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INTERNATIONAL A LEVEL MATHEMATICS • PURE 3

Pure Mathematics 3

Exam Questions by Topic: Mark Scheme

Pearson Edexcel International Advanced Level (WMA13/01) • Calculator allowed

Every question in the booklet, marked line by line the way an examiner marks it

Chapters (tap a chapter to jump to it)	Qs	Marks	Page	My score
1 Algebraic Methods Algebraic fractions, improper fractions and algebraic division	10	69	own PDF	
2 Functions and Graphs Modulus, composite and inverse functions, transformations	10	76	own PDF	
3 Trigonometric Functions sec, cosec, cot, identities, inverse trigonometric functions	10	72	3	
4 Trigonometric Addition Formulae Addition and double-angle formulae, $R\cos(\theta \pm \alpha)$	10	75	own PDF	
5 Exponentials and Logarithms e^x , $\ln x$, modelling, logarithmic graphs	10	78	own PDF	
6 Differentiation Chain, product and quotient rules, trigonometric and exponential functions	10	76	own PDF	
7 Integration Standard integrals, $f(ax+b)$, trigonometric identities, reverse chain rule	10	77	own PDF	
8 Numerical Methods Locating roots, iteration	10	77	own PDF	

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How to use this mark scheme

The same marking rules Pearson examiners use for WMA13.

Mark every line, not just the answer. Method marks are given for a correct method even when the final answer is wrong. If you lost marks, find the *first* line where your working differs from the scheme. That is the thing to fix.

Code	Meaning
M1	Method mark: for knowing a correct method and attempting to apply it. Small slips in the working do not lose this mark.
dM1	Dependent method mark: can only be given if the method mark before it has been given.
A1	Accuracy mark: for a correct answer or line of working. It can only be given if the method mark it depends on has been given.
B1	Independent mark: for a correct result or statement, whatever method was used.
*	The answer is given in the question. Every step must be shown, and the answer must follow from correct working (cso).
cao	Correct answer only.
cso	Correct solution only: no errors anywhere in the working.
ft	Follow through: the mark can be given for correct work using your own earlier (wrong) answer.
oe	Or equivalent: any correct equivalent form is accepted.

“Solutions relying on calculator technology are not acceptable.” When a question says this, a correct final answer with no algebraic working scores **no marks**. The notes under each scheme line say when this applies.



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Chapter 3: Trigonometric Functions

Mark scheme. Marks are shown on the right of each line.

Question 1

5 marks

Part	Scheme	Marks
(a)	$\tan^2 \theta = \sec^2 \theta - 1 = 8$ $\tan \theta = \pm 2\sqrt{2}$ Obtuse, so $\tan \theta < 0$: $-2\sqrt{2}$	M1 A1 A1 (3)
(b)	$\cos \theta = -\frac{1}{3}$, so $\sin \theta = \frac{2\sqrt{2}}{3}$ (positive for an obtuse angle) $\operatorname{cosec} \theta = \frac{3}{2\sqrt{2}} = \frac{3}{4}\sqrt{2}$	M1 A1 (2)
Total for Question 1: 5 marks		

Question 2

6 marks

Part	Scheme	Marks
	Uses $\operatorname{cosec}^2 x = 1 + \cot^2 x$: $3 \cot^2 x - 4 \cot x - 1 = 0$ Correct quadratic $\cot x = \frac{4 \pm \sqrt{28}}{6} = \frac{2 \pm \sqrt{7}}{3}$ $\tan x = 0.6458 \dots$ or $\tan x = -4.645 \dots$ Two correct solutions $x = 0.573, 1.78, 3.71, 4.92$ <i>Working in degrees, or giving only the principal values, loses the last two marks.</i>	M1 A1 M1 A1 A1 A1 (6)
Total for Question 2: 6 marks		



Question 3

5 marks

Part	Scheme	Marks
(a)	U shape between $x = 0$ and $x = \frac{\pi}{2}$, $\cap$ shape between $\frac{\pi}{2}$ and $\frac{3\pi}{2}$, U shape after Asymptotes $x = \frac{\pi}{2}$ and $x = \frac{3\pi}{2}$ Turning points $(0, 1)$, $(\pi, -1)$, $(2\pi, 1)$	B1 B1 B1 (3)
(b)	$\sec x \geq 1$ gives $f(x) \leq 1$; $\sec x \leq -1$ gives $f(x) \geq 5$ $f(x) \leq 1$ or $f(x) \geq 5$	M1 A1 (2)
Total for Question 3: 5 marks		

Question 4

5 marks

Part	Scheme	Marks
(a)	$\frac{1}{\cos x} - \cos x = \frac{1 - \cos^2 x}{\cos x}$ $= \frac{\sin^2 x}{\cos x}$ $= \sin x \times \frac{\sin x}{\cos x} = \sin x \tan x$ *	M1 A1 A1* (3)
(b)	Similarly $\operatorname{cosec} x - \sin x = \frac{\cos^2 x}{\sin x}$ $\frac{\cos^2 x}{\sin x} \times \frac{\sin^2 x}{\cos x} = \sin x \cos x$ *	M1 A1* (2)
Total for Question 4: 5 marks		



Question 5

5 marks

Part	Scheme	Marks
(a)	Decreasing curve from $(-1, \pi)$ to $(1, 0)$ Crosses the y -axis at $(0, \frac{\pi}{2})$	B1 B1 (2)
(b)	$2x - 1 = \frac{1}{2} \therefore x = \frac{3}{4}$	B1 (1)
(c)	Let $\theta = \arccos x$, so $\cos \theta = x$ with $0 \leq \theta \leq \pi$; $\sin^2 \theta = 1 - x^2$ $\sin \theta \geq 0$ on $[0, \pi]$, so $\sin(\arccos x) = \sqrt{1 - x^2}$ *	M1 A1* (2)
Total for Question 5: 5 marks		

Question 6

9 marks

Part	Scheme	Marks
(a)	$\tan^2 \theta = \sec^2 \theta - 1 = (\sec \theta - 1)(\sec \theta + 1)$ Uses the identity and factorises correctly Cancels $(\sec \theta - 1)$: $\sec \theta + 1$ *	M1 A1 A1* (3)
(b)	$\sec 2x + 1 = 3 \Rightarrow \sec 2x = 2$ $\cos 2x = \frac{1}{2}$ $2x = \frac{\pi}{3}, \frac{5\pi}{3}$ (in $0 < 2x < 2\pi$) $x = \frac{\pi}{6}, \frac{5\pi}{6}$	M1 A1 M1 A1 (4)
(c)	$\sec \theta + 1 = 2$ gives $\sec \theta = 1$ but $\sec \theta \neq 1$ (the denominator $\sec \theta - 1$ would be zero), so there are no solutions	M1 A1 (2)
Total for Question 6: 9 marks		



Question 7

9 marks

Part	Scheme	Marks
(a)	$3(1 + \tan^2 x) - 5 \tan x - 5 = 0$ $3 \tan^2 x - 5 \tan x - 2 = 0$ *	M1 A1* (2)
(b)	$(3 \tan x + 1)(\tan x - 2) = 0$ $\tan x = 2: 63.4^\circ, 243.4^\circ$ $\tan x = -\frac{1}{3}: 161.6^\circ, 341.6^\circ$ $63.4^\circ, 161.6^\circ, 243.4^\circ, 341.6^\circ$	M1 A1 A1 A1 (4)
(c)	$\theta > 0$ means $2\theta - 30^\circ > -30^\circ$, so $2\theta - 30^\circ = -18.43^\circ$ is also a solution Smallest solutions: $\theta = 5.8^\circ$, then $\theta = \frac{63.43^\circ + 30^\circ}{2}$ 46.7° <i>Missing the negative value of $2\theta - 30^\circ$ gives 95.8° as the "second smallest" and scores M0 M1 A0.</i>	M1 M1 A1 (3)
Total for Question 7: 9 marks		



Question 8

9 marks

Part	Scheme	Marks
(a)	Increasing curve through the origin Asymptotes $y = \pm \frac{\pi}{2}$	B1 B1 (2)
(b)	$-\frac{\pi}{2} < \arctan 2x < \frac{\pi}{2}$, so adds $\frac{\pi}{4}$ $-\frac{\pi}{4} < h(x) < \frac{3\pi}{4}$	M1 A1 (2)
(c)	$x = \frac{\pi}{4} + \arctan 2y \Rightarrow 2y = \tan\left(x - \frac{\pi}{4}\right)$ $h^{-1}(x) = \frac{1}{2} \tan\left(x - \frac{\pi}{4}\right)$ Domain $-\frac{\pi}{4} < x < \frac{3\pi}{4}$	M1 A1 B1ft (3)
(d)	$\arctan 2x = \frac{\pi}{4}$, so $2x = 1$ $x = \frac{1}{2}$	M1 A1 (2)
Total for Question 8: 9 marks		



Question 9

9 marks

Part	Scheme	Marks
(a)	$\sin y = x$ with $-\frac{\pi}{2} \leq y \leq \frac{\pi}{2}$, and $\cos\left(\frac{\pi}{2} - y\right) = \sin y = x$ $0 \leq \frac{\pi}{2} - y \leq \pi$, which is the range of $\arccos$ So $\arccos x = \frac{\pi}{2} - y$, giving $\arcsin x + \arccos x = \frac{\pi}{2}$ *	M1 B1 A1* (3)
(b)	$\arcsin x = 2\left(\frac{\pi}{2} - \arcsin x\right)$ $3 \arcsin x = \pi$, so $\arcsin x = \frac{\pi}{3}$ $x = \frac{\sqrt{3}}{2}$	M1 A1 A1 (3)
(c)	$\theta = \arccos \frac{1}{3}$: right-angled triangle with adjacent 1 and hypotenuse 3 Opposite = $\sqrt{9 - 1} = 2\sqrt{2}$ $2\sqrt{2}$	M1 A1 A1 (3)
Total for Question 9: 9 marks		

Question 10

10 marks

Part	Scheme	Marks
(a)	U shape for $0 < x < \pi$ with minimum $\left(\frac{\pi}{2}, 3\right)$; $\cap$ shape for $\pi < x < 2\pi$ with maximum $\left(\frac{3\pi}{2}, 1\right)$ Asymptotes $x = 0, x = \pi, x = 2\pi$ Range $f(x) \leq 1$ or $f(x) \geq 3$	B1 B1 B1 (3)
(b)	$\operatorname{cosec} x = 3$, so $\sin x = \frac{1}{3}$ $x = 0.340$ $x = 0.340, 2.80$	M1 A1 A1 (3)
(c)	Uses the gap in the range between 1 and 3 $1 < k < 3$	M1 A1 (2)
(d)	$f(x) = 2$ is impossible ($\operatorname{cosec} x \neq 0$); $f(x) = -2$ gives $\sin x = -\frac{1}{4}$ 2 solutions, both in $\pi < x < 2\pi$	M1 A1 (2)
Total for Question 10: 10 marks		

