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A-LEVEL MATHEMATICS • STATISTICS 2

Approximations

Worked Solutions

Pearson Edexcel International A Level (WST02) • Calculator and statistical tables allowed: show every step

Full worked solutions for the free Approximations worksheet, A-Level and International A-Level Maths

A **Examples** & Your turn page 2

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Tap a section to jump straight to it

Full worked solutions for every question on the Statistics 2 Approximations worksheet, in the same order and with the same question numbers.

How to mark your work. Tick each step you got right, not just the final answer. At A-level most marks are for method, so a wrong final answer can still earn most of the marks. If you lost marks, find the *first* line where your working is different from ours. That is the thing to fix.

In Section E: **M** = method mark, **dM** = method mark that depends on the previous M mark, **A** = accuracy mark (needs the M mark before it), **B** = independent mark.

Question sheet | Exam questions by topic | Statistics 2 exam questions | More worksheets

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A Your turn answers

Compare each line of your working with the solution.

CORE

A1 Po(4): $1 - \mathrm{P}(X \leqslant 2) = 1 - 0.2381$: **0.7619**

A2 N(36, 19.8): $\mathrm{P}(Y > 40.5)$, $z = 1.01$; $1 - 0.8438$: **0.1562**

A3 N(50, 50): $\mathrm{P}(Y > 59.5)$, $z = 1.34$; $1 - 0.9099$: **0.0901**

STRETCH

A4 N(30, 15): $29.5 < Y < 30.5$, $z = \pm 0.13$; $2(0.5517) - 1$: **0.1034**

A5 Late parcels B(300, 0.01) $\approx$ Po(3); more than 296 on time means at most 3 late: **0.6472**

A6 N(81, 81): $75.5 < Y < 89.5$, $z = -0.61$ and 0.94; $0.8264 - 0.2709$: **0.5555**

A7 p is not small and n is large, so N(150, 105): $\mathrm{P}(Y < 139.5)$, $z = -1.02$: **0.1539**

B Fluency solutions

One mark for each correct part. The notes in italics explain why each sequence was chosen.

CORE

B1 **(a)** $\mathrm{P}(Y < 20.5)$ **(b)** $\mathrm{P}(Y < 19.5)$ **(c)** $\mathrm{P}(Y > 19.5)$ **(d)** $\mathrm{P}(Y > 20.5)$

Decide which whole numbers are included, then go half a unit beyond them.

B2 **(a)** $\binom{100}{2}(0.03)^2(0.97)^{98}$: **0.2252** **(b)** Po(3): **0.2240** **(c)** **0.4232** **(d)** **0.4198**

With n large and p small the Poisson values are close to the exact ones.

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B3 N(100, 50)

(a) $Y < 110.5$, $z = 1.48$: **0.9306** **(b)** $Y < 89.5$: **0.0694** **(c)** $99.5 < Y < 100.5$, $z = \pm 0.07$: **0.0558**

(d) $94.5 < Y < 105.5$, $z = \pm 0.78$: **0.5646**

A single value becomes an interval one unit wide.

B4 N(49, 49)

(a) $Y > 55.5$, $z = 0.93$: **0.1762** **(b)** $Y < 40.5$, $z = -1.21$: **0.1131**

(c) $45.5 < Y < 54.5$, $z = -0.5, 0.79$: **0.4767** **(d)** $Y > 49.5$, $z = 0.07$: **0.4721**

The mean and the variance of the normal are both λ

B5 **(a)** n large, p small: Po(2) **(b)** n large, $p = 0.5$: N(40, 20)

(c) **Neither**: n is small; use the tables for the failures, B(30, 0.4)

An approximation is only needed when the exact distribution cannot be used.

B6 **(a)** N(90, 57.6) **(b)** N(75, 75) **(c)** Po(2) **(d)** Po(8)

Find the exact distribution first, then decide on the approximation.

STRETCH

B7 Cracked eggs B(150, 0.04) $\approx$ Po(6)

(a) e^{-6}: **0.0025** **(b)** cracked $\leqslant 5$: **0.4457** **(c)** cracked $\geqslant 11$, $1 - 0.9574$: **0.0426**

(d) **p is not close to 0.5**, and the number cracked has a small mean, so the distribution is skewed

Count whichever outcome is rare.

B8 N(60, 30)

(a) $Y > 65.5$, $z = 1.00$: **0.1587** **(b)** $\dfrac{k - 0.5 - 60}{\sqrt{30}} > 1.6449$, $k > 69.5$: $k = 70$

(c) $\dfrac{m + 0.5 - 60}{\sqrt{30}} < -1.2816$, $m < 52.5$: $m = 52$ **(d)** $52.5 < Y < 69.5$; $0.9582 - 0.0853$: **0.8729**

Write the continuity-corrected boundary, then use the percentage points.

B9 **(a)** Po(36) $\approx$ N(36, 36): $Y > 40.5$, $z = 0.75$: **0.2266** **(b)** $Y < 30.5$, $z = -0.92$: **0.1788**

(c) Po(25) $\approx$ N(25, 25): $Y > 29.5$, $z = 0.9$: **0.1841** **(d)** $19.5 < Y < 30.5$, $z = \pm 1.1$: **0.7286**

Build the Poisson total first; approximate it afterwards.

B10 **(a)** **0.2148** **(b)** N(20, 10): $Y < 17.5$, $z = -0.79$: **0.2148** **(c)** $Y < 17$, $z = -0.95$: **0.1711**

(d) **The continuity correction makes the approximation much closer**

The correction matters most when the standard deviation is small.

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C Reasoning solutions

Your wording can be different. Check that your reason makes the same point.

CORE

C1 No continuity correction: $X \geqslant 48$ becomes $Y > 47.5$; $z = 1.53$, $1 - 0.9370$: **0.0630**

C2 $X < 42$ means $X \leqslant 41$, so $Y < 41.5$; $z = -1.20$: **0.1151**

C3 p is close to 1, not 0.5, so a normal model is poor. Count the unusable bolts: B(120, 0.05) $\approx$ Po(6)

at least 117 usable means at most 3 unusable: **0.1512**

C4 The mean is too small for a normal approximation, and the tables give the exact value: **0.2381**

STRETCH

C5 A Poisson approximation also needs p small; here $p = 0.4$. Use N(80, 48): $Y < 69.5$, $z = -1.52$: **0.0643**

C6 The variance is $np(1-p) = 24$, not np. N(30, 24): $Y > 35.5$, $z = 1.12$; $1 - 0.8686$: **0.1314**

C7 λ is large, so use N(30, 30): $Y < 25.5$, $z = -0.82$: **0.2061**

C8 **(a)** $Y \sim \text{N}(\frac{n}{2}, \frac{n}{4})$; $X \leqslant \frac{n}{2}$ becomes $Y < \frac{n}{2} + \frac{1}{2}$, so $z = \frac{1/2}{\sqrt{n}/2} = \frac{1}{\sqrt{n}}$, as required

(b) $\Phi(z) = 0.5398$ gives $z = 0.10$; $\frac{1}{\sqrt{n}} = 0.1$: $n = 100$

D Problem solving solutions

There is often more than one correct method. Any complete, correct method scores full marks.

CORE

D1 **(a)** B(200, 0.015) $\approx$ Po(3): $1 - \mathrm{P}(X \leqslant 4) = 1 - 0.8153$: **0.1847**

(b) **n is large and p is small**

D2 B(120, 0.35) $\approx$ N(42, 27.3); $34.5 < Y < 50.5$, $z = -1.44$ and 1.63

$0.9484 - 0.0749$: **0.8735**

D3 Po(54) $\approx$ N(54, 54); $Y < 44.5$, $z = -1.29$: **0.0985**

D4 **(a)** $\dfrac{110.5 - 120}{\sigma} = -1.12$, $\sigma = 8.482$, $\sigma^2 = 71.95 = 120(1 - p)$: $p = 0.4$

(b) $\dfrac{120}{0.4}$: $n = 300$

D5 **(a)** Wrongly scanned B(400, 0.015) $\approx$ Po(6); at least 396 correct means at most 4 wrong: **0.2851**

(b) **p is close to 1, not 0.5**, so the distribution is very skewed

D6 **(a)** Po(28) $\approx$ N(28, 28); $Y > 35.5$, $z = 1.42$; $1 - 0.9222$: **0.0778**

(b) $(0.9222)^{10}$: **0.445**

D7 **(a)** $\mathrm{P}(Y > t - 0.5) = 0.8944$, so $\dfrac{t - 0.5 - 100}{10} = -1.25$: $t = 88$

(b) $87.5 < Y < 88.5$: $\Phi(-1.15) - \Phi(-1.25) = 0.1251 - 0.1056$: **0.0195**

D8 **(a)** $1 - \mathrm{P}(X \leqslant 3)$ for B(20, 0.05): $1 - 0.9841$: **0.0159**

(b) B(200, 0.0159) $\approx$ Po(3.18); $\mathrm{e}^{-3.18}\left(1 + 3.18 + \dfrac{3.18^2}{2}\right)$: **0.384**

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E Exam practice solutions

Mark-scheme notes are shown on the right of each line.

CORE *Exam questions for everyone*

E1 **(a)** B(20, 0.25): $1 - \text{P}(X \leqslant 7) = 1 - 0.8982$ *M1*

0.1018 *A1*

(b) N(75, 56.25) *B1*

$\text{P}(Y > 80.5)$ *M1*

$z = \dfrac{80.5 - 75}{7.5} \approx 0.73$ *M1*

$1 - 0.7673$: **0.2327** *A1*

(c) $\text{P}(64.5 < Y < 85.5)$ *M1*

$z = \pm 1.4$; $2(0.9192) - 1$ *M1*

0.8384 *A1*

(d) n is large and p is close to 0.5 *B1*

$np = 75$ and $n(1-p) = 225$ are both large *B1*

E2 **(a)** Po(3.5): $\text{P}(X \leqslant 1)$ *M1*

0.1359 *A1*

(b) Po(63) $\approx$ N(63, 63) *B1*

$\text{P}(Y > 75.5)$ *M1*

$z = \dfrac{75.5 - 63}{\sqrt{63}} \approx 1.57$ *M1*

$1 - 0.9418$: **0.0582** *A1*

(c) $\dfrac{n - 0.5 - 63}{\sqrt{63}} < -1.6449$ *M1 M1*

$n < 50.4$: $n = 50$ *A1*

(d) The mean, 63, is large *B1*

and is beyond the range of the Poisson tables *B1*

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E3 **(a)** B(250, 0.04) $\approx$ Po(10) *B1*

$1 - \mathrm{P}(X \leqslant 15) = 1 - 0.9513$ *M1*

0.0487 *A1*

(b) At least 245 valid means at most 5 invalid: $\mathrm{P}(X \leqslant 5)$ *M1*

0.0671 *A1*

(c) N(200, 192) *B1*

$\mathrm{P}(Y > 225.5)$ *M1*

$z = \dfrac{25.5}{\sqrt{192}} \approx 1.84$ *M1*

$1 - 0.9671$: **0.0329** *A1*

(d) $\mathrm{P}(Y > k - 0.5) < 0.025$: $\dfrac{k - 0.5 - 200}{\sqrt{192}} > 1.96$ *M1 M1*

$k > 227.7$: $k = 228$ *A1*

(e) n is large *B1*

and $np = 200$ and $n(1 - p) = 4800$ are both large (greater than 5) *B1*

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