4$; $2.4 - 2.25$: **0.15**

STRETCH

C5 $f(x) = 0$ finds where the density is zero, the least likely values. Solve $f'(x) = \frac{3}{32}(4 - 2x) = 0$: **mode 2**

C6 She has not added $F(2) = \frac{1}{2}$: $\frac{1}{2} + \int_2^x \frac{1}{2}\,dx$: $\frac{x}{2} - \frac{1}{2}$, which gives $F(3) = 1$

C7 $F(3) = \frac{3}{4} > 0.5$, so the median is in the first piece; her 2.59 is not in $[3, 4]$

$\frac{m^2}{12} = 0.5$: $m = \sqrt{6} = 2.45$

C8 **(a)** $\frac{k}{n+1} = 1$, so $k = n + 1$; $E(X) = \int_0^1 (n+1)x^{n+1}\,dx = \frac{n+1}{n+2}$, as required

(b) $E(X^2) = \frac{n+1}{n+3}$; $\frac{n+1}{n+3} - \frac{(n+1)^2}{(n+2)^2} = \frac{(n+1)\left[(n+2)^2 - (n+1)(n+3)\right]}{(n+2)^2(n+3)} = \frac{n+1}{(n+2)^2(n+3)}$, as required

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D Problem solving solutions

There is often more than one correct method. Any complete, correct method scores full marks.

CORE

D1 $F(1) = \frac{2}{7}$, $F(1.5) = \frac{9}{16}$; $\frac{\frac{9}{16} - \frac{2}{7}}{1 - \frac{2}{7}} = \frac{31/112}{5/7}$: $\boxed{\frac{31}{80} = 0.3875}$

D2 **(a)** $\frac{c}{2} + \frac{c}{3}\left[4x - \frac{1}{2}x^2\right]_1^4 = \frac{c}{2} + \frac{3c}{2} = 2c = 1$: $\boxed{c = \frac{1}{2}}$

(b) $\int_0^1 \frac{x^2}{2}\,dx + \int_1^4 \frac{4x - x^2}{6}\,dx = \frac{1}{6} + \frac{1}{6}\left[2x^2 - \frac{1}{3}x^3\right]_1^4 = \frac{1}{6} + \frac{3}{2}$: $\boxed{\frac{5}{3}}$

D3 **(a)** $\boxed{F(x) = 0 \text{ for } x < 0, \frac{x^2 + 4x}{12} \text{ for } 0 \leqslant x \leqslant 2, 1 \text{ for } x > 2}$

(b) $m^2 + 4m - 6 = 0$, $m = -2 + \sqrt{10}$: $\boxed{1.16}$

D4 $E(X) = \frac{12}{5}$, $E(X^2) = 6$; $5(6) - 2\left(\frac{12}{5}\right) + 1$: $\boxed{26.2}$

STRETCH

D5 **(a)** Area: $3a + 9b = 1$; $E(X) = \int_0^3 (ax + bx^3)\,dx = \frac{9}{2}a + \frac{81}{4}b = 2$

$a = \frac{1 - 9b}{3}$: $\frac{3}{2} - \frac{27}{2}b + \frac{81}{4}b = 2$, $b = \frac{2}{27}$, $a = \frac{1}{9}$, as required

(b) $1 - \left[\frac{x}{9} + \frac{2x^3}{81}\right]_0^2 = 1 - \frac{34}{81}$: $\boxed{\frac{47}{81} \approx 0.580}$

D6 **(a)** $\frac{8m^2 - m^4}{16} = \frac{1}{2}$, so $m^4 - 8m^2 + 8 = 0$, as required

(b) $m^2 = 4 \pm 2\sqrt{2}$; $m = \sqrt{4 + 2\sqrt{2}} = 2.61$ is outside $[0, 2]$: $\boxed{m = \sqrt{4 - 2\sqrt{2}} = 1.08}$

D7 **(a)** $F(2.7) = 0.718 < 0.75 < F(2.8) = 0.803$, so $\boxed{\text{the upper quartile lies between 2.7 and 2.8}}$

(b) $f(x) = \frac{3x^2 - 6x + 4}{12}$; $f'(x) = 0$ at $x = 1$ is a minimum of f; f is greatest at the end: $f(3) = \frac{13}{12} > f(0) = \frac{1}{3}$:

$\boxed{\text{mode } 3}$

D8 **(a)** $1 - \left[\frac{(x-1)^3}{8}\right]_1^{2.5} = 1 - \frac{27}{64} = \frac{37}{64}$, as required

(b) $Y \sim \text{B}\left(8, \frac{37}{64}\right)$: $\text{P}(Y = 6) + \text{P}(Y = 7) + \text{P}(Y = 8)$: **0.271**

E Exam practice solutions

Mark-scheme notes are shown on the right of each line.

CORE *Exam questions for everyone*

E1 **(a)** Area $= \frac{1}{2}(4)(3k) = 6k = 1$ *M1*

$k = \frac{1}{6}$ *A1**

(b) $\int_0^2 \frac{x}{6}\,\text{d}x = \left[\frac{x^2}{12}\right]_0^2$ *M1*

$\frac{1}{3}$ *A1*

(c) $\int_0^3 \frac{x^2}{6}\,\text{d}x + \int_3^4 \frac{x(4-x)}{2}\,\text{d}x$ *M1*

$= \frac{3}{2} + \frac{1}{2}\left[2x^2 - \frac{1}{3}x^3\right]_3^4 = \frac{3}{2} + \frac{5}{6}$ *M1*

$\frac{7}{3}$ *A1*

(d) $\text{F}(3) = \frac{3}{4} > 0.5$, so $\frac{m^2}{12} = 0.5$ *M1*

$m = \sqrt{6} = 2.45$ *A1*

(e) **Mode 3** *B1*

mean $<$ median $<$ mode, so **negative skew** *B1*

E2 **(a)** $k\left[16x-\frac{1}{3}x^3\right]_0^4 = k\left(64-\frac{64}{3}\right) = \frac{128}{3}k = 1$ *M1*

$\boxed{k=\frac{3}{128}}$ *A1**

(b) $\frac{3}{128}\int_0^4 (16x-x^3)\,\mathrm{d}x$ *M1*

$= \frac{3}{128}\left[8x^2-\frac{1}{4}x^4\right]_0^4 = \frac{3}{128}(128-64)$ *M1*

$\boxed{\frac{3}{2}}$ *A1*

(c) $\mathrm{E}(X^2) = \frac{3}{128}\left[\frac{16}{3}x^3-\frac{1}{5}x^5\right]_0^4 = \frac{16}{5}$ *M1*

$\frac{16}{5}-\frac{9}{4}$: $\boxed{\frac{19}{20}}$ *A1*

(d) $\mathrm{F}(m) = \frac{3}{128}\left(16m-\frac{1}{3}m^3\right) = \frac{48m-m^3}{128} = \frac{1}{2}$ *M1*

$m^3-48m+64=0$, as required *A1**

(e) $\mathrm{g}(1.3)=3.797>0$, $\mathrm{g}(1.4)=-0.456<0$; sign change, so **the median lies between 1.3 and 1.4** *M1 A1*

STRETCH

E3 **(a)** $\mathrm{F}(2)=1$: $4a+8b=1$ *M1*

$\mathrm{f}(x)=2ax+3bx^2$; mode: $\mathrm{f}'(1.2)=2a+7.2b=0$ *M1 A1*

$a=-3.6b$, $-6.4b=1$: $b=-\frac{5}{32}$, $a=\frac{9}{16}$, as required *A1**

(b) $1-\mathrm{F}(1.5) = 1-\frac{189}{256}$ *M1*

$\boxed{\frac{67}{256}\approx 0.262}$ *A1*

(c) $\mathrm{F}(0.7)=0.222$ and $\mathrm{F}(0.8)=0.28$ *M1 A1*

$0.222<0.25<0.28$, so **the lower quartile lies between 0.7 and 0.8** *A1*

(d) $Y\sim \mathrm{B}(10, 0.2617)$ *B1*

$\binom{10}{3}(0.2617)^3(0.7383)^7$ *M1*

$\boxed{\textbf{0.257}}$ *A1*

(e) $\mathrm{E}(X)=\frac{9}{8}$, $\mathrm{E}(X^2)=\frac{3}{2}$ *M1*

$\frac{3}{2}-\frac{81}{64}$: $\boxed{\frac{15}{64}\approx 0.234}$ *A1*