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INTERNATIONAL A LEVEL MATHEMATICS • PURE 4 (WMA14)

Pure 4 Practice Paper 3

Mark Scheme

AIMING FOR A/A*

Written in the style of the Pearson Edexcel International Advanced Level paper (WMA14/01) • 75 marks

Every question marked line by line the way an examiner marks it, with notes on the common errors

Mark every line, not just the answer. Method marks are given for a correct method even when the final answer is wrong. If you lost marks, find the *first* line where your working differs from the scheme. That is the thing to fix.

Code	Meaning
M1	Method mark: for knowing a correct method and attempting to apply it. Small slips in the working do not lose this mark.
dM1	Dependent method mark: can only be given if the method mark before it has been given.
A1	Accuracy mark: for a correct answer or line of working. It depends on the method mark before it.
B1	Independent mark: for a correct result or statement, whatever method was used.
*	The answer is given in the question. Every step must be shown, and the answer must follow from correct working (cso).
cao / cso	Correct answer only / correct solution only, with no errors anywhere in the working.
ft / oe	Follow through from your own earlier answer / or any correct equivalent form.

"Solutions relying on calculator technology are not acceptable." Every question in this paper says this. There, a correct final answer with no algebraic working scores **no marks**.

Rough grade guide (out of 75): A* **57** • A **50** • B **42** • C **34**

Based on recent WMA14 grade boundaries. Real boundaries change every series, so treat this as a guide, not a grade.

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What to revise next

Write in your score for each question. Where you dropped marks, tap the chapter to practise ten more exam questions on it.

Q	What it tested	Marks	Practise it	Score
1	The general term from partial fractions; summing a series at $x = \frac{1}{2}$	9	Ch 2 Partial Fractions; Ch 4 Binomial Expansion	
2	Parametric area; a parametric volume needing $\cos^4\theta$	10	Ch 3 Coordinate Geometry; Ch 6 Integration	
3	Solving a differential equation with partial fractions; its long-term limit	10	Ch 2 Partial Fractions; Ch 6 Integration	
4	The foot of a perpendicular; an equilateral triangle on a line	9	Ch 7 Vectors	
5	Proof by contradiction using odd and even cases	9	Ch 1 Proof	
6	Implicit differentiation; horizontal and vertical tangents; symmetry	9	Ch 5 Differentiation	
7	Parametric differentiation; an angle by scalar product; a tangent	9	Ch 3 Coordinate Geometry; Ch 5 Differentiation; Ch 7 Vectors	
8	Volume of revolution; connected rates of change; a differential equation	10	Ch 5 Differentiation; Ch 6 Integration	
	Total	**75**		

Where did the marks go? If you lost the first marks of a question, the method needs work: go back to that chapter's key skills page. If you lost the final marks, the method is fine and the gap is the trap: a value of x inside the range of validity, a point to reject, a parameter outside the range, or a limit that is not the obvious one. At grade A* the method is rarely the problem; checking every answer against the question is.

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Question 1

9 marks

Part	Scheme	Marks
(a)	$f(x) \equiv \frac{A}{1-x} + \frac{B}{(1-x)^2} + \frac{C}{2-x}$	M1
	$3 - x \equiv A(1-x)(2-x) + B(2-x) + C(1-x)^2$: $A = -1$, $B = 2$, $C = 1$	A1
	$(1-x)^{-1} = \sum x^n$ and $(1-x)^{-2} = \sum (n+1)x^n$	M1
	$(2-x)^{-1} = \frac{1}{2}\left(1 - \frac{x}{2}\right)^{-1} = \sum \frac{x^n}{2^{n+1}}$	M1
	Coefficient $= -1 + 2(n+1) + \frac{1}{2^{n+1}} = \boxed{2n + 1 + \frac{1}{2^{n+1}}}$	A1
	Working out the first few terms and guessing the pattern scores at most M1 A1. The general term of each expansion is needed.	(5)
(b)	$\boxed{\lvert x \rvert < 1}$	B1
	$(2-x)^{-1}$ *is valid for* $\lvert x \rvert < 2$*, but the other two factors need* $\lvert x \rvert < 1$	(1)
(c)	$x = \frac{1}{2}$ is inside $\lvert x \rvert < 1$, so $f\left(\frac{1}{2}\right) = \frac{20}{3} = \sum (2n+1)\frac{1}{2^n} + \sum \frac{1}{2^{n+1}} \cdot \frac{1}{2^n}$	M1
	$\sum \frac{1}{2}\left(\frac{1}{4}\right)^n = \frac{\frac{1}{2}}{1 - \frac{1}{4}} = \frac{2}{3}$	M1
	$\frac{20}{3} - \frac{2}{3} = \boxed{6}$	A1
		(3)
		Total for Question 1: 9 marks

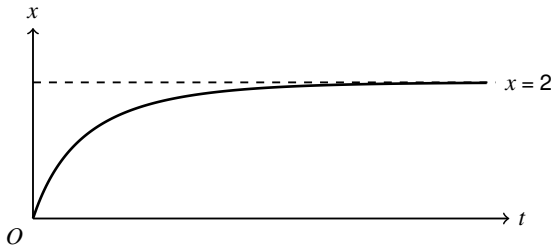
Question 2

10 marks

Part	Scheme	Marks
(a)	Area $= \int y\frac{\mathrm{d}x}{\mathrm{d}\theta}\,\mathrm{d}\theta = \int \cos^3\theta \times 2\sec^2\theta\,\mathrm{d}\theta$	M1
	$= \int 2\cos\theta\,\mathrm{d}\theta$	A1
	Limits $\theta = 0$ and $\theta = \frac{\pi}{4}$ (from $2\tan\theta = 2$), and integrates to $2\sin\theta$	M1
	$\boxed{\sqrt{2}}$	A1
		(4)
(b)	$V = \pi\int y^2\frac{\mathrm{d}x}{\mathrm{d}\theta}\,\mathrm{d}\theta = \pi\int \cos^6\theta \times 2\sec^2\theta\,\mathrm{d}\theta = 2\pi\int \cos^4\theta\,\mathrm{d}\theta$	M1
	$\cos^4\theta = \frac{1}{4}(1+\cos 2\theta)^2 = \frac{1}{4}\left(1+2\cos 2\theta+\cos^2 2\theta\right)$	M1
	Uses $\cos^2 2\theta = \frac{1}{2}(1+\cos 4\theta)$: $\cos^4\theta = \frac{3}{8}+\frac{1}{2}\cos 2\theta+\frac{1}{8}\cos 4\theta$	M1
	$\int \cos^4\theta\,\mathrm{d}\theta = \frac{3}{8}\theta+\frac{1}{4}\sin 2\theta+\frac{1}{32}\sin 4\theta$	A1
	Limits 0 and $\frac{\pi}{4}$: $2\pi\left(\frac{3\pi}{32}+\frac{1}{4}+0\right)$	dM1
	$\boxed{\frac{\pi}{2}+\frac{3\pi^2}{16}}$	A1
	Using $y^2 = \cos^6\theta$ with $\mathrm{d}x$ instead of $(\mathrm{d}x/\mathrm{d}\theta)\,\mathrm{d}\theta$ scores M0. The double-angle formula is needed twice: once for $\cos^2\theta$ and again for $\cos^2 2\theta$	
		(6)
		Total for Question 2: 10 marks

Question 3

10 marks

Part	Scheme	Marks
(a)	Separates: $\int \frac{8}{(4-x)(2-x)}\,\mathrm{d}x = \int 1\,\mathrm{d}t$	M1
	$\frac{8}{(4-x)(2-x)} \equiv \frac{4}{2-x} - \frac{4}{4-x}$	B1
	Integrates to $-4\ln(2-x)+4\ln(4-x)$	M1
	$4\ln\left(\frac{4-x}{2-x}\right) = t + c$	A1
	$t = 0$, $x = 0$: $c = 4\ln 2$, so $\frac{4-x}{2-x} = 2\mathrm{e}^{\frac{t}{4}}$	M1
	$\boxed{x = \frac{4\left(\mathrm{e}^{\frac{t}{4}}-1\right)}{2\mathrm{e}^{\frac{t}{4}}-1}}$ oe	A1
	The signs of the logarithms come from the derivative of $\ln(2-x)$, which is $-1/(2-x)$. Getting them wrong usually gives $x = 0$ at every time.	(6)
(b)		B1
	An increasing curve from O, concave, levelling off below the asymptote	
	Asymptote $\boxed{x = 2}$	B1
	As $t \to \infty$, $x \to \frac{4}{2} = 2$. The asymptote is not $x = 4$: the model only holds for $0 \leqslant x < 2$	(2)
(c)	$x = 1$: $\frac{4-1}{2-1} = 3 = 2\mathrm{e}^{\frac{t}{4}}$	M1
	$\boxed{t = 4\ln\frac{3}{2}}$ minutes	A1
		(2)
		Total for Question 3: 10 marks

Question 4

9 marks

Part	Scheme	Marks
(a)	General point F on l: $\overrightarrow{AF} = \begin{pmatrix} 2\lambda - 2 \\ \lambda - 4 \\ -\lambda - 2 \end{pmatrix}$ and $\overrightarrow{AF} \cdot \begin{pmatrix} 2 \\ 1 \\ -1 \end{pmatrix} = 6\lambda - 6 = 0$	M1
	$\lambda = 1$, foot of the perpendicular $F(3, -1, 2)$	A1
	$AF = \sqrt{0+9+9} = 3\sqrt{2}$	M1
	F is the midpoint of PQ and angle $FAP = 30°$: $FP = AF\tan 30° = \sqrt{6}$	M1
	$\lvert\mathbf{d}\rvert = \sqrt{6}$, so P and Q are at $\lambda = 1 \pm 1$	M1
	$(1, -2, 3)$	A1
	$(5, 0, 1)$	A1
	Alternative: $AP^2 = 6\lambda^2 - 12\lambda + 24$ *(the distance from* A *to the point with parameter* λ*), and* PQ *is twice* FP*. Setting* $AP = PQ$ *with* P*,* Q *symmetric about* F *leads to* $\lambda = 0$ *and* $\lambda = 2$	(7)
(b)	Side $2\sqrt{6}$: $\frac{\sqrt{3}}{4} \times 24$, or $\frac{1}{2} \times PQ \times AF = \frac{1}{2} \times 2\sqrt{6} \times 3\sqrt{2}$	M1
	$6\sqrt{3}$	A1
		(2)
		Total for Question 4: 9 marks

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Question 5

9 marks

Part	Scheme	Marks
(a)	Assume there is a rational solution $x = \frac{p}{q}$, where p and q are integers with no common factor, so they are not both even	B1
	Multiplies by q^2: $ap^2 + bpq + cq^2 = 0$	M1
	Considers the cases for the parity of p and q	M1
	p and q both odd: odd + odd + odd = odd	A1
	p even, q odd: even + even + odd = odd	A1
	p odd, q even: odd + even + even = odd. In every case the left side is odd, so it cannot be 0. This contradicts the assumption, so there are no rational solutions	A1
	All three cases are needed. "Both even" is ruled out by the assumption, and this must be said or used.	(6)
(b)	Assume $\alpha = \frac{p}{q}$ in lowest terms: $p^3 + pq^2 + q^3 = 0$	B1
	Considers the three parity cases	M1
	Each gives odd = 0 (odd + odd + odd, even + even + odd, odd + even + even), a contradiction, so α is irrational	A1
		(3)
		Total for Question 5: 9 marks

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Question 6

9 marks

Part	Scheme	Marks
(a)	$3x^2 + 3y^2\frac{dy}{dx} = \ldots$ Product rule on $3xy$: $\ldots = 3y + 3x\frac{dy}{dx}$ $\boxed{\frac{dy}{dx} = \frac{y - x^2}{y^2 - x}}$	M1 M1 A1 (3)
(b)	$\frac{dy}{dx} = 0$: $y = x^2$ Substitutes: $x^3 + x^6 = 3x^3$, so $x^3\left(x^3 - 2\right) = 0$ $x = 0$ gives O, where $y^2 - x = 0$ too, so it is rejected; $x = \sqrt[3]{2}$ $\boxed{\left(2^{\frac{1}{3}}, 2^{\frac{2}{3}}\right)}$	M1 M1 A1 A1 (4)
(c)	$\boxed{\left(2^{\frac{2}{3}}, 2^{\frac{1}{3}}\right)}$ The equation is unchanged when x and y are swapped, so C is symmetrical in $y = x$. Reflecting in $y = x$ turns a horizontal tangent into a vertical one *Setting $y^2 - x = 0$ and solving again is correct but is not "write down": it still scores B1 for the point, with B1 for a correct reason.*	B1 B1 (2)
		Total for Question 6: 9 marks

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Question 7

9 marks

Part	Scheme	Marks
(a)	$\frac{dx}{dt} = e^t(\cos t - \sin t)$, $\frac{dy}{dt} = e^t(\sin t + \cos t)$	M1
	$\boxed{\frac{dy}{dx} = \frac{\sin t + \cos t}{\cos t - \sin t}}$	A1
		(2)
(b)	Tangent direction $\begin{pmatrix}\cos t - \sin t \\ \sin t + \cos t\end{pmatrix}$ and direction of OP $\begin{pmatrix}\cos t \\ \sin t\end{pmatrix}$	M1
	Scalar product $= \cos^2 t - \sin t \cos t + \sin^2 t + \sin t \cos t = 1$	M1
	Magnitudes $\sqrt{(\cos t - \sin t)^2 + (\sin t + \cos t)^2} = \sqrt{2}$ and 1	M1
	$\cos\phi = \frac{1}{\sqrt{2}}$ for every t, so the angle is $45°$	A1*
	Checking one or two values of t is not a proof: M0. The working must hold for every t.	(4)
(c)	$\frac{dy}{dt} = 0$: $\sin t + \cos t = 0$, so $\tan t = -1$	M1
	$t = \frac{3\pi}{4}$, the only value in $0 \leqslant t \leqslant \pi$ (and $\cos t - \sin t \neq 0$ there)	A1
	$\boxed{\left(-\frac{\sqrt{2}}{2}e^{\frac{3\pi}{4}}, \frac{\sqrt{2}}{2}e^{\frac{3\pi}{4}}\right)}$	A1
	A calculator gives $\tan^{-1}(-1) = -\frac{\pi}{4}$, which is outside the range. Using it scores M1 A0 A0.	(3)
		Total for Question 7: 9 marks

Question 8

10 marks

Part	Scheme	Marks
(a)	$V = \pi \int_0^h y^2 \,\mathrm{d}x = \pi \int_0^h (x+1)\,\mathrm{d}x$	M1
	$= \pi \left[\frac{1}{2}x^2 + x\right]_0^h = \pi\left(\frac{1}{2}h^2 + h\right)$	A1*
		(2)
(b)	$\frac{\mathrm{d}V}{\mathrm{d}t} = 4\pi - \pi h$	M1
	$\frac{\mathrm{d}V}{\mathrm{d}h} = \pi(h+1)$, so $\pi(h+1)\frac{\mathrm{d}h}{\mathrm{d}t} = \pi(4-h)$	M1
	Separates: $\int \frac{h+1}{4-h}\,\mathrm{d}h = \int 1\,\mathrm{d}t$	M1
	$\frac{h+1}{4-h} = -1 + \frac{5}{4-h}$, integrated: $-h - 5\ln(4-h)$	M1
	$t = -h - 5\ln(4-h) + c$	A1
	$t = 0$, $h = 0$: $c = 5\ln 4$, $\boxed{t = 5\ln\left(\frac{4}{4-h}\right) - h}$ oe	A1
	The rate in is not $\mathrm{d}h/\mathrm{d}t$: it is $\mathrm{d}V/\mathrm{d}t$, and the depth rate comes from the chain rule. The fraction must be divided before integrating.	(6)
(c)	$h = 3$: $t = 5\ln 4 - 3 = \boxed{3.93}$ seconds	B1
	As $h \to 4$, $\ln\left(\frac{4}{4-h}\right) \to \infty$, so $t \to \infty$. At $h = 4$ water leaks out as fast as it is poured in	B1
		(2)
		Total for Question 8: 10 marks

TOTAL FOR PAPER: 75 MARKS

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