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A-LEVEL MATHEMATICS • MECHANICS 2

Statics of Rigid Bodies

Worksheet

Pearson Edexcel International A Level (WME02) • Calculator allowed: take $g = 9.8\,\text{m}\,\text{s}^{-2}$ and give answers to 2 or 3 s.f.

Free Statics of Rigid Bodies worksheet for A-Level and International A-Level Maths, with full worked solutions

A **Examples** & Your turn page 2

B **Fluency** core skills page 4

C **Reasoning** explain it page 6

D **Problem** solving page 7

E **Exam** practice page 10

Tap a section to jump straight to it

By the end of this worksheet you can: take moments of forces at an angle • solve rods held by strings, pegs and hinges • find the magnitude and direction of a hinge force • solve ladder problems with friction • *Stretch:* rough walls, struts and rails, and ranges of positions, forces and μ

Which questions are for me?

CORE The methods every A-level student must secure. Start here.

STRETCH Hinge forces in size and direction, struts, rails, rough walls, and ranges of positions and forces.

Every section has both, clearly labelled. Questions get harder as you go, and every skill in Sections A to D is needed in Section E.

Track your progress

Section	B	C	D	E
Core marks	23	10	13	20
Stretch marks	16	14	17	13
My score				

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Name ..... Class ..... Date .....

A Worked examples & Your turn

Read each example carefully. Cover it up, then answer the question beside it using the same method.

CORE *Secure the method*

Worked example A1: a force at an angle

A uniform rod AB, 4 m long and of weight 60 N, is hinged at A and held horizontal by a string at B inclined at 30° to the rod. Find the tension.

Moments about A: perpendicular distance $4\sin 30°$

$T\sin 30° \times 4 = 60 \times 2$

$T = 60\,\text{N}$

Your turn A1

A uniform rod AB, 6 m long and of weight 80 N, is hinged at A and held horizontal by a string attached at C, where $AC = 4$ m, inclined at 40° to the rod. Find the tension.

Worked example A2: the hinge force

For the rod in Worked example A1, find the magnitude of the force at the hinge and the angle it makes with the rod.

Resolve: $X = T\cos 30°$, $Y = 60 - T\sin 30°$

$X = 51.96$, $Y = 30$; $\sqrt{X^2+Y^2}$; $\tan^{-1}\dfrac{Y}{X}$

60 N at 30° to the rod

Your turn A2

For the rod in Your turn A1, find the magnitude of the force at the hinge and the angle it makes with the rod.

Worked example A3: a ladder

A uniform ladder of weight 200 N rests on rough ground against a smooth wall, at θ to the ground with $\tan\theta = \frac{4}{3}$. Find the reaction at the wall and the least possible μ

Moments about the foot; the wall reaction is horizontal

$S(2l\sin\theta) = 200(l\cos\theta)$, $S = \dfrac{200}{2\tan\theta}$; $\mu \geqslant \dfrac{S}{200}$

75 N, $\mu \geqslant 0.375$

Your turn A3

A uniform ladder of weight 240 N rests on rough ground against a smooth wall, at θ to the ground with $\tan\theta = \frac{12}{5}$. Find the reaction at the wall and the least possible μ

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STRETCH

Worked example A4: a smooth peg

A uniform 6 m rod AB of weight W has A on rough ground and rests on a smooth peg at C, $AC = 5$ m, at 30° to the ground. Find the least μ

The peg reaction N is perpendicular to the rod

$5N = 3W\cos 30^\circ$; $F = N\sin 30^\circ$, $R = W - N\cos 30^\circ$

$\mu \geqslant 0.472$

Your turn A4

A uniform 4 m rod AB of weight W has A on rough ground and rests on a smooth peg at C, $AC = 3$ m, at θ to the ground, $\tan\theta = \frac{3}{4}$. Find the least μ

Worked example A5: a hinge and a wall

A uniform rod AB, 2 m long and of weight 30 N, with a 20 N load at B, is hinged to a wall at A and held horizontal by a string from B to a point 1.5 m above A. Find T and the hinge force.

$T(0.6)(2) = 30(1) + 20(2)$; $X = 0.8T$, $Y = 50 - 0.6T$

58.3 N, 49.0 N

Your turn A5

A uniform rod AB, 3 m long and of weight 40 N, with a 10 N load at B, is hinged to a wall at A and held horizontal by a string from B to a point 4 m above A. Find T and the hinge force.

Worked example A6: the limiting angle

A uniform ladder rests against a smooth wall with its foot on rough ground, $\mu = 0.4$. Find the least angle between the ladder and the ground.

$S = \mu W = \dfrac{W}{2\tan\theta}$, so $\tan\theta = \dfrac{1}{2\mu} = 1.25$

51.3°

Your turn A6

A uniform ladder rests against a smooth wall with its foot on rough ground, $\mu = 0.3$. Find the least angle between the ladder and the ground.

Worked example A7: a non-uniform rod

A non-uniform rod AB, 4 m long and of weight 50 N, is hinged at A and held horizontal by a string at B at 60° to the rod with tension 40 N. How far from A is its centre of mass?

$40\sin 60^\circ \times 4 = 50x$

2.77 m

Your turn A7

A non-uniform rod AB, 5 m long and of weight 60 N, is hinged at A and held horizontal by a string at B at 45° to the rod with tension 50 N. How far from A is its centre of mass?

B Fluency

Each question is a designed sequence. From one part to the next, ask yourself: what changed, and what did it do to the answer?

CORE *Secure the method*

B1 A force of 50 N acts at the end B of a rod AB of length 2 m. Find its moment about A when it acts

(a) perpendicular to the rod

(b) at 30° to the rod

(c) at 60° to the rod

(d) along the rod

[4 marks]

B2 A uniform ladder of weight 120 N rests on rough ground against a smooth wall. Find the reaction at the wall when the ladder makes an angle θ with the ground, where

(a) $\theta = 60°$

(b) $\theta = 45°$

(c) $\tan\theta = 3$

(d) Find the least μ for part (b).

[4 marks]

B3 A uniform rod AB, 2 m long and of weight 40 N, is hinged to a wall at A and held horizontal by a string at B to a point on the wall above A. The string makes an angle θ with the rod.

(a) $\theta = 30°$: find T

(b) $\theta = 60°$: find T

(c) For $\theta = 30°$, find the components of the hinge force.

(d) For $\theta = 30°$, find its magnitude.

[4 marks]

B4 A uniform rod AB of length 8 m and weight W has A on rough ground and rests on a smooth peg at C, $AC = 6$ m, at 45° to the ground.

(a) Find the reaction at the peg.

(b) Find the friction force at A

(c) Find the normal reaction at A

(d) Find the least possible μ

[4 marks]

B5 A non-uniform rod AB, 6 m long and of weight 90 N, is hinged at A and held horizontal by a string at B at 30° to the rod with tension 60 N.

(a) Find the moment of the tension about A

(b) Find the distance of the centre of mass from A

(c) Find the vertical component of the hinge force.

[3 marks]

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B6 A uniform ladder rests on rough ground against a smooth wall and is about to slip. Find the angle between the ladder and the ground when

(a) $\mu = 0.5$

(b) $\mu = 0.25$

(c) $\mu = \frac{1}{\sqrt{3}}$

(d) Describe what happens to $\tan\theta$ when μ is halved.

[4 marks]

STRETCH

B7 A uniform ladder of weight W rests with its foot on rough ground ($\mu = 0.5$) and its top against a rough wall ($\mu' = 0.2$). It is about to slip at both ends.

(a) Find the normal reaction at the ground.

(b) Find the normal reaction at the wall.

(c) Find $\tan\theta$, where θ is the angle with the ground.

(d) Find θ

[4 marks]

B8 A uniform rod AB, 4 m long and of weight 60 N, is hinged to a wall at A with a 30 N particle at B. It is held horizontal by a string from B at 30° to the rod, attached to the wall above A

(a) Find T

(b) Find the components of the hinge force.

(c) Find its magnitude.

(d) Find the angle it makes with the rod.

[4 marks]

B9 A uniform rod AB, 2 m long and of weight 20 N, has A against a rough vertical wall ($\mu = 0.5$) and is held horizontal by a string from B to the wall above A, at 30° to the rod. A particle of weight 10 N is at a distance x m from A

(a) Find T in terms of x

(b) Find the normal reaction at the wall in terms of x

(c) Find the friction needed in terms of x

(d) Find the range of x for equilibrium.

[4 marks]

B10 A uniform ladder of weight W rests against a smooth wall with its foot on rough ground ($\mu = 0.1$), at θ to the ground with $\tan\theta = 3$

(a) Find the reaction at the wall.

(b) Find the friction needed for equilibrium.

(c) Show that the ladder cannot rest on its own.

(d) Find the least horizontal force towards the wall at the foot that holds it.

[4 marks]

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C Reasoning

Show your working and explain your thinking. A correct answer without a reason does not score full marks.

CORE *Secure the method*

C1 A force of 30 N acts at the end of a 3 m rod at 40° to the rod. Amir gives its moment about the other end as $30 \times 3\cos 40° = 68.9$ N m. Explain his error, and find the moment. **[2 marks]**

C2 A ladder rests against a smooth vertical wall. Bea draws the reaction at the top of the ladder perpendicular to the ladder. Explain her error. **[2 marks]**

C3 A uniform ladder of weight 150 N rests against a smooth wall on rough ground, with $\tan\theta = 2$. Cal draws the friction at the foot acting away from the wall.

Explain his error, and find the friction force. **[3 marks]**

C4 A uniform ladder at 75° to the ground is about to slip, against a smooth wall. Dev rounds $\tan 75°$ to 3.7 and gives $\mu = 0.135$

Explain why this loses the mark, and find μ **[3 marks]**

STRETCH

C5 A uniform rod AB, 3 m long and of weight 60 N, is hinged at A and held horizontal by a string at B at 30° to the rod. Eve finds the horizontal component of the hinge force, 52.0 N, and gives this as the force at the hinge.

Explain her error, and find the magnitude of the hinge force. **[3 marks]**

C6 A rod is hinged at A and inclined at 25° above the horizontal. The hinge force acts at 65° above the horizontal. Asked for the angle between the hinge force and the rod, Fay writes 65°

Explain her error, and give the angle. **[3 marks]**

C7 A uniform ladder of length 4 m and weight W rests at 60° to the ground against a smooth wall. Gus takes moments about the foot as $S \times 4 = W \times 2$ and finds $S = 0.5W$

Explain his error, and find S **[3 marks]**

C8 A uniform ladder of length $2a$ and weight W rests at an angle θ to the ground, against a smooth vertical wall, with its foot on rough horizontal ground. The coefficient of friction is μ

(a) Show that the ladder is on the point of slipping when $\tan\theta = \dfrac{1}{2\mu}$

(3)

(b) A person of weight W stands at the top of the ladder. Show that it is now on the point of slipping when $\tan\theta = \dfrac{3}{4\mu}$

(2)

[5 marks]

D Problem solving

You decide the method. Read the whole question first, and show every step.

CORE *Secure the method*

D1 A uniform rod AB of length $6a$ and weight W has its end A freely hinged to a fixed point on a vertical wall. A particle of weight $\frac{1}{2}W$ is attached to the rod at B. The rod is held in equilibrium in a horizontal position by a light inextensible string CD, where C is on the rod with $AC = 4a$ and D is on the wall vertically above A with $AD = 3a$, as shown in Figure 1.

Show that the tension in the string is $\frac{5}{2}W$

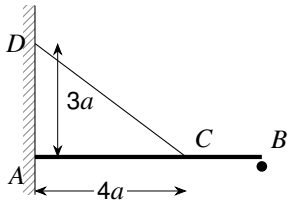


Figure 1

[3 marks]

D2 A uniform beam AB, of weight 60 N and length 8 m, rests with its end A on rough horizontal ground and rests on a smooth horizontal peg at C, with $AC = 5$ m, as shown in Figure 2. The beam is inclined at an angle θ to the ground, where $\sin\theta = \frac{3}{5}$, and lies in a vertical plane perpendicular to the peg.

(a) Find the magnitude of the normal reaction between the beam and the peg.

(2)

(b) Find the least possible value of the coefficient of friction between the beam and the ground.

(2)

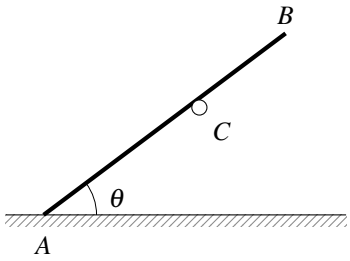


Figure 2

[4 marks]

D3 A uniform ladder AB, of length 5 m and weight 300 N, rests with its foot A on rough horizontal ground and its top B against a smooth vertical wall. The foot of the ladder is 1.4 m from the wall, as shown in Figure 3, and the ladder lies in a vertical plane perpendicular to the wall.

(a) Find the magnitude of the reaction at the wall.

(2)

(b) Find the least possible value of the coefficient of friction between the ladder and the ground.

(1)

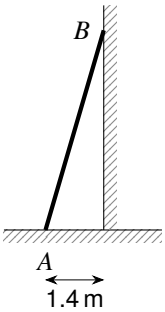


Figure 3

[3 marks]

D4 A uniform rod AB of length $6a$ and weight W is freely hinged to a vertical wall at A. It is held horizontal by a light string from B to the point D on the wall vertically above A, where $AD = 6a$, as shown in Figure 4.

(a) Find the tension in the string.

(2)

(b) Show that the force on the rod at the hinge has magnitude $\frac{\sqrt{2}}{2}W$ and acts at 45° to the rod.

(1)

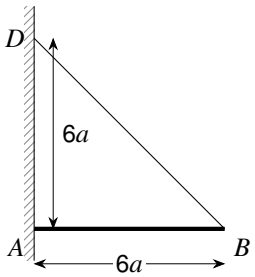


Figure 4

[3 marks]

STRETCH

D5 A uniform ladder AB, of length 6 m and weight 240 N, rests with its foot A on rough horizontal ground and its top B against a rough vertical wall, at 70° to the ground, as shown in Figure 5. The coefficient of friction between the ladder and the wall is 0.25. The ladder is about to slip down, and is in limiting equilibrium at both ends.

(a) Find the magnitude of the normal reaction at the wall.

(2)

(b) Find the coefficient of friction between the ladder and the ground.

(2)

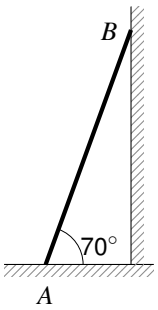


Figure 5

[4 marks]

D6 A uniform rod AB of length $2a$ and weight W has its end A freely hinged to a fixed point on horizontal ground. The rod is held at 60° to the ground by a light string BC, where C is on the ground with $AC = 2a$ and A lies between C and the point of the ground vertically below B, as shown in Figure 6. The rod and the string lie in the same vertical plane.

(a) Find the tension in the string in terms of W

(2)

(b) Find the magnitude of the force exerted on the rod by the hinge.

(2)

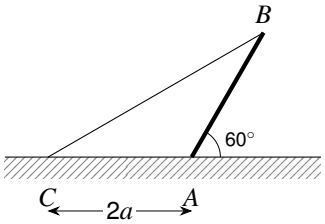


Figure 6

[4 marks]

D7 A uniform pole AB, of length 5 m and weight 80 N, has its end A freely hinged to a vertical wall. A particle of weight W newtons is attached at B. The pole is held horizontal by a light rod CD, where C is on the pole with $AC = 3$ m and D is on the wall vertically below A with $AD = 4$ m, as shown in Figure 7. The thrust in the light rod is 150 N.

(a) Find the value of W

(2)

(b) Find the magnitude of the resultant force acting on the pole at A

(2)

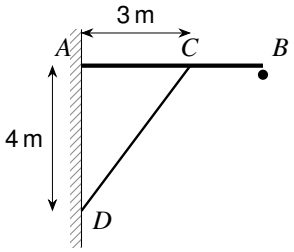


Figure 7

[4 marks]

D8 A uniform rod AB, of length $4a$ and weight W, has its end A on rough horizontal ground and rests against a smooth horizontal rail at C, where $AC = 2.5a$, as shown in Figure 8. The rod is inclined at θ to the ground and lies in a vertical plane at right angles to the rail. The coefficient of friction between the rod and the ground is $\frac{1}{2}$

(a) Show that the reaction at C has magnitude $0.8W\cos\theta$

(2)

(b) Given that the rod is in limiting equilibrium, show that $5\tan^2\theta - 8\tan\theta + 1 = 0$

(2)

(c) Given also that $\theta > 45°$, find θ

(1)

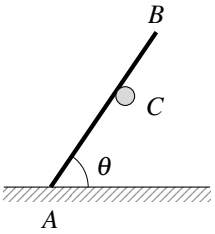


Figure 8

[5 marks]

E Exam practice

Full-size exam questions in the style of WME02. Write your answers in the spaces, and show every step.

CORE *Exam questions for everyone*

E1 A uniform rod AB has length $8a$ and weight $2W$. The end A of the rod is freely hinged to a fixed point on a vertical wall, and the rod lies in a vertical plane that is perpendicular to the wall. A particle of weight W is attached to the rod at B. A light inextensible string has one end attached to the rod at the point C, where $AC = 6a$, and the other end attached to the wall at the point D, vertically above A, where $AD = 8a$. The rod rests in equilibrium in a horizontal position, as shown in Figure 9.

(a) Show that the tension in the string is $\frac{10}{3}W$

(3)

(b) Find the horizontal and vertical components of the force exerted on the rod by the hinge at A

(3)

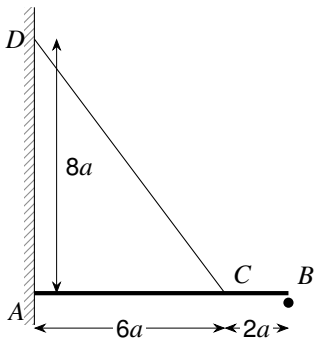


Figure 9

(c) Find the angle between the force exerted on the rod by the hinge and the rod.

(2)

The string will break if its tension exceeds $3W$. The particle is moved to the point P on the rod, and the string does not break.

(d) Find the greatest possible value of AP

(2)

(Total for Question E1 is 10 marks)

E2 A uniform ladder AB, of length 6 m and weight 150 N, rests with its foot A on rough horizontal ground and its top B against a smooth vertical wall. The ladder makes an angle α with the ground, where $\alpha = 65^\circ$, and lies in a vertical plane perpendicular to the wall. The coefficient of friction between the ladder and the ground is 0.4. A painter of weight 650 N climbs the ladder. The painter is modelled as a particle at P, where $AP = x$ metres, as shown in Figure 10.

(a) Find the normal reaction at A

(1)

(b) Show that the reaction at the wall is $\dfrac{450 + 650x}{6\tan 65^\circ}$ newtons.

(3)

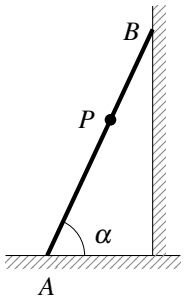


Figure 10

(c) Find the greatest distance up the ladder that the painter can climb before the ladder slips.

(3)

(d) The ladder is moved so that it makes a different angle α with the ground. Find the least value of α for which the painter can reach the top of the ladder without it slipping.

(3)

(Total for Question E2 is 10 marks)

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STRETCH

E3 A uniform beam AB, of length $10a$ and weight W, rests with its end A on rough horizontal ground and rests on a smooth horizontal peg at C, where $AC = 8a$. The beam is inclined at an angle θ to the ground, where $\tan\theta = \frac{3}{4}$, and lies in a vertical plane perpendicular to the peg. A package of weight W is placed on the beam at the point P, where $AP = x$, and the package is modelled as a particle, as shown in Figure 11.

(a) Show that the normal reaction at C has magnitude

$$\frac{W(5a+x)}{10a}$$

(3)

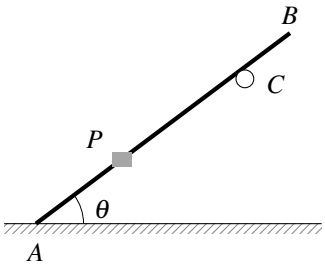


Figure 11

The coefficient of friction between the beam and the ground is $\frac{1}{2}$

(b) Show that the beam rests in equilibrium only if $x \leqslant 5a$

(4)

The package is now moved to B, and the coefficient of friction between the beam and the ground is μ

(c) Find the least value of μ for which the beam can rest in equilibrium.

(3)

In fact $\mu = \frac{1}{2}$. With the package at B, the beam is held in equilibrium by a horizontal force of magnitude P applied at A, acting in the vertical plane containing the beam.

(d) Find the least possible value of P in terms of W

(3)

(Total for Question E3 is 13 marks)

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