eClassroom

Probability

Worked Solutions

Suitable for:

Pearson Edexcel International A-Level (iAL) — Statistics 1

Question 1 — Worked Solution

(a) P(product even)

Product is odd only when BOTH dice show odd. Odd faces: 1,3,5 on each die.

$$P(\text{both odd}) = \frac{3}{6} \times \frac{3}{6} = \frac{9}{36}$$

$$\therefore P(\text{product even}) = 1 - \frac{9}{36} = \frac{27}{36} = \frac{3}{4}$$

(b) P(product > 12)

List pairs giving product > 12:
(3,5),(3,6),(4,4),(4,5),(4,6),(5,3),(5,4),(5,5),(5,6),(6,3),(6,4),(6,5),(6,6) — 13 pairs.

$$\therefore P(\text{product} > 12) = \frac{13}{36}$$

(c) P(product is a perfect square)

Perfect squares from 1 to 36: 1,4,9,16,25,36.

Pairs: (1,1),(1,4),(4,1),(2,2),(3,3),(4,4),(5,5),(6,6) — 8 pairs.

$$\therefore P(\text{perfect square}) = \frac{8}{36} = \frac{2}{9}$$

Question 2 — Worked Solution

(a) Sample space

{(1,H), (1,T), (2,H), (2,T), (3,H), (3,T), (4,H), (4,T)} — 8 equally likely outcomes.

(b) P(H and even)

Favourable: (2,H), (4,H) — 2 outcomes.

$$\therefore P = \frac{2}{8} = \frac{1}{4}$$

(c) P(T and > 2)

Favourable: (3,T), (4,T) — 2 outcomes.

$$\therefore P = \frac{2}{8} = \frac{1}{4}$$

Question 3 — Worked Solution

(a)

$$P(A \cup B) = P(A) + P(B) - P(A \cap B) = 0.4 + 0.35 - 0.15 = 0.60$$

(b)

$$P(A' \cap B) = P(B) - P(A \cap B) = 0.35 - 0.15 = 0.20$$

Question 4 — Worked Solution

(a) Find P(B)

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$0.8 = 0.6 + P(B) - 0.2 \Rightarrow P(B) = 0.4$$

(b)

$$P(A \mid B) = \frac{P(A \cap B)}{P(B)} = \frac{0.2}{0.4} = 0.5$$

$$\therefore P(A' \mid B) = 1 - P(A \mid B) = 1 - 0.5 = 0.5$$

Question 5 — Worked Solution

(a)

Mutually exclusive means no overlap:

$$P(A \cup B) = P(A) + P(B) = 0.3 + 0.45 = 0.75$$

(b)

$$P(A' \cap B') = P((A \cup B)') = 1 - P(A \cup B) = 1 - 0.75 = 0.25$$

Question 6 — Worked Solution

(a) Find p

All regions must sum to 1:

$$3p + p + 2p + 0.1 = 1 \Rightarrow 6p = 0.9 \Rightarrow p = 0.15$$

$$\therefore p = \frac{3}{20}$$

(b)

$$P(A \cap B) = p = \frac{3}{20} \qquad P(B) = 2p + p = 3p = \frac{9}{20}$$

$$\therefore P(A \mid B) = \frac{P(A \cap B)}{P(B)} = \frac{3/20}{9/20} = \frac{1}{3}$$

(c) Independence check

$$P(A) = 3p + p = 4p = \frac{3}{5} \qquad P(B) = \frac{9}{20}$$

$$P(A) \times P(B) = \frac{3}{5} \times \frac{9}{20} = \frac{27}{100}$$

$$P(A \cap B) = \frac{3}{20} = \frac{15}{100}$$

Since $P(A) \times P(B) \neq P(A \cap B)$, A and B are not independent.

Question 7 — Worked Solution

(a) Find p and q

All regions sum to 1:

$$p + 0.25 + q + 0.15 = 1 \Rightarrow p + q = 0.60 \quad \cdots(1)$$

P(A) = P(B):

$$p + 0.25 = q + 0.25 \Rightarrow p = q \quad \cdots(2)$$

Substitute (2) into (1):

$$2p = 0.60 \Rightarrow p = 0.30$$

$$\therefore p = q = \frac{3}{10}$$

(b)

$$P(B) = q + 0.25 = \frac{3}{10} + \frac{1}{4} = \frac{11}{20}$$

$$P(A' \cap B) = P(B \text{ only}) = q = \frac{3}{10}$$

$$\therefore P(A' \mid B) = \frac{P(A' \cap B)}{P(B)} = \frac{3/10}{11/20} = \frac{3}{10} \times \frac{20}{11} = \frac{6}{11}$$

Question 8 — Worked Solution

(a) Form equations

All regions sum to 1:

$$p + \frac{1}{5} + q + \frac{1}{10} = 1 \Rightarrow p + q = \frac{7}{10} \quad \cdots(1) \checkmark$$

P(A) = p + 1/5, P(B) = q + 1/5. Given P(B)=2P(A):

$$q + \frac{1}{5} = 2\left(p + \frac{1}{5}\right) \Rightarrow q - 2p = \frac{1}{5} \quad \cdots(2) \checkmark$$

(b) Solve

(1) – (2): 3p = 7/10 – 1/5 = 1/2

$$p = \frac{1}{6}$$

$$q = \frac{7}{10} - \frac{1}{6} = \frac{21}{30} - \frac{5}{30} = \frac{8}{15}$$

$$\therefore p = \frac{1}{6} \qquad q = \frac{8}{15}$$

(c) Independence

$$P(A) = \frac{1}{6} + \frac{1}{5} = \frac{11}{30} \qquad P(B) = \frac{8}{15} + \frac{1}{5} = \frac{11}{15}$$

$$P(A) \times P(B) = \frac{11}{30} \times \frac{11}{15} = \frac{121}{450} \neq \frac{1}{5} = P(A \cap B)$$

Not independent.

Question 9 — Worked Solution

(a) Show p + q = 0.20

$$P(B) = 0.10 + 0.20 + p + q = 0.30 + (p + q)$$

$$P(C) = 0.05 + p + q + 0.15 = 0.20 + (p + q)$$

B and C independent:

$$P(B \cap C) = P(B) \times P(C)$$

$$p + q = (0.30 + p + q)(0.20 + p + q)$$

Let s = p+q. Also: all regions sum to 1:

$$0.20 + 0.10 + 0.20 + 0.05 + s + 0.15 + 0.10 = 1 \Rightarrow s = 0.20 \checkmark$$

Verify independence: P(B)=0.50, P(C)=0.40, P(B∩C)=0.20=0.50×0.40 ✓

(b) Find p and q

P(A∩B∩C) = p = 0.04 (given).

$$\therefore p = 0.04 \qquad q = 0.20 - 0.04 = 0.16$$

(c)

$$\therefore P(A \cup B \cup C) = 1 - 0.10 = \frac{9}{10}$$

Question 10 — Worked Solution

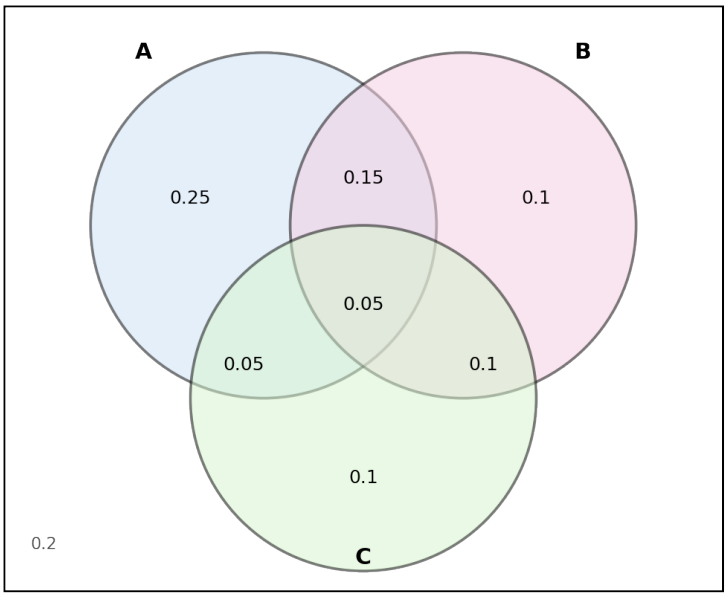
(a) Venn diagram regions

Work outward from centre:

$r_{ABC} = 0.05 \quad r_{AB} = 0.20 - 0.05 = 0.15 \quad r_{AC} = 0.10 - 0.05 = 0.05 \quad r_{BC} = 0.15 - 0.05 = 0.10$

$r_A = 0.50 - 0.15 - 0.05 - 0.05 = 0.25 \quad r_B = 0.40 - 0.15 - 0.10 - 0.05 = 0.10 \quad r_C = 0.30 - 0.05 - 0.10 - 0.05 = 0.10$

$$r_{\text{none}} = 1 - 0.80 = 0.20$$



(b)

$$\therefore P(A \cup B \cup C) = 1 - 0.20 = 0.80$$

(c)

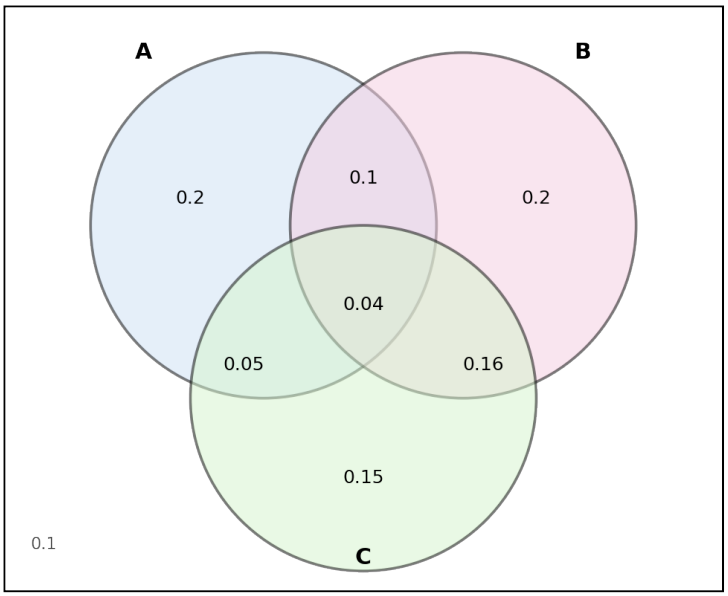
$$P(A \mid B \cap C) = \frac{P(A \cap B \cap C)}{P(B \cap C)} = \frac{0.05}{0.15} = \frac{1}{3}$$

Question 11 — Worked Solution

(a) Position of D

D must not overlap A (mutually exclusive with A) but must overlap B.

D could sit in the B only region, or overlap B and C but not A.



(b)

P(B) = 0.10 + 0.20 + 0.04 + 0.16 = 0.50.

Since D does not overlap A, D lies entirely outside A.

The portion of D within B: P(B∩D) = P(D) = 0.10 (if D ⊂ B, which satisfies A∩D=∅ and P(B∩D)>0).

$$P(D' \cap B) = P(B) - P(B \cap D) = 0.50 - 0.10 = 0.40$$

$$\therefore P(D' \mid B) = \frac{P(D' \cap B)}{P(B)} = \frac{0.40}{0.50} = \frac{4}{5}$$

Question 12 — Worked Solution

(a)

$$P(A \mid B) = \frac{P(A \cap B)}{P(B)} = \frac{0.12}{0.3} = 0.4$$

(b) Independence check

$$P(A) \times P(B) = 0.4 \times 0.3 = 0.12 = P(A \cap B) \checkmark$$

Since P(A)×P(B) = P(A∩B), A and B are independent.

Question 13 — Worked Solution

$$P(A) \times P(B) = 0.5 \times 0.6 = 0.3 = P(A \cap B) \checkmark$$

Since P(A)×P(B) = P(A∩B) = 0.3, A and B are independent.

Question 14 — Worked Solution

Since A and B are independent:

$$P(A \cap B) = P(A) \times P(B) = 0.4\,P(B)$$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$0.7 = 0.4 + P(B) - 0.4\,P(B) = 0.4 + 0.6\,P(B)$$

$$0.6\,P(B) = 0.3$$

$$\therefore P(B) = 0.5$$

Question 15 — Worked Solution

Let P(A)=a, P(B)=b. Since A and B are independent:

$$P(A \cap B') = P(A)(1 - P(B)) = a(1 - b) = 0.3 \quad \cdots(1)$$

$$P(A' \cap B) = (1 - P(A))P(B) = (1 - a)b = 0.2 \quad \cdots(2)$$

(1)–(2): a–ab–b+ab = 0.1 $\Rightarrow$ a–b = 0.1 $\Rightarrow$ a = b+0.1

Sub into (1): (b+0.1)(1-b) = 0.3

$$b + 0.1 - b^2 - 0.1b = 0.3 \Rightarrow b^2 - 0.9b + 0.2 = 0$$

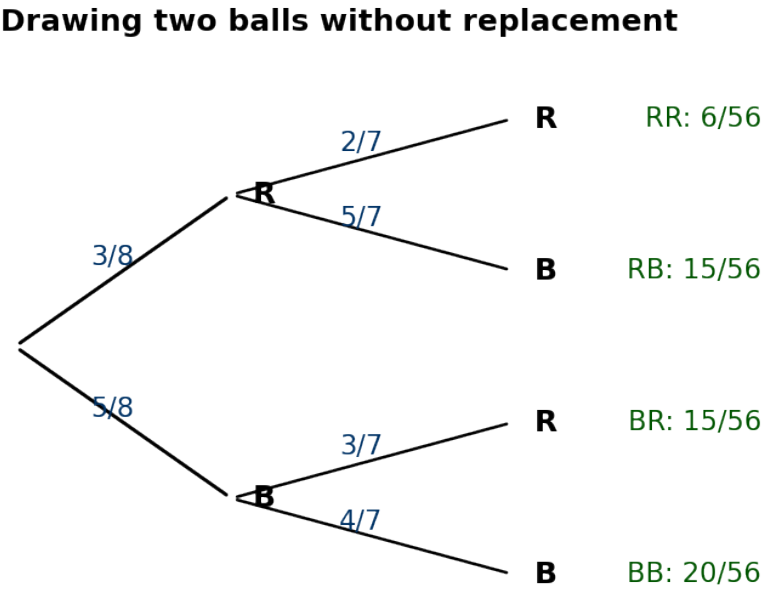
$$(b - 0.4)(b - 0.5) = 0 \Rightarrow b = 0.4 \text{ or } b = 0.5$$

If b=0.4: a=0.5. If b=0.5: a=0.6. Both are valid solutions.

P(A)=0.6, P(B)=0.5 or P(A)=0.5, P(B)=0.4

Question 16 — Worked Solution

(a) Tree diagram



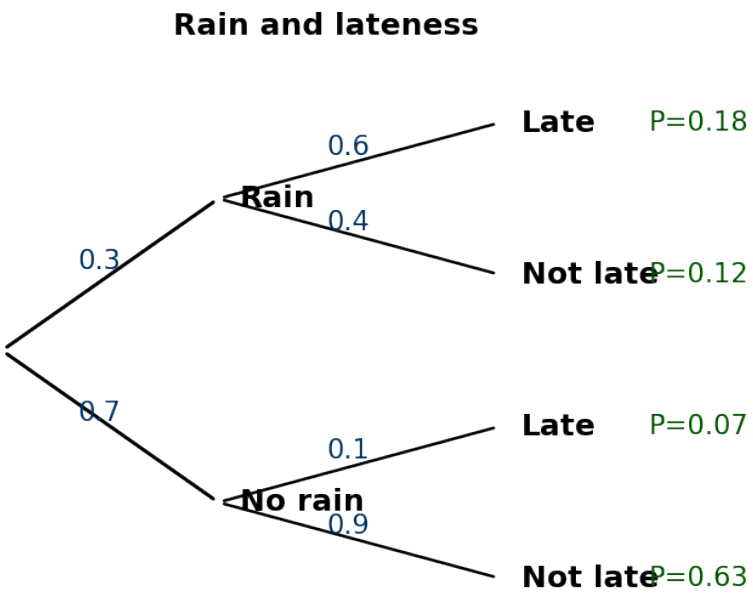
(b) P(same colour)

$$P(\text{same}) = P(RR) + P(BB) = \frac{6}{56} + \frac{20}{56} = \frac{26}{56}$$

$$\therefore = \frac{13}{28}$$

Question 17 — Worked Solution

(a) Tree diagram



(b) P(late)

$$P(\text{late}) = 0.3 \times 0.6 + 0.7 \times 0.1 = 0.18 + 0.07 = 0.25$$

(c) P(rain | late)

$$P(\text{rain} \mid \text{late}) = \frac{P(\text{rain} \cap \text{late})}{P(\text{late})} = \frac{0.18}{0.25}$$

$$\therefore = 0.72$$

Question 18 — Worked Solution

(a) P(exactly 2 heads)

Sequences with exactly 2 heads: HHT, HTH, THH — 3 equally probable sequences.

$$P(\text{exactly 2H}) = 3 \times (0.6)^2 \times (0.4) = 3 \times 0.36 \times 0.4 = 0.432$$

(b) P(at least one tail)

$$P(\text{at least 1T}) = 1 - P(\text{all heads}) = 1 - (0.6)^3 = 1 - 0.216$$

$$\therefore = 0.784$$