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A-LEVEL MATHEMATICS • MECHANICS 2

Work and Energy

Worksheet

Pearson Edexcel International A Level (WME02) • Calculator allowed: take $g = 9.8\,\text{m s}^{-2}$ and give answers to 2 or 3 s.f.

Free Work and Energy worksheet for A-Level and International A-Level Maths, with full worked solutions

A → **B** → **C** → **D** → **E**

Examples & Your turn — page 2

Fluency core skills — page 4

Reasoning explain it — page 6

Problem solving — page 7

Exam practice — page 10

Tap a section to jump straight to it

By the end of this worksheet you can: find the work done by and against forces • use kinetic and potential energy • apply the work–energy principle on rough surfaces and slopes • use $P = Fv$ for the driving force of an engine • *Stretch:* maximum speeds and accelerations of vehicles, towing, and lifting.

Which questions are for me?

CORE The methods every A-level student must secure. Start here.

STRETCH Rough slopes both ways, towing, journeys with height and speed changes, and lifting at constant power.

Every section has both, clearly labelled. Questions get harder as you go, and every skill in Sections A to D is needed in Section E.

Track your progress

Section	B	C	D	E
Core marks	23	10	13	19
Stretch marks	16	14	17	14
My score				

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A Worked examples & Your turn

Read each example carefully. Cover it up, then answer the question beside it using the same method.

CORE *Secure the method*

Worked example A1: work done

A box is pulled 12 m along a horizontal floor by a rope inclined at 30° above the horizontal. The tension is 40 N. Find the work done by the tension.

Only the component along the motion does work

$40\cos 30° \times 12$

416 J

Your turn A1

A sledge is pulled 15 m along horizontal snow by a rope inclined at 25° above the horizontal. The tension is 60 N. Find the work done by the tension.

Worked example A2: energy

A ball of mass 2 kg is thrown vertically upwards at $14\,\text{m s}^{-1}$. Find its kinetic energy at the start and its speed when it is 5 m higher.

KE lost = PE gained = mgh

$\frac{1}{2}(2)(14^2) = 196;\ 196 - 2(9.8)(5) = 98 = \frac{1}{2}(2)v^2$

$196\,\text{J},\ 9.90\,\text{m s}^{-1}$

Your turn A2

A particle of mass 3 kg is released from rest and slides down a smooth slope through a vertical height of 2.5 m. Find its speed at the bottom.

Worked example A3: driving force

A car of mass 1200 kg moves on a level road against a resistance of 1000 N. Its engine works at 24 kW. Find the driving force and the acceleration when its speed is $15\,\text{m s}^{-1}$

$P = Fv$, then $F - R = ma$

$F = \dfrac{24000}{15} = 1600;\ 1600 - 1000 = 1200a$

$1600\,\text{N},\ 0.5\,\text{m s}^{-2}$

Your turn A3

A van of mass 1500 kg moves on a level road against a resistance of 900 N. Its engine works at 30 kW. Find the driving force and the acceleration when its speed is $20\,\text{m s}^{-1}$

STRETCH

Worked example A4: a rough surface

A 4 kg block slides on rough horizontal ground at $6\,\text{m s}^{-1}$, $\mu = 0.3$. Find how far it slides before stopping.

Work against friction = KE lost

$F = 0.3(4)(9.8) = 11.76$; $11.76d = \frac{1}{2}(4)(6^2)$

6.12 m

Your turn A4

A 5 kg block slides on rough horizontal ground at $8\,\text{m s}^{-1}$, $\mu = 0.25$. Find how far it slides before stopping.

Worked example A5: a rough slope

A 2 kg particle is projected at $10\,\text{m s}^{-1}$ up a line of greatest slope of a rough plane inclined at α, where $\sin\alpha = \frac{3}{5}$, $\mu = 0.5$. Find how far it moves up the plane.

KE lost = PE gained + work against friction

$100 = 2(9.8)(0.6)d + 0.5(2)(9.8)(0.8)d$

5.10 m

Your turn A5

A 3 kg particle is released from rest on a rough plane inclined at α, where $\sin\alpha = \frac{5}{13}$, $\mu = 0.2$. Find its speed after it has slid 4 m down the plane.

Worked example A6: maximum speed

A 900 kg car has maximum power 36 kW and resistance 1200 N. Find its maximum speed on a level road, and up a hill with $\sin\alpha = \frac{1}{20}$

At top speed $a = 0$: driving force = total resistance

$\frac{36000}{1200}$; $\frac{36000}{1200+441}$

$30\,\text{m s}^{-1}$, $21.9\,\text{m s}^{-1}$

Your turn A6

A 1200 kg car has maximum power 45 kW and resistance 1800 N. Find its maximum speed on a level road, and up a hill with $\sin\alpha = \frac{1}{15}$

Worked example A7: acceleration on a slope

A 4000 kg lorry moves up a hill with $\sin\alpha = \frac{1}{40}$ against a resistance of 2000 N, its engine at 60 kW. Find its acceleration at $12\,\text{m s}^{-1}$

$5000 - 2000 - 980 = 4000a$

$0.505\,\text{m s}^{-2}$

Your turn A7

A 2500 kg van moves *down* a hill with $\sin\alpha = \frac{1}{25}$ against a resistance of 3000 N, its engine at 50 kW. Find its acceleration at $20\,\text{m s}^{-1}$

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B Fluency

Each question is a designed sequence. From one part to the next, ask yourself: what changed, and what did it do to the answer?

CORE *Secure the method*

B1 A box is moved 6 m along a straight line. Find the work done

(a) by a 50 N force along the direction of motion

(b) by a 50 N force at 60° to the direction of motion

(c) against a friction force of 20 N

(d) against gravity in lifting a 5 kg box vertically through 6 m

[4 marks]

B2 A particle has mass 1.5 kg.

(a) Find its KE at $4\,\mathrm{m\,s^{-1}}$

(b) Find its KE at $8\,\mathrm{m\,s^{-1}}$

(c) Find the PE it gains when raised 3 m

(d) Find its speed after falling 3 m from rest.

[4 marks]

B3 An engine works at a constant rate.

(a) Find the power when the driving force is 600 N at $15\,\mathrm{m\,s^{-1}}$

(b) Find the driving force when the power is 12 kW at $20\,\mathrm{m\,s^{-1}}$

(c) A car moves at constant speed against 800 N with its engine at 20 kW. Find its speed.

(d) The same engine works at 20 kW at $16\,\mathrm{m\,s^{-1}}$. Find the driving force.

[4 marks]

B4 A block of mass 2 kg moves at $6\,\mathrm{m\,s^{-1}}$ on rough horizontal ground, $\mu = 0.25$

(a) Find the friction force.

(b) Find the work done against friction in 2 m

(c) Find its speed after 2 m

(d) Find the distance it travels before stopping.

[4 marks]

B5 A car of mass 1000 kg has maximum power 40 kW, and the resistance from non-gravitational forces is 800 N. Find its maximum speed

(a) on a level road.

(b) up a road inclined at α, where $\sin\alpha = \frac{1}{49}$

(c) down the same road.

[3 marks]

B6 A van of mass 1500 kg moves on a level road at a constant power of 30 kW, against a resistance of 1000 N. Find its acceleration when its speed is

(a) $10\,\text{m}\,\text{s}^{-1}$

(b) $20\,\text{m}\,\text{s}^{-1}$

(c) $30\,\text{m}\,\text{s}^{-1}$

(d) $25\,\text{m}\,\text{s}^{-1}$

[4 marks]

STRETCH

B7 A particle of mass 4 kg is on a rough plane inclined at 30° to the horizontal, $\mu = 0.4$

(a) Find the normal reaction.

(b) Find the friction force when it moves.

(c) Find the total work done against friction and gravity in moving it 3 m up the plane.

(d) Find the least speed of projection up the plane for it to travel 3 m

[4 marks]

B8 A cyclist and her bicycle have total mass 70 kg. She rides 50 m along a straight road against a constant resistance of 30 N, and her speed increases from $4\,\text{m}\,\text{s}^{-1}$ to $10\,\text{m}\,\text{s}^{-1}$ in 8 seconds.

(a) The road is level. Find the work she does.

(b) Find her average rate of working.

(c) The road instead rises 5 m over the 50 m. Find the work she does.

(d) Find her average rate of working now.

[4 marks]

B9 A ball of mass 2 kg is released from rest at A, 5 m above the ground, and slides down a curved ramp.

(a) The ramp is smooth. Find its speed at the bottom.

(b) Find its speed when it is 1.8 m above the ground.

(c) It is projected from A at $3\,\text{m}\,\text{s}^{-1}$ instead. Find its speed at the bottom.

(d) The ramp is rough and it reaches the bottom at $8\,\text{m}\,\text{s}^{-1}$ from rest. Find the work done against resistance.

[4 marks]

B10 A car of mass 900 kg tows a trailer of mass 300 kg along a level road. The resistances to the car and the trailer are 400 N and 200 N. The engine works at 24 kW.

(a) Find the driving force at $15\,\text{m}\,\text{s}^{-1}$

(b) Find the acceleration at $15\,\text{m}\,\text{s}^{-1}$

(c) Find the tension in the towbar at $15\,\text{m}\,\text{s}^{-1}$

(d) Find the maximum speed.

[4 marks]

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C Reasoning

Show your working and explain your thinking. A correct answer without a reason does not score full marks.

CORE *Secure the method*

C1 A 2 kg particle speeds up from $3\,\text{m s}^{-1}$ to $5\,\text{m s}^{-1}$. Amir writes the gain in kinetic energy as $\frac{1}{2}(2)(5-3)^2 = 4\,\text{J}$. Explain his error, and find the gain. **[2 marks]**

C2 A car's engine works at 30 kW at $15\,\text{m s}^{-1}$. Bea writes the driving force as $\dfrac{30}{15} = 2\,\text{N}$. Explain her error, and find the driving force. **[2 marks]**

C3 A 5 kg box is pulled at constant speed 6 m up a rough plane inclined at 30°, against a friction force of 10 N. Cal writes the work done as $mgh + (mg\sin 30^\circ)(6) + 10(6) = 354\,\text{J}$.

Explain his error, and find the work done. **[3 marks]**

C4 A car's engine works at 20 kW. At the instant its speed is $10\,\text{m s}^{-1}$ it is accelerating at $0.3\,\text{m s}^{-2}$ on a level road; its mass is 1000 kg. Dev says the resistance is $\dfrac{20000}{10} = 2000\,\text{N}$.

Explain his error, and find the resistance. **[3 marks]**

STRETCH

C5 A 2 kg block is on a rough plane inclined at α, where $\tan\alpha = \frac{3}{4}$, and $\mu = 0.5$. Eve writes $R = 2g\sin\alpha$ and finds the friction as 5.88 N.

Explain her error, and find the friction. **[3 marks]**

C6 A 3 kg particle A on a smooth table is joined by a light string over a smooth pulley at the edge to a 2 kg particle B hanging freely. They are released from rest. Fay finds the speed after B falls 1.5 m from $\frac{1}{2}(2)v^2 = 2(9.8)(1.5)$

Explain her error, and find the speed. **[3 marks]**

C7 A car moves up a hill at a constant $25\,\text{m s}^{-1}$ for 40 seconds with its engine working at 30 kW. Gus says that, since its speed does not change, the engine does no work.

Explain his error, and find the work done by the engine. **[3 marks]**

C8 A car of mass m has an engine with maximum power P. The resistance from non-gravitational forces is a constant R

(a) Show that its maximum speed up a road inclined at α is $v = \dfrac{P}{R + mg\sin\alpha}$

(2)

(b) Show that, on a level road and at this speed v, working at power P, the acceleration of the car is $g\sin\alpha$

(3)

[5 marks]

D Problem solving

You decide the method. Read the whole question first, and show every step.

CORE *Secure the method*

D1 A fixed rough plane is inclined to the horizontal at an angle α, where $\tan\alpha = \frac{3}{4}$. A box P of mass 3 kg is pushed, with constant speed, a distance of 8 m up a line of greatest slope of the plane by a force acting parallel to the plane, as shown in Figure 1. The coefficient of friction between P and the plane is $\frac{1}{4}$. The box is modelled as a particle and air resistance is negligible.

Use the model to find the total work done in pushing P the 8 m up the plane.

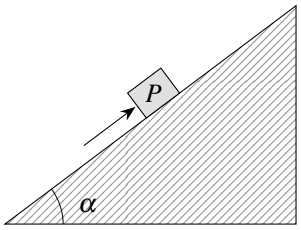


Figure 1

[3 marks]

D2 A truck of mass 2000 kg is moving on a straight horizontal road. The engine of the truck is working at a constant rate of 40 kW, and the resistance to its motion is modelled as a constant force of magnitude R newtons. At the instant when its speed is 16 m s^{-1}, its acceleration is 0.5 m s^{-2}

(a) Find the value of R

(2)

Later the truck moves up a straight road inclined at α to the horizontal, where $\sin\alpha = \frac{1}{14}$. The resistance from non-gravitational forces is still R newtons and the engine still works at 40 kW.

(b) Find the maximum speed of the truck up this road.

(2)

[4 marks]

D3 A rough ramp is fixed to horizontal ground and inclined at α, where $\sin\alpha = 0.28$. The points A and B lie on a line of greatest slope, with B above A and $AB = 5$ m, as shown in Figure 2. A block of mass 2 kg is released from rest at B and slides down to A. The coefficient of friction is 0.15

(a) Find the work done against friction as the block moves from B to A

(1)

(b) Use the work–energy principle to find the speed of the block at A

(2)

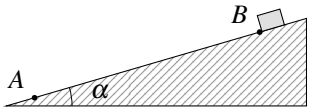


Figure 2

[3 marks]

D4 A car of mass 1200 kg moves up a straight road inclined at α to the horizontal, where $\sin\alpha = \frac{1}{12}$. The resistance to motion from non-gravitational forces is 600 N, as shown in Figure 3. At the instant when the speed of the car is 25 m s^{-1}, its engine is working at a rate of 10 kW.

Find the deceleration of the car at this instant.

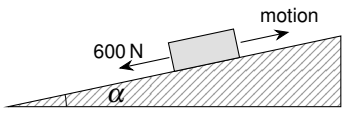


Figure 3 (not to scale)

[3 marks]

STRETCH

D5 A van of mass 1100 kg tows a trailer of mass 400 kg up a straight road inclined at α, where $\sin\alpha = \frac{1}{20}$, as shown in Figure 4. The towbar is light and parallel to the road. The resistances from non-gravitational forces are 300 N on the van and 150 N on the trailer. The engine of the van works at 18 kW.

(a) Find the acceleration of the van at the instant when its speed is 12 m s^{-1}

(2)

(b) Find the tension in the towbar at this instant.

(2)

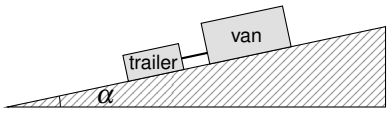


Figure 4 (not to scale)

[4 marks]

D6 A cyclist rides from rest at a point A to a point B along a straight road of length 2 km. The point B is 40 m higher than A, and the cyclist arrives at B with speed 6 m s^{-1}. The total mass of the cyclist and her bicycle is 75 kg, and the resistance to her motion from non-gravitational forces is modelled as a constant force of magnitude 25 N.

(a) Using the model, find the total work done by the cyclist in riding from A to B

(3)

(b) The journey from A to B takes 400 seconds. Find her average rate of working.

(1)

[4 marks]

D7 A car of mass 800 kg passes the top of a hill at 10 m s^{-1} with its engine switched off. It moves 200 m down a straight road inclined at α, where $\sin\alpha = \frac{1}{20}$, against a constant resistance from non-gravitational forces of 300 N.

(a) Use the work–energy principle to find its speed after it has moved 200 m

(3)

(b) Find the resistance for which the car would instead move down this road at constant speed.

(1)

[4 marks]

D8 A rough ramp is inclined at θ to the horizontal, where $\tan\theta = \frac{7}{24}$. A package of mass 1.2 kg is projected up a line of greatest slope from the point A with speed $U\,\text{m s}^{-1}$ and first comes to instantaneous rest at B, where $AB = 3\,\text{m}$, as shown in Figure 5. The coefficient of friction is 0.25

(a) Find the work done against friction from A to B

(1)

(b) Use the work–energy principle to find U

(2)

(c) Show that the package slides back down, and find its speed when it returns to A

(2)

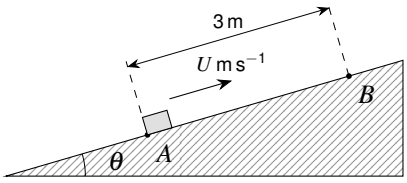


Figure 5

[5 marks]

E Exam practice

Full-size exam questions in the style of WME02. Write your answers in the spaces, and show every step.

CORE *Exam questions for everyone*

E1 A straight road is inclined to the horizontal at an angle α, where $\sin\alpha = \frac{1}{18}$. A car of mass 900 kg moves up a line of greatest slope of the road. The car is modelled as a particle, and the road is modelled as a straight line in a vertical plane. The total resistance to the motion of the car from non-gravitational forces is modelled as a constant force of magnitude 300 N. The engine of the car is working at a constant rate of 15 kW.

(a) Find the constant speed, in m s^{-1}, at which the car could move up the road with its engine working at this rate.

(3)

(b) Find the acceleration of the car at the instant when it is moving up the road at $12\,\text{m s}^{-1}$

(3)

At this instant, the car is at the point A and the engine of the car is switched off. The total resistance to the motion of the car from non-gravitational forces remains a constant force of magnitude 300 N.

Using the work–energy principle,

(c) find how far up the road from A the car would travel before it first comes to rest. You may assume that the car does not reach the top of the hill before it comes to rest.

(4)

(Total for Question E1 is 10 marks)

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E2 A skier of mass 60 kg, together with her skis, starts from rest at the point A on a straight slope and skis down a line of greatest slope to the point B at the bottom of the slope. She does not use her ski poles to push herself along at any point of the motion. The point A is 20 m vertically above B. From B she continues along a straight horizontal stretch of snow BC, as shown in Figure 6. The skier is modelled as a particle.

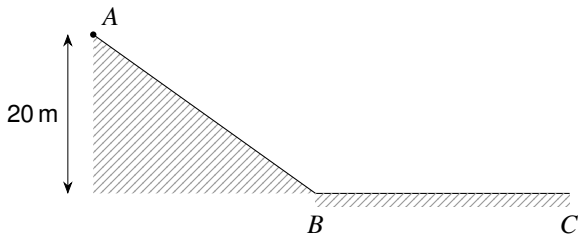


Figure 6 (not to scale)

(a) Assuming that the slope is smooth and that there is no air resistance, find the speed of the skier at B

(2)

In fact the skier reaches B with speed $15\,\text{m}\,\text{s}^{-1}$

(b) Find the total work done against the resistance to her motion as she moves from A to B

(2)

The coefficient of friction between the skier and the horizontal snow is 0.1, and all other resistances are ignored on BC

(c) Use the work–energy principle to find the distance she travels along BC before coming to rest.

(3)

(d) The length of the slope AB is 50 m. Assuming that the total resistance to the motion of the skier on AB is a constant force acting parallel to the slope, find the magnitude of this resistance.

(2)

(Total for Question E2 is 9 marks)

STRETCH

E3 A crane lifts a load of mass 500 kg vertically upwards, using a light vertical cable, as shown in Figure 7. The resistance to the motion of the load from non-gravitational forces is modelled as a constant force of magnitude 300 N. The load is modelled as a particle.

The crane works at a constant rate of 12 kW.

(a) Find the greatest possible speed at which the load could be lifted.

(2)

(b) Find the acceleration of the load at the instant when it is rising at $1.5\,\text{m s}^{-1}$

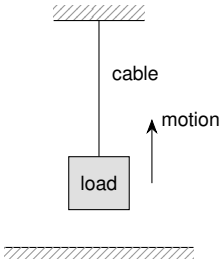


Figure 7

(3)

On another occasion the crane raises the load from rest through a vertical distance of 10 m, and the load is then rising at $2\,\text{m s}^{-1}$. The lift takes 6 seconds, and the rate at which the crane works is not constant.

(c) Find the average rate at which the crane works during this lift.

(4)

At the instant when the load is rising at $2\,\text{m s}^{-1}$, the cable is released so that it no longer exerts a force on the load. The resistance from non-gravitational forces is still 300 N, and it always opposes the motion.

Using the work–energy principle,

(d) find the further height through which the load rises,

(3)

(e) find the speed of the load when it has fallen back to the height at which the cable was released.

(2)

(Total for Question E3 is 14 marks)

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