



eClassroom

A-LEVEL MATHEMATICS • PURE 3

Trigonometric Formulae

Worksheet

Pearson Edexcel International A Level (WMA13) & A Level (9MA0) • Calculator allowed: show every step

Free Trigonometric Formulae worksheet for A-Level and International A-Level Maths, with full worked solutions



Tap a section to jump straight to it

By the end of this worksheet you can: addition and double angle formulae • equations • *Stretch:* R forms and proofs.

Which questions are for me?

CORE The methods every A-level student must secure. Start here.

STRETCH Exact values, proofs with double angles, R forms and their maxima, and equations.

Every section has both, clearly labelled. Questions get harder as you go, and every skill in Sections A to D is needed in Section E.

Track your progress

Section	B	C	D	E
Core marks	23	10	13	11
Stretch marks	16	14	17	8
My score				

Full worked solutions

Exam questions by topic

Trigonometry revision engine

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Name

Class

Date



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A

Worked examples & Your turn

Read each example carefully. Cover it up, then answer the question beside it using the same method.

CORE

Secure the method

Worked example A1

Find the exact value of $\tan 75^\circ$

$$75^\circ = 45^\circ + 30^\circ \text{ and } \tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$$

$$\frac{1 + \frac{1}{\sqrt{3}}}{1 - \frac{1}{\sqrt{3}}} = \frac{\sqrt{3} + 1}{\sqrt{3} - 1}$$

$$2 + \sqrt{3}$$

Your turn A1

Find the exact value of $\cos 105^\circ$

Worked example A2

$\sin A = \frac{3}{5}$, A acute. Find $\sin 2A$ and $\cos 2A$

$$\cos A = \frac{4}{5}; \sin 2A = 2 \sin A \cos A, \cos 2A = \cos^2 A - \sin^2 A$$

$$\frac{24}{25}, \frac{7}{25}$$

Your turn A2

$\cos A = -\frac{5}{13}$, A obtuse. Find $\sin 2A$ and $\cos 2A$

Worked example A3

Solve $\sin 2x = \sin x$ for $0 \leq x < 360^\circ$

$2 \sin x \cos x - \sin x = 0$: factorise, do not divide.

$$\sin x(2 \cos x - 1) = 0$$

$$0^\circ, 60^\circ, 180^\circ, 300^\circ$$

Your turn A3

Solve $\cos 2x = \cos x$ for $0 \leq x < 360^\circ$



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STRETCH

Worked example A4: an R form

Write $3\sin x + 4\cos x$ as $R\sin(x + \alpha)$, $0 < \alpha < 90^\circ$

$$R\cos\alpha = 3, R\sin\alpha = 4: R = \sqrt{3^2 + 4^2}, \tan\alpha = \frac{4}{3}$$

$$5\sin(x + 53.13^\circ)$$

Your turn A4

Write $5\cos x - 12\sin x$ as $R\cos(x + \alpha)$, $0 < \alpha < 90^\circ$

Worked example A5: solving with R

Solve $3\sin x + 4\cos x = 2.5$ for $0 \leq x < 360^\circ$

$$\sin(x + 53.13^\circ) = 0.5, \text{ with } 53.13^\circ \leq x + 53.13^\circ < 413.13^\circ$$

$$x + 53.13^\circ = 150^\circ \text{ or } 390^\circ$$

$$x = 96.9^\circ, 336.9^\circ$$

Your turn A5

Solve $5\cos x - 12\sin x = 6.5$ for $0 \leq x < 360^\circ$

Worked example A6: a maximum

Find the maximum of $3\sin x + 4\cos x$ and the smallest positive x where it occurs.

$5\sin(x + 53.13^\circ)$ is greatest when $x + 53.13^\circ = 90^\circ$

$$5 \text{ at } x = 36.9^\circ$$

Your turn A6

Find the minimum of $5\cos x - 12\sin x$ and the smallest positive x where it occurs.

Worked example A7: a proof

Prove that $\frac{1 - \cos 2x}{\sin 2x} \equiv \tan x$

Choose $\cos 2x = 1 - 2\sin^2 x$ so that the 1 cancels.

$$\frac{2\sin^2 x}{2\sin x \cos x} = \frac{\sin x}{\cos x}$$

$$= \tan x$$

Your turn A7

Prove that $\frac{\sin 2x}{1 + \cos 2x} \equiv \tan x$



B**Fluency**

Each question is a designed sequence. From one part to the next, ask yourself: what changed, and what did it do to the answer?

CORE*Secure the method*

B1 Expand and simplify, giving exact coefficients.

(a) $\sin(x + 30^\circ)$

(b) $\cos(x - 60^\circ)$

(c) $\tan(x + 45^\circ)$

(d) $\sin\left(x - \frac{\pi}{4}\right)$

[4 marks]

B2 Find the exact value of

(a) $\sin 15^\circ$

(b) $\cos 165^\circ$

(c) $\tan 15^\circ$

(d) $\sin 105^\circ$

[4 marks]

B3 Write as a single trigonometric function.

(a) $2 \sin 3x \cos 3x$

(b) $\cos^2 2x - \sin^2 2x$

(c) $1 - 2 \sin^2 \frac{x}{2}$

(d) $\frac{2 \tan 2x}{1 - \tan^2 2x}$

[4 marks]

B4 $\tan A = \frac{3}{4}$ and $\cos B = \frac{5}{13}$, where A and B are acute. Find the exact value of

(a) $\sin(A + B)$

(b) $\cos(A - B)$

(c) $\sin 2A$

(d) $\cos 2B$

[4 marks]



CORE continued: equations and R forms**B5** Solve for $0 \leq x < 360^\circ$

(a) $\cos 2x = \sin x$

(b) $\sin 2x = \cos x$

(c) $\cos 2x + 3 \cos x + 2 = 0$

[3 marks]

B6 Write in the form given, with $R > 0$ and $0 < \alpha < 90^\circ$

(a) $\sin x + \sqrt{3} \cos x = R \sin(x + \alpha)$

(b) $\sqrt{3} \cos x - \sin x = R \cos(x + \alpha)$

(c) $2 \sin x - 2 \cos x = R \sin(x - \alpha)$

(d) $8 \cos x + 15 \sin x = R \cos(x - \alpha)$

[4 marks]

STRETCH**B7** $f(x) = 5 \sin x - 12 \cos x = 13 \sin(x - 67.38^\circ)$. Find**(a)** the maximum of $f(x)$ **(b)** the minimum of $f(x)$ **(c)** the maximum of $20 + \frac{26}{20 + f(x)}$ **(d)** the maximum of $\frac{26}{20 + f(x)}$

[4 marks]

B8 Prove that

(a) $(\sin x + \cos x)^2 \equiv 1 + \sin 2x$

(b) $\cos^4 x - \sin^4 x \equiv \cos 2x$

(c) $\cot x - \tan x \equiv 2 \cot 2x$

(d) $\sin 3x \equiv 3 \sin x - 4 \sin^3 x$

[4 marks]

B9 Solve for $0 \leq x < 360^\circ$

(a) $\tan(x + 45^\circ) = 3$

(b) $\sin(x + 30^\circ) = 2 \cos x$

(c) $\cos(x - 60^\circ) = \sin x$

(d) $2 \sin(x + 60^\circ) = \cos(x + 30^\circ)$

[4 marks]

B10**(a)** Write $\cos^2 x$ in terms of $\cos 2x$ **(b)** Write $\sin^2 x$ in terms of $\cos 2x$ **(c)** Find the exact value of $\cos^2 15^\circ$ **(d)** Find the exact value of $\sin^2 22.5^\circ$

[4 marks]



C**Reasoning**

Show your working and explain your thinking. A correct answer without a reason does not score full marks.

CORE

Secure the method

- C1** Sam writes $\sin 75^\circ = \sin 45^\circ + \sin 30^\circ$
Explain Sam's mistake, and find $\sin 75^\circ$ exactly. [2 marks]
- C2** Lily writes $\cos 2x = 1 - \sin^2 x$
Explain Lily's mistake, and give all three correct forms of $\cos 2x$ [2 marks]
- C3** Kai writes $7 \cos x - 24 \sin x$ as $R \cos(x + \alpha)$ and uses $\tan \alpha = \frac{7}{24}$
Explain Kai's mistake, and find R and α in radians to 3 decimal places. [3 marks]
- C4** $g(x) = \frac{5}{30 + 13 \cos x}$. Jo says the minimum of $g(x)$ occurs when $\cos x = -1$
Explain Jo's mistake, and find the minimum value of $g(x)$ [3 marks]



STRETCH

- C5** Max solves $2 \sin 2x = 3 \sin x$ for $0 \leq x < 360^\circ$ by dividing by $\sin x$, and gives 41.4° and 318.6° .
Explain what Max has lost, and give all the solutions. [3 marks]
- C6** Ali solves $\cos(2x + 30^\circ) = 0.5$ for $0 \leq x < 180^\circ$, writes $2x + 30^\circ = 60^\circ$, and gives $x = 15^\circ$ only.
Explain what Ali has missed, and give all the solutions. [3 marks]
- C7** To prove $\cos 2A \equiv 1 - 2 \sin^2 A$, Ali writes “ $\cos 2A = 1 - 2 \sin^2 A$, and $1 - 2 \sin^2 A = \cos 2A$, so it is true.”
Explain why this is not a proof, and give a correct proof starting from $\cos(A + A)$ [3 marks]
- C8** (a) Starting from the formulae for $\sin(A + B)$ and $\cos(A + B)$, prove that $\tan(A + B) \equiv \frac{\tan A + \tan B}{1 - \tan A \tan B}$ (3)
- (b) Hence write down an identity for $\tan 2A$ (1)
- (c) Hence find the exact value of $\tan \frac{\pi}{12}$, using $\frac{\pi}{12} = \frac{\pi}{3} - \frac{\pi}{4}$ (1)
- [5 marks]



D**Problem solving**

You decide the method. Read the whole question first, and show every step.

CORE

Secure the method

- D1** *In this question you must show all stages of your working. Solutions relying on calculator technology are not acceptable.*

Given that $\sin(x + 30^\circ) = 2 \cos(x + 60^\circ)$, use the addition formulae to show that $\tan x = \frac{\sqrt{3}}{9}$, showing each step of your working clearly and stating any exact values that you use.

[3 marks]

- D2** *In this question you must show all stages of your working. Solutions relying on calculator technology are not acceptable.*

Solve, for $0 < \theta < \pi$, the equation

$$5 \sin 2\theta = 3 \tan \theta$$

giving your answers in radians to 3 significant figures. You must show how you avoid losing any solutions, and explain why $\sin \theta = 0$ gives none in this interval.

[4 marks]

- D3** *In this question you must show all stages of your working. Solutions relying on calculator technology are not acceptable.*

Solve, for $0 \leq x < 360^\circ$, the equation $\cos 2x = 5 \sin x - 2$

You must use a double angle formula to form a quadratic in $\sin x$, and explain why one root of the quadratic gives no solutions.

[3 marks]



- D4** *In this question you must show all stages of your working. Solutions relying on calculator technology are not acceptable.*

The angles A and B are both acute, with $\tan A = \frac{1}{2}$ and $\tan B = \frac{1}{3}$

Using the formula for $\tan(A + B)$, show that $A + B = 45^\circ$, explaining why the value of $A + B$ cannot be 225°

[3 marks]

STRETCH

- D5** *In this question you must show all stages of your working. Solutions relying on calculator technology are not acceptable.*

The angles x and y are such that $\tan x = 2$ and $\tan(x + y) = -1$

(a) Using the formula for $\tan(A + B)$, find the exact value of $\tan y$

(2)

(b) Find the exact value of $\tan 2x$, and hence the exact value of $\tan(2x + y)$

(2)

[4 marks]



D6 In this question you must show all stages of your working. Solutions relying on calculator technology are not acceptable.

(a) Prove that $\frac{\sin 3x}{\sin x} - \frac{\cos 3x}{\cos x} \equiv 2$ for all x where both sides are defined.

(3)

(b) State the values of x in $0 \leq x < 2\pi$ for which the left-hand side is not defined.

(1)

[4 marks]

D7 $f(x) = 6 \sin x \cos x + 8 \cos^2 x + 1$

(a) Write $f(x)$ in the form $a \sin 2x + b \cos 2x + c$, where a , b and c are integers.

(2)

(b) Hence, by writing $a \sin 2x + b \cos 2x$ in the form $R \sin(2x + \alpha)$, state the maximum value of $f(x)$

(1)

(c) Find the smallest positive value of x , in radians to 3 decimal places, at which this maximum occurs.

(1)

[4 marks]

D8 In this question you must show all stages of your working. Solutions relying on calculator technology are not acceptable.

(a) Using double angle formulae twice, show that $\cos 4x \equiv 8 \cos^4 x - 8 \cos^2 x + 1$

(3)

(b) Hence, by using $x = \frac{\pi}{8}$, show that $\cos \frac{\pi}{8} = \frac{\sqrt{2+\sqrt{2}}}{2}$, explaining why you choose the positive roots.

(2)

[5 marks]



E**Exam practice**

Exam-style questions in the style of WMA13. Write your answers in the spaces, and show every step.

CORE

Exam questions for everyone

E1 *In this question you must show all stages of your working. Solutions relying on calculator technology are not acceptable.*

(a) Using a suitable double angle formula, show that the equation

$$3 \cos 2x + 7 \cos x + 5 = 0$$

can be written as $6 \cos^2 x + 7 \cos x + 2 = 0$

(2)

(b) Hence solve, for $0 \leq x < 360^\circ$, the equation $3 \cos 2x + 7 \cos x + 5 = 0$, giving your answers to one decimal place where appropriate.

(3)

(Total for Question E1 is 5 marks)



E2 In this question you must show all stages of your working. Solutions relying on calculator technology are not acceptable.

$$f(x) = 2 \sin x - 5 \cos x$$

(a) Express $f(x)$ in the form $R \sin(x - \alpha)$, where $R > 0$ and $0 < \alpha < 90^\circ$. Give the exact value of R and the value of α to one decimal place.

.....
(3)

(b) Hence solve, for $0 \leq x < 360^\circ$, the equation $f(x) = 3$, giving your answers to one decimal place and showing clearly how you deal with the interval for $x - \alpha$

.....
(3)

(Total for Question E2 is 6 marks)



STRETCH

E3 In this question you must show all stages of your working. Solutions relying on calculator technology are not acceptable.

$$f(\theta) = 4\cos^2\theta - 3\sin 2\theta$$

(a) Show that $f(\theta) = 2 + 2\cos 2\theta - 3\sin 2\theta$

(2)

(b) Write $2\cos 2\theta - 3\sin 2\theta$ in the form $R\cos(2\theta + \alpha)$, where $R > 0$ and $0 < \alpha < \frac{\pi}{2}$, giving the exact value of R and α to 4 decimal places.

(2)

(c) State the maximum value of $f(\theta)$, and find the smallest positive value of θ at which it occurs, to 3 decimal places.

(2)

(d) Without solving it, find the number of solutions of $f(\theta) = 5$ in the interval $0 \leq \theta < \pi$, giving a reason.

(2)

(Total for Question E3 is 8 marks)

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