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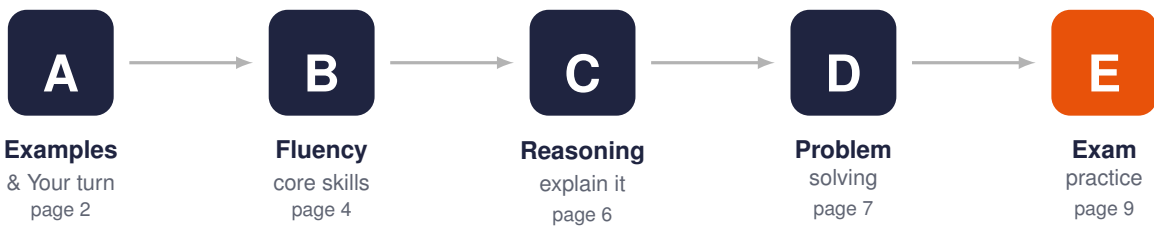
A-LEVEL MATHEMATICS • PURE 4

Binomial Expansion

Worksheet

Pearson Edexcel International A Level (WMA14) & A Level (9MA0) • Calculator allowed: show every step

Free Binomial Expansion worksheet for A-Level and International A-Level Maths, with full worked solutions



Tap a section to jump straight to it

By the end of this worksheet you can: expand $(1+x)^n$ for negative and fractional n • state the range of validity • take out the factor a^n to expand $(a+bx)^n$ • expand after partial fractions • *Stretch:* combine and multiply expansions, find unknown constants, and use a small x to approximate roots.

Which questions are for me?

CORE The methods every A-level student must secure. Start here.

STRETCH Factors taken out of $(a+bx)^n$, combined validity, expansions multiplied together, and approximations where the choice of x matters.

Every section has both, clearly labelled. Questions get harder as you go, and every skill in Sections A to D is needed in Section E.

Track your progress

Section	B	C	D	E
Core marks	23	10	13	11
Stretch marks	16	14	17	8
My score				

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A

Worked examples & Your turn

Read each example carefully. Cover it up, then answer the question beside it using the same method.

CORE

Secure the method

Worked example A1: negative and fractional powers

Find the expansion of $(1 + 2x)^{-2}$ up to the term in x^3

Use $1 + nx + \frac{n(n-1)}{2!}x^2 + \frac{n(n-1)(n-2)}{3!}x^3$ with $n = -2$, and $2x$ for x

$$1 + (-2)(2x) + \frac{(-2)(-3)}{2}(2x)^2 + \frac{(-2)(-3)(-4)}{6}(2x)^3$$

$$1 - 4x + 12x^2 - 32x^3$$

Your turn A1

Find the expansion of $(1 - 3x)^{\frac{1}{2}}$ up to the term in x^3

Worked example A2: the range of validity

Expand $\left(1 + \frac{x}{3}\right)^{-1}$ up to the term in x^2 , and state the range of validity.

The expansion of $(1 + y)^n$ is valid for $|y| < 1$, here $y = \frac{x}{3}$

$$1 - \frac{x}{3} + \frac{x^2}{9}, \text{ valid for } \left|\frac{x}{3}\right| < 1$$

$$1 - \frac{x}{3} + \frac{x^2}{9}, |x| < 3$$

Your turn A2

Expand $\left(1 - \frac{2x}{5}\right)^{-\frac{1}{2}}$ up to the term in x^2 , and state the range of validity.

Worked example A3: expanding $(a + bx)^n$

Expand $\sqrt{4+x}$ up to the term in x^2 , and state the range of validity.

Take out $4^{\frac{1}{2}} = 2$: $\sqrt{4+x} = 2\left(1 + \frac{x}{4}\right)^{\frac{1}{2}}$

$$2\left(1 + \frac{x}{8} - \frac{x^2}{128}\right), \text{ valid for } \left|\frac{x}{4}\right| < 1$$

$$2 + \frac{x}{4} - \frac{x^2}{64}, |x| < 4$$

Your turn A3

Expand $\sqrt{9-2x}$ up to the term in x^2 , and state the range of validity.



STRETCH

Worked example A4: a power in the denominator

Expand $\frac{1}{(2-x)^2}$ up to the term in x^3

$$\frac{1}{4} \left(1 - \frac{x}{2}\right)^{-2} = \frac{1}{4} \left(1 + x + \frac{3}{4}x^2 + \frac{1}{2}x^3\right)$$

$$\frac{1}{4} + \frac{x}{4} + \frac{3x^2}{16} + \frac{x^3}{8}$$

Your turn A4

Expand $\frac{1}{\sqrt{4+3x}}$ up to the term in x^2 , and state the range of validity.

Worked example A5: with partial fractions

$f(x) = \frac{4-x}{(1-x)(2+x)}$. Expand $f(x)$ up to x^2

Partial fractions first: $f(x) = \frac{1}{1-x} + \frac{2}{2+x}$

$(1+x+x^2) + \left(1 - \frac{x}{2} + \frac{x^2}{4}\right)$, valid for $|x| < 1$ and $|x| < 2$

$$2 + \frac{x}{2} + \frac{5x^2}{4}, \quad |x| < 1$$

Your turn A5

$g(x) = \frac{5+x}{(1+x)(3-x)}$. Expand $g(x)$ up to the term in x^2 , and state the range of validity.

Worked example A6: approximating a root

Use the expansion of $(1-2x)^{\frac{1}{2}}$ up to x^2 , with $x = 0.01$, to estimate $\sqrt{2}$

$x = 0.01$ gives $\sqrt{0.98} = \frac{7\sqrt{2}}{10}$, and $1 - x - \frac{x^2}{2} = 0.98995$

$$\sqrt{2} \approx \frac{10}{7} \times 0.98995 \approx 1.41421$$

Your turn A6

Use the expansion of $(1+x)^{\frac{1}{3}}$ up to x^2 , with $x = 0.008$, to estimate $\sqrt[3]{126}$ to 4 decimal places.

Worked example A7: an unknown constant

In the expansion of $(1+kx)^{-3}$, where $k > 0$, the coefficient of x^2 is 54. Find k and the coefficient of x^3

$$6k^2 = 54, \text{ so } k = 3; \quad x^3: \frac{(-3)(-4)(-5)}{6} \times 3^3$$

$$k = 3, \quad -270$$

Your turn A7

In the expansion of $(1-kx)^{\frac{1}{2}}$, where $k > 0$, the coefficient of x^2 is -2 . Find k and the coefficient of x^3



B**Fluency**

Each question is a designed sequence. From one part to the next, ask yourself: what changed, and what did it do to the answer?

CORE

Secure the method

B1 Find the first three terms, in ascending powers of x , of each expansion.

(a) $(1+x)^{-1}$

(b) $(1+x)^{\frac{1}{2}}$

(c) $(1-x)^{-2}$

(d) $(1+x)^{-\frac{1}{3}}$

[4 marks]

B2 Find the first three terms, in ascending powers of x , of each expansion, simplifying each coefficient.

(a) $(1+2x)^{-1}$

(b) $(1-3x)^{\frac{1}{3}}$

(c) $\left(1+\frac{x}{2}\right)^{-2}$

(d) $(1-4x)^{\frac{3}{2}}$

[4 marks]

B3 State the range of values of x for which each expansion is valid.

(a) $(1+3x)^{-2}$

(b) $\left(1-\frac{x}{4}\right)^{\frac{1}{2}}$

(c) $(2+x)^{-1}$

(d) $(9-4x)^{\frac{1}{2}}$

[4 marks]

B4 Expand each expression up to the term in x^2 , taking out the factor a^n first.

(a) $(4+x)^{-1}$

(b) $(9+x)^{\frac{1}{2}}$

(c) $(2-x)^{-3}$

(d) $(8+2x)^{\frac{1}{3}}$

[4 marks]

B5 (a) Express $\frac{2}{(1-x)(1+x)}$ in partial fractions, and hence expand it up to the term in x^2

(b) Express $\frac{3}{(1-2x)(1+x)}$ in partial fractions, and hence expand it up to the term in x^2

(c) State the range of validity of each expansion in (a) and (b).

[3 marks]

B6 Use the first three terms of a suitable expansion of $(1 \pm x)^n$ to estimate each value, to 4 decimal places.

(a) $\sqrt{1.04}$

(b) $\frac{1}{0.98}$

(c) $\sqrt[3]{0.97}$

(d) $\frac{1}{1.01^2}$

[4 marks]



STRETCH

B7 Expand each expression up to the term in x^2

(a) $\frac{1}{(3+x)^2}$

(b) $\sqrt{25-10x}$

(c) $\frac{1}{(x-4)^2}$

(d) $\frac{2+x}{(1-x)^2}$

[4 marks]

B8 State the range of values of x for which each expansion is valid.

(a) $(3-2x)^{-\frac{1}{2}}$

(b) $\frac{\sqrt{1+x}}{2-x}$

(c) $(1+x^2)^{-1}$

(d) $\frac{\sqrt{x+4}}{1-3x}$

[4 marks]

B9 (a) Expand $\frac{3x+5}{(1+x)(1+3x)}$ up to the term in x^2 , using partial fractions.

(b) Expand $\frac{4}{(1-x)^2(1+x)}$ up to the term in x^2 , using partial fractions.

(c) Expand $\frac{x^2+1}{1-x^2}$ up to the term in x^2

(d) Find the coefficient of x^3 in the expansion of $\frac{2}{(1-x)(1-2x)}$

[4 marks]

B10 (a) Expand $(1-x)^{\frac{1}{2}}$ up to the term in x^2

(b) Use $x = 0.01$ to estimate $\sqrt{0.99}$

(c) Show that $\sqrt{0.99} = \frac{3\sqrt{11}}{10}$, and hence estimate $\sqrt{11}$ to 4 decimal places.

(d) Explain why a small value of x is chosen.

[4 marks]



C

Reasoning

Show your working and explain your thinking. A correct answer without a reason does not score full marks.

CORE

Secure the method

C1 Asif writes $(4+x)^{\frac{1}{2}} = 4\left(1+\frac{x}{4}\right)^{\frac{1}{2}}$

Explain his error, and write the correct first line.

[2 marks]

C2 Beth says that the expansion of $(1-3x)^{-2}$ is valid for $|x| < 3$

Explain her error and give the correct range of validity.

[2 marks]

C3 Carl expands $(1+4x)^{\frac{1}{2}}$ and writes $1+2x-\frac{x^2}{2}+\dots$

Find his error in the x^2 term, and give the correct expansion up to x^2

[3 marks]

C4 Dee expands $(1-x)^{-\frac{1}{2}} \approx 1+\frac{x}{2}+\frac{3x^2}{8}$ and says that $x=0.04$ gives an estimate of $\sqrt{0.96}$

Explain what her estimate really approximates, and use it to estimate $\sqrt{0.96}$

[3 marks]

STRETCH

C5 Eli writes $(x-2)^{-1} = x^{-1}\left(1-\frac{2}{x}\right)^{-1} = \frac{1}{x} + \frac{2}{x^2} + \frac{4}{x^3} + \dots$

Explain why this does not answer “expand $(x-2)^{-1}$ in ascending powers of x ”, and give the correct expansion up to x^2 , with its validity.

[3 marks]

C6 Fern writes $\frac{3}{(1-2x)(1+x)} = \frac{2}{1-2x} + \frac{1}{1+x}$, expands it, and says the expansion is valid for $|x| < 1$

Explain her error and give the correct range of validity.

[3 marks]

C7 Gus says that $x=1$ cannot be used in the expansion of $(4+x)^{\frac{1}{2}}$ because “ x is not small”.

Explain whether he is right, and comment on the accuracy of the estimate of $\sqrt{5}$ from three terms.

[3 marks]

C8 (a) Show that, for $|x| < 1$, the coefficient of x^r in the expansion of $(1-x)^{-2}$ is $r+1$

(2)

(b) Hence show that $1 + \frac{2}{2} + \frac{3}{4} + \frac{4}{8} + \frac{5}{16} + \dots = 4$

(2)

(c) Explain why the same method cannot be used with $x=2$

(1)

[5 marks]



D**Problem solving**

You decide the method. Read the whole question first, and show every step.

CORE*Secure the method*

- D1** Find the first three terms, in ascending powers of x , of the binomial expansion of

$$(1 + 6x)^{-\frac{1}{3}}$$

giving each coefficient in its simplest form, and state the range of values of x for which the expansion is valid. [3 marks]

- D2** (a) Find the binomial expansion of $(1 - x)^{-3}$, in ascending powers of x , up to and including the term in x^2 (2)

(b) Hence find the expansion of $\frac{(1 + 2x)^2}{(1 - x)^3}$ up to and including the term in x^2 , and state the range of values of x for which it is valid.

(2)
[4 marks]

- D3** (a) Find the binomial expansion of $(1 + x)^{\frac{1}{3}}$, in ascending powers of x , up to and including the term in x^2 (1)

(b) By substituting $x = \frac{1}{8}$ into your expansion, find an estimate for $\sqrt[3]{9}$, giving your answer to 3 decimal places. You must show how you use $\sqrt[3]{\frac{9}{8}}$

(2)
[3 marks]

- D4** Given that, for $|x| < \frac{1}{2}$, $(1 - 2x)^{-1} = 1 + 2x + 4x^2 + \dots$

find the expansion of $\frac{3+x}{1-2x}$ in ascending powers of x , up to and including the term in x^2 , simplifying each coefficient, and state the range of values of x for which it is valid. [3 marks]



STRETCH

- D5** In the binomial expansion of $(1+ax)^{-\frac{1}{2}}$, in ascending powers of x , the coefficient of x^2 is 6 and the coefficient of x is negative.

Find the value of the constant a , the coefficient of x^3 , and the range of values of x for which the expansion is valid. [4 marks]

D6 $g(x) = \sqrt{1+2x} - \sqrt{1-2x}$, $|x| < \frac{1}{2}$

- (a) Show that, up to and including the term in x^3 , $g(x) \approx 2x + x^3$, explaining why there is no term in x^2

(3)

- (b) Hence find an estimate for $\sqrt{1.2} - \sqrt{0.8}$, giving your answer to 3 decimal places.

(1)

[4 marks]

- D7** In this question you must show all stages of your working. Solutions relying on calculator technology are not acceptable.

- (a) Find the binomial expansion of $(1-4x)^{-\frac{1}{2}}$, in ascending powers of x , up to and including the term in x^2

(2)

- (b) Hence find the expansion of $\frac{2+x}{\sqrt{1-4x}}$ up to and including the term in x^2 , and state the range of values of x for which it is valid.

(2)

[4 marks]

D8 $f(x) = \frac{3+2x+3x^2}{(1-x)(1+x)^2}$, $|x| < 1$

- (a) Express $f(x)$ in partial fractions.

(3)

- (b) Hence find the expansion of $f(x)$ in ascending powers of x , up to and including the term in x^2 , and find the coefficient of x^3

(2)

[5 marks]



E**Exam practice**

Exam-style questions in the style of WMA14. Write your answers in the spaces, and show every step.

CORE

Exam questions for everyone

- E1** (a) Find the first three terms, in ascending powers of x , of the binomial expansion of

$$(27 - 9x)^{\frac{1}{3}}$$

giving each coefficient as a simplified fraction, and state the range of values of x for which the expansion is valid.

(3)

- (b) Use your answer to part (a), with a suitable value of x , to find an estimate for $\sqrt[3]{26}$, giving your answer to 4 decimal places. State the value of x that you use.

(2)

(Total for Question E1 is 5 marks)



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E2 $f(x) = \frac{4+3x}{(2-x)(1+2x)}, \quad |x| < \frac{1}{2}$

(a) Express $f(x)$ in partial fractions.

(2)

(b) Hence, or otherwise, find the series expansion of $f(x)$, in ascending powers of x , up to and including the term in x^2 . Give each coefficient as a simplified fraction.

(3)

(c) Explain why the expansion is valid only for $|x| < \frac{1}{2}$

(1)

(Total for Question E2 is 6 marks)



STRETCH

- E3** (a) Find the binomial expansion of $(1 + 8x)^{\frac{1}{2}}$, in ascending powers of x , up to and including the term in x^3 , simplifying each coefficient.

(3)

- (b) Show that $\sqrt{1.08} = \frac{3\sqrt{3}}{5}$. Hence, by substituting $x = 0.01$ into your expansion, find an estimate for $\sqrt{3}$, giving your answer to 6 decimal places.

(2)

- (c) By substituting $x = -\frac{1}{32}$ into your expansion, find a second estimate for $\sqrt{3}$, giving your answer to 6 decimal places.

(2)

- (d) Without using the true value of $\sqrt{3}$, explain which of your two estimates you would expect to be more accurate.

(1)

(Total for Question E3 is 8 marks)

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