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INTERNATIONAL A LEVEL MATHEMATICS • PURE 4

Pure Mathematics 4

Exam Questions by Topic

Pearson Edexcel International Advanced Level (WMA14/01) • Calculator allowed

Multi-step exam questions at the difficulty of the real WMA14 paper, with a full mark scheme

Chapters (tap a chapter to jump to it)	Qs	Marks	Page	My score
1 Proof Proof by contradiction: irrational numbers, primes, integers, inequalities	10	73	own PDF	
2 Partial Fractions Linear and repeated factors, improper fractions	10	78	own PDF	
3 Coordinate Geometry in the (x, y) Plane Parametric equations and conversion to Cartesian form	10	76	own PDF	
4 Binomial Expansion $(a + bx)^n$ for rational n , validity, using partial fractions	10	81	own PDF	
5 Differentiation Parametric and implicit differentiation, rates of change	10	81	2	
Integration Volumes of revolution, substitution, parts, partial fractions, differential equations	10	86	own PDF	
7 Vectors 3D vectors, vector equations of lines, intersections, scalar product	10	82	own PDF	

[Full mark scheme](#)

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CHAPTER 5 • KEY SKILLS

Differentiation

Specification P4: Differentiation • 10 questions • 81 marks • about 97 minutes

1 Parametric

$\frac{dy}{dx} = \frac{dy/dt}{dx/dt}$. Use the value of t at the point, not x .

2 Tangents and normals

Normal gradient = $-\frac{1}{m}$. Give the form asked for: $ax + by + c = 0$ needs integers and “= 0”.

3 Implicit

$y^2 \rightarrow 2y \frac{dy}{dx}$; $xy \rightarrow y + x \frac{dy}{dx}$ (product rule); a constant $\rightarrow 0$. Collect the $\frac{dy}{dx}$ terms.

4 Exponentials a^x

$\frac{d}{dx}(a^x) = a^x \ln a$, e.g. $2^x \rightarrow 2^x \ln 2$ (since $2^x = e^{x \ln 2}$)

5 Taking logarithms

For $y = x^{\cos x}$: $\ln y = \cos x \ln x$, then $\frac{1}{y} \frac{dy}{dx} = \dots$ and multiply by y .

6 Stationary and vertical

$\frac{dy}{dx} = 0$: numerator = 0, then use the curve's equation. Vertical tangent: denominator = 0.

7 Connected rates

$\frac{dS}{dt} = \frac{dS}{dr} \times \frac{dr}{dt}$. Quote the formula you use first.

8 Forming equations

Net rate = rate in – rate out.
“Proportional to” needs a constant k .
Similar triangles link r and h .

9 Interpreting

A negative rate means decreasing. A level stops changing when the rate is 0. Give units.

Skills map every skill above is tested at least once; interleaved skills come from GCSE or other chapters

Skill	Q1	Q2	Q3	Q4	Q5	Q6	Q7	Q8	Q9	Q10
Marks	6	6	5	7	6	10	10	10	10	11
1 Parametric		•				•			•	
2 Tangents and normals	•	•			•	•			•	
3 Implicit	•			•	•		•			•
4 Exponentials a^x					•					
5 Taking logarithms										•
6 Stationary and vertical				•			•			•
7 Connected rates			•					•		
8 Forming equations								•		
9 Interpreting			•					•		•
Interleaved: trigonometric identities, double angles (P3)		•							•	
Interleaved: e, ln and numerical methods (P3)					•					•
Interleaved: quadratics, discriminant, coordinate geometry (P1)	•			•		•	•			
Interleaved: proof and parametric curves (Chapters 1, 3)						•	•		•	

Exam difficulty. Questions 1 to 5 check each skill on its own. Questions 6 to 10 are multi-step questions built like the last questions on a real WMA14 paper, where one part leads into the next and skills from different topics are combined.

Must learn. In implicit work every y term gets a $\frac{dy}{dx}$; do not write “ $\frac{dy}{dx} =$ ” at the start of the line. Check tangent or normal, and the form of the answer. For rates, write the chain rule in full before substituting.



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1. The curve C has equation

$$2x^2 + xy - y^3 = 9$$

The point $P(2, 1)$ lies on C .

Find an equation of the normal to C at P , giving your answer in the form $ax + by + c = 0$, where a , b and c are integers.

(6)

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Question 1 continued

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(Total for Question 1 is 6 marks)



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2. The curve C has parametric equations

$$x = 4 \sin^2 t \quad y = 3 \tan t \quad 0 < t < \frac{\pi}{2}$$

(a) Find $\frac{dy}{dx}$ in terms of t , giving your answer in its simplest form.

(3)

(b) Find an equation of the tangent to C at the point where $t = \frac{\pi}{6}$, giving your answer in the form $y = mx + c$, where m and c are exact constants.

(3)

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Question 2 continued

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(Total for Question 2 is 6 marks)



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3. A spherical balloon is being inflated. At time t seconds the radius of the balloon is r cm, its volume is V cm³ and its surface area is S cm², where

$$V = \frac{4}{3}\pi r^3 \quad S = 4\pi r^2$$

The volume of the balloon increases at a constant rate of $50 \text{ cm}^3 \text{ s}^{-1}$.

Find the rate at which the surface area of the balloon is increasing at the instant when $r = 5$

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Question 3 continued

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(Total for Question 3 is 5 marks)**Join students already acing their exams.**Book your FREE lesson at www.eclassroom.org.uk

4. In this question you must show all stages of your working.

Solutions relying on calculator technology are not acceptable.

The curve C has equation

$$x^3 - 3xy + y^2 = k$$

where k is a constant.

(a) Find $\frac{dy}{dx}$ in terms of x and y .

(4)

The point $P(p, 4)$, where $p > 0$, is a stationary point of C .

(b) Find the value of p and the value of k .

(3)

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Question 4 continued

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5. The curve C has equation

$$2^x + xy = y^2 \quad y > 0$$

- (a) Show that $\frac{dy}{dx} = \frac{2^x \ln 2 + y}{2y - x}$

(4)

The point $P(0, 1)$ lies on C . The tangent to C at P meets the x -axis at the point Q .

- (b) Find the exact x coordinate of Q .

(2)

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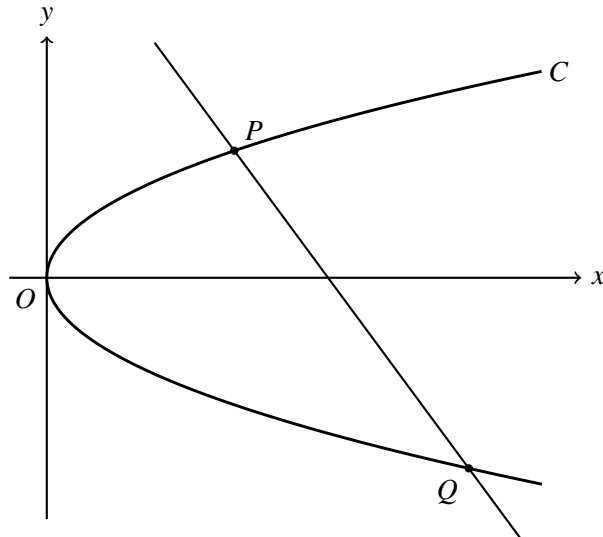


Figure 1

Figure 1 shows a sketch of the curve C with parametric equations

$$x = t^2 \quad y = 2t \quad t \in \mathbb{R}$$

The point P on C has parameter $t = 2$. The normal to C at P meets C again at the point Q .

- (a) Find $\frac{dy}{dx}$ in terms of t . (2)
- (b) Show that the normal to C at P has equation $y = 12 - 2x$ (3)
- (c) Use algebra to find the coordinates of Q . (3)
- (d) Find the area of triangle OPQ , where O is the origin. (2)



Question 6 continued

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Question 6 continued

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(Total for Question 6 is 10 marks)



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7. In this question you must show all stages of your working.
Solutions relying on calculator technology are not acceptable.

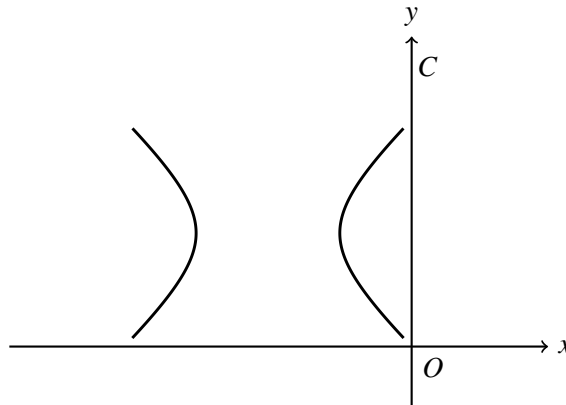
**Figure 2**

Figure 2 shows a sketch of the curve C with equation

$$y^2 - 6y = 3x^2 + 12x$$

The curve is not defined for values of x in the interval $p < x < q$

- (a) Find $\frac{dy}{dx}$ in terms of x and y . (3)
- (b) Using your answer to part (a), or otherwise, find the value of p and the value of q . (4)
- (c) Use proof by contradiction to prove that C has no stationary points. (3)



Question 7 continued

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Question 7 continued

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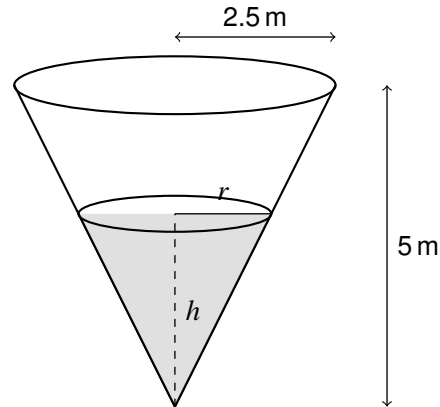
**Figure 3**

Figure 3 shows a container in the shape of an inverted right circular cone, with height 5 m and top radius 2.5 m. At time t minutes, the depth of water in the container is h metres, the radius of the water surface is r metres and the volume of water is $V \text{ m}^3$

- (a) Show that $V = \frac{\pi h^3}{12}$ (2)

Water flows into the container at a constant rate of $2\pi \text{ m}^3$ per minute and leaks out at a rate of $\frac{1}{2}\pi h \text{ m}^3$ per minute.

- (b) Show that $\frac{dh}{dt} = \frac{8-2h}{h^2}$ (4)

- (c) Find the rate at which the depth of water is increasing when $h = 2$ (1)

- (d) Find the depth at which, according to the model, the water level would stop rising. (2)

- (e) Explain whether, according to the model, the container will overflow. (1)



Question 8 continued

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Question 8 continued

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9. In this question you must show all stages of your working.
Solutions relying on calculator technology are not acceptable.

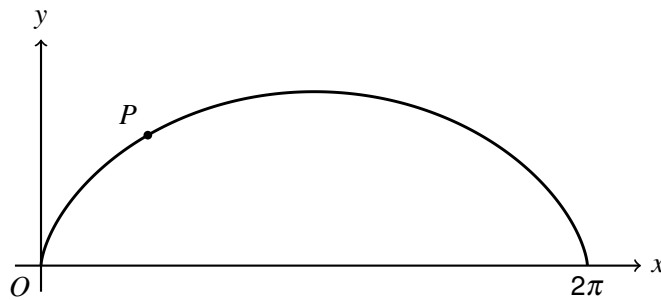


Figure 4

Figure 4 shows a sketch of the curve C with parametric equations

$$x = t - \sin t \quad y = 1 - \cos t \quad 0 \leq t \leq 2\pi$$

- (a) Show that $\frac{dy}{dx} = \frac{\sin t}{1 - \cos t}$, $t \neq 0, 2\pi$ (2)

- (b) Hence show that $\frac{dy}{dx} = \cot\left(\frac{t}{2}\right)$ (2)

The point P on C has parameter $t = \frac{2\pi}{3}$

- (c) Find an equation of the tangent to C at P , giving your answer in the form $y = mx + c$, where m and c are exact. (3)
- (d) Show that, for $0 < t < 2\pi$, the normal to C at the point with parameter t passes through the point $(t, 0)$ (3)



Question 9 continued

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Question 9 continued

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(Total for Question 9 is 10 marks)



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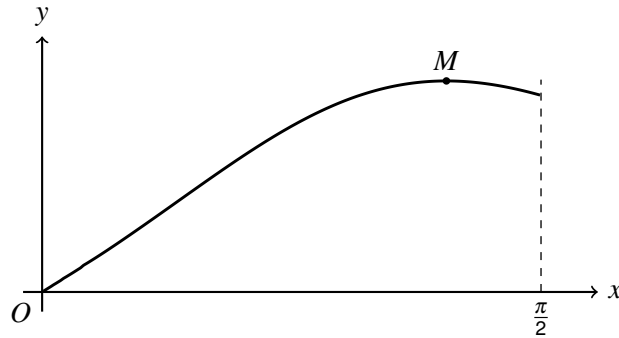


Figure 5

Figure 5 shows a sketch of the curve C with equation

$$y = x^{\cos x} \quad 0 < x < \frac{\pi}{2}$$

The curve has a maximum point M .

- (a) By first taking natural logarithms, show that $\frac{dy}{dx} = x^{\cos x} \left(\frac{\cos x}{x} - \sin x \ln x \right)$ (4)

- (b) Hence show that the x coordinate of M is a solution of the equation $x \tan x \ln x = 1$ (2)

The x coordinate of M is α .

- (c) Show that α lies between 1.2 and 1.3 (2)

- (d) Show that $\alpha = 1.273$ correct to 3 decimal places. (2)

- (e) Hence find the y coordinate of M , giving your answer to 3 significant figures. (1)



Question 10 continued

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Question 10 continued

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(Total for Question 10 is 11 marks)

TOTAL FOR CHAPTER 5 IS 81 MARKS

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