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A-LEVEL MATHEMATICS • FURTHER PURE 1

Roots of Quadratic Equations

Worksheet

Pearson Edexcel International A Level (WFM01) • Calculator allowed: show every step

Free Roots of Quadratic Equations worksheet for A-Level and International A-Level Maths, with full worked solutions

A **Examples** & Your turn page 2

B **Fluency** core skills page 4

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Tap a section to jump straight to it

By the end of this worksheet you can: use the sum and product of the roots • evaluate expressions in α and β • form equations with transformed roots • *Stretch:* unknown coefficients, roots related to another equation, and harder transformed roots, all without solving.

Which questions are for me?

CORE The methods every A-level student must secure. Start here.

STRETCH Unknown coefficients, roots given as reciprocals, and new equations whose roots are not simple shifts.

Every section has both, clearly labelled. Questions get harder as you go, and every skill in Sections A to D is needed in Section E.

Track your progress

Section	B	C	D	E
Core marks	23	10	13	13
Stretch marks	16	14	17	9
My score				

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A Worked examples & Your turn

Read each example carefully. Cover it up, then answer the question beside it using the same method.

CORE *Secure the method*

Worked example A1: sum and product

$3x^2 - 7x + 2 = 0$ has roots α and β. Write down $\alpha + \beta$ and $\alpha\beta$

$\alpha + \beta = -\frac{b}{a}$, $\alpha\beta = \frac{c}{a}$: watch the sign of the sum

$\frac{7}{3}, \frac{2}{3}$

Your turn A1

$5x^2 + 4x - 3 = 0$ has roots α and β. Write down $\alpha + \beta$ and $\alpha\beta$

Worked example A2: the sum of squares

$x^2 - 5x + 3 = 0$ has roots α and β. Find $\alpha^2 + \beta^2$ without solving the equation.

$\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$

$5^2 - 2(3)$

19

Your turn A2

$2x^2 + 3x - 4 = 0$ has roots α and β. Find $\alpha^2 + \beta^2$ without solving the equation.

Worked example A3: shifted roots

$x^2 - 4x + 1 = 0$ has roots α and β. Find the equation with roots $\alpha + 2$ and $\beta + 2$

Sum: $4 + 4 = 8$; product: $\alpha\beta + 2(\alpha + \beta) + 4 = 13$

$x^2 - 8x + 13 = 0$

Your turn A3

$x^2 + 3x - 5 = 0$ has roots α and β. Find the equation with roots $\alpha - 1$ and $\beta - 1$

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STRETCH

Worked example A4: the sum of cubes

$x^2 - 3x - 2 = 0$ has roots α and β. Find $\alpha^3 + \beta^3$

$\alpha^3 + \beta^3 = (\alpha + \beta)^3 - 3\alpha\beta(\alpha + \beta)$

$27 - 3(-2)(3)$

45

Your turn A4

$3x^2 - x + 2 = 0$ has roots α and β. Find $\alpha^3 + \beta^3$

Worked example A5: reciprocal roots

$2x^2 + 5x - 3 = 0$ has roots α and β. Find the equation, with integer coefficients, with roots $\frac{1}{\alpha}$ and $\frac{1}{\beta}$

Sum: $\frac{\alpha + \beta}{\alpha\beta} = \frac{-5/2}{-3/2} = \frac{5}{3}$; product: $\frac{1}{\alpha\beta} = -\frac{2}{3}$

$3x^2 - 5x - 2 = 0$

Your turn A5

$4x^2 - 3x + 6 = 0$ has roots α and β. Find the equation, with integer coefficients, with roots $\frac{1}{\alpha}$ and $\frac{1}{\beta}$

Worked example A6: the difference of the roots

$x^2 - 6x + 4 = 0$ has roots $\alpha > \beta$. Find $\alpha - \beta$ without solving.

$(\alpha - \beta)^2 = (\alpha + \beta)^2 - 4\alpha\beta$

$36 - 16 = 20$

$2\sqrt{5}$

Your turn A6

$2x^2 - 8x + 3 = 0$ has roots $\alpha > \beta$. Find $\alpha - \beta$ without solving.

Worked example A7: an unknown coefficient

The roots of $x^2 - 9x + k = 0$ differ by 3. Find k

$(\alpha - \beta)^2 = 81 - 4k = 9$

$k = 18$

Your turn A7

One root of $x^2 + kx + 12 = 0$ is three times the other. Find the possible values of k

B Fluency

Each question is a designed sequence. From one part to the next, ask yourself: what changed, and what did it do to the answer?

CORE *Secure the method*

B1 $2x^2 - 6x + 1 = 0$ has roots α and β. Without solving the equation, find

(a) $\alpha + \beta$

(b) $\alpha\beta$

(c) $\alpha^2 + \beta^2$

(d) $\dfrac{1}{\alpha} + \dfrac{1}{\beta}$

[4 marks]

B2 $x^2 + 4x - 2 = 0$ has roots α and β. Without solving the equation, find

(a) $\alpha^2 + \beta^2$

(b) $\alpha^3 + \beta^3$

(c) $\alpha^2\beta + \alpha\beta^2$

(d) $\dfrac{1}{\alpha^2} + \dfrac{1}{\beta^2}$

[4 marks]

B3 $x^2 - 5x + 2 = 0$ has roots α and β. Find the equation, with integer coefficients, whose roots are

(a) 3α and 3β

(b) $\alpha + 1$ and $\beta + 1$

(c) α^2 and β^2

(d) $\dfrac{1}{\alpha}$ and $\dfrac{1}{\beta}$

[4 marks]

B4 Write down a quadratic equation, with integer coefficients, whose roots have

(a) sum 5 and product 6

(b) sum $-\dfrac{3}{2}$ and product $\dfrac{1}{4}$

(c) sum 0 and product -7

(d) sum $\dfrac{2}{3}$ and product $-\dfrac{5}{9}$

[4 marks]

B5 $3x^2 + x - 4 = 0$ has roots α and β. Without solving the equation, find

(a) $\alpha + \beta$ and $\alpha\beta$

(b) $(\alpha - \beta)^2$

(c) $\alpha^2 - \beta^2$, given that $\alpha > \beta$

[3 marks]

B6 $2x^2 + x - 5 = 0$ has roots α and β. Without solving the equation, find

(a) $\dfrac{\alpha}{\beta} + \dfrac{\beta}{\alpha}$

(b) $(\alpha + 1)(\beta + 1)$

(c) $\alpha^2\beta^2$

(d) $(2\alpha - 1)(2\beta - 1)$

[4 marks]

STRETCH

B7 $x^2+3x+1=0$ has roots α and β. Find the equation, with integer coefficients, whose roots are

(a) α^3 and β^3

(b) $\frac{\alpha}{\beta}$ and $\frac{\beta}{\alpha}$

(c) $\alpha+2\beta$ and $\beta+2\alpha$

(d) $\alpha^2\beta$ and $\alpha\beta^2$

[4 marks]

B8 $x^2-6x+k=0$ has roots α and β. Find k when

(a) the roots differ by 4

(b) one root is twice the other

(c) $\alpha^2+\beta^2=30$

(d) $\frac{1}{\alpha}+\frac{1}{\beta}=3$

[4 marks]

B9 $2x^2-kx+3=0$ has roots α and β

(a) Write down $\alpha+\beta$

(b) Write down $\alpha\beta$

(c) Find $\alpha^2+\beta^2$ in terms of k

(d) Find the positive value of k for which $\alpha^2+\beta^2=1$

[4 marks]

B10 $x^2-3x-1=0$ has roots α and β

(a) Find the equation with roots $2\alpha-1$ and $2\beta-1$

(b) Check part (a) by substituting $x=\frac{y+1}{2}$

(c) Find the equation with roots α^2+1 and β^2+1

(d) Find the equation with roots $\alpha-\beta$ and $\beta-\alpha$

[4 marks]

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C Reasoning

Show your working and explain your thinking. A correct answer without a reason does not score full marks.

CORE *Secure the method*

C1 $3x^2 + 6x - 1 = 0$ has roots α and β. Amir writes $\alpha + \beta = 2$. Explain his error, and give $\alpha + \beta$ **[2 marks]**

C2 $x^2 - 4x + 1 = 0$ has roots α and β. Bea writes $\alpha^2 + \beta^2 = (\alpha + \beta)^2 = 16$. Explain her error, and find $\alpha^2 + \beta^2$ **[2 marks]**

C3 $x^2 - x - 4 = 0$ has roots α and β. For the equation with roots $\alpha + 3$ and $\beta + 3$, Cal finds the sum 7 and the product $\alpha\beta + 9 = 5$

Explain his error, and find the equation. **[3 marks]**

C4 Dev is asked for a quadratic equation, with integer coefficients, whose roots have sum $\frac{5}{2}$ and product $\frac{1}{2}$.

He writes $x^2 - \frac{5}{2}x + \frac{1}{2}$

Explain why his answer is incomplete, and give the required equation. **[3 marks]**

STRETCH

C5 The equation $3x^2 - 6x + 2 = 0$ has roots $\frac{1}{p}$ and $\frac{1}{q}$. Eve says that $p + q = 2$

Explain her error, and find $p + q$ and pq **[3 marks]**

C6 $x^2 - 2x - 3 = 0$ has roots α and β. Fay writes $\alpha^3 + \beta^3 = (\alpha + \beta)^3 - 3\alpha\beta = 8 + 9 = 17$

Explain her error, and find $\alpha^3 + \beta^3$ **[3 marks]**

C7 A question asks for $\alpha^3 + \beta^3$, without solving the equation, where α and β are the roots of $x^2 - 4x + 1 = 0$. Gus solves the equation on his calculator and gives $\alpha^3 + \beta^3 \approx 52.0$

Explain why this would not score full marks, and find the exact value correctly. **[3 marks]**

C8 $ax^2 + bx + c = 0$, where $a \neq 0$, has roots α and β

(a) Show that $(\alpha - \beta)^2 = \frac{b^2 - 4ac}{a^2}$

(2)

(b) Hence show that the roots are equal if and only if $b^2 = 4ac$, and that, when the roots are real, they differ by $\frac{\sqrt{b^2 - 4ac}}{|a|}$

(3)

[5 marks]

D Problem solving

You decide the method. Read the whole question first, and show every step.

CORE *Secure the method*

D1 The quadratic equation $4x^2 - 3x - 5 = 0$ has roots α and β, where α and β are real. Without solving the equation,

(a) write down the value of $\alpha + \beta$ and the value of $\alpha\beta$

(1)

(b) determine the exact value of $\alpha^2 + \beta^2$, giving your answer as a fraction in its simplest form.

(2)

[3 marks]

D2 The equation $2x^2 + kx - 6 = 0$, where k is a constant, has roots α and β. Given that $\alpha^2 + \beta^2 = 10$, and without solving the equation $2x^2 + kx - 6 = 0$ at any stage,

(a) determine the possible values of k

(2)

(b) determine the exact value of $\frac{1}{\alpha} + \frac{1}{\beta}$ when k takes the positive value found in part (a).

(2)

[4 marks]

D3 The quadratic equation $x^2 + kx + 2 = 0$, where k is a constant, has roots α and β. Without solving the equation, and showing your working clearly, determine, in simplest form in terms of k, the value of $\frac{\alpha^2}{\beta} + \frac{\beta^2}{\alpha}$, writing your answer as a single fraction. **[3 marks]**

D4 The quadratic equation $3x^2 - 4x + 2 = 0$ has roots α and β. Without solving the equation, and giving each answer as an exact fraction in its simplest form, determine the value of

(a) $\alpha^2 + \beta^2$

(1)

(b) $\alpha^3 + \beta^3$, using the identity $\alpha^3 + \beta^3 = (\alpha + \beta)^3 - 3\alpha\beta(\alpha + \beta)$

(2)

[3 marks]

STRETCH

D5 The quadratic equation $x^2 - 3x + 5 = 0$ has roots α and β. Without solving the equation, find a quadratic equation which has roots $(2\alpha - \beta)$ and $(2\beta - \alpha)$, giving your answer in the form $px^2 + qx + r = 0$, where p, q and r are integers to be determined. **[4 marks]**

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D6 The quadratic equation $Ax^2 - 2x - 3 = 0$, where A is a constant, has roots α and β. The equation $x^2 - 6x + B = 0$, where B is a constant, has roots $(\alpha + 2\beta)$ and $(\beta + 2\alpha)$

Without solving either equation,

(a) determine the value of A, using the sum of the roots of each equation

(2)

(b) determine the value of B, using the product of the roots of the second equation

(2)

[4 marks]

D7 The quadratic equation $2x^2 + 3x + k = 0$, where k is a non-zero integer, has roots α and β. Without solving the equation, determine a quadratic equation which has roots $\frac{\alpha}{\beta^2}$ and $\frac{\beta}{\alpha^2}$, giving your answer in the form $px^2 + qx + r = 0$, where p, q and r are integers in terms of k **[4 marks]**

D8 The equation $4x^2 - 5x + 3 = 0$ has roots $\frac{1}{p}$ and $\frac{1}{q}$

(a) Without solving the equation, show that $pq = \frac{4}{3}$, and determine the value of $p + q$

(2)

(b) Hence, without finding the values of p and q, determine a quadratic equation which has roots p^2 and q^2, giving your answer in the form $ax^2 + bx + c = 0$, where a, b and c are integers.

(3)

[5 marks]

E Exam practice

Exam-style questions in the style of WFM01. Write your answers in the spaces, and show every step.

CORE *Exam questions for everyone*

E1 The quadratic equation $3x^2 - 5x + 4 = 0$ has roots α and β

Without solving the equation,

(a) write down the value of $(\alpha + \beta)$ and the value of $\alpha\beta$

(1)

(b) determine the value of $\alpha^2 + \beta^2$

(2)

(c) determine a quadratic equation which has roots $\alpha^2\beta$ and $\alpha\beta^2$, giving your answer in the form $px^2 + qx + r = 0$, where p, q and r are integers to be determined.

(3)

(Total for Question E1 is 6 marks)

E2 The quadratic equation $x^2 - 4x + 7 = 0$ has roots α and β

Without solving the equation,

(a) write down the value of $(\alpha + \beta)$ and the value of $\alpha\beta$

(1)

(b) determine the value of $\alpha^2 + \beta^2$

(2)

(c) find a quadratic equation which has roots $\left(\alpha + \frac{1}{\alpha^2}\right)$ and $\left(\beta + \frac{1}{\beta^2}\right)$, giving your answer in the form $px^2 + qx + r = 0$, where p, q and r are integers to be determined.

(4)

(Total for Question E2 is 7 marks)

STRETCH

E3 $f(x) = 2x^2 + px + 5$, where p is a positive constant. The equation $f(x) = 0$ has roots α and β

(a) By first expanding $(\alpha + \beta)^3$, prove that $\alpha^3 + \beta^3 = (\alpha + \beta)^3 - 3\alpha\beta(\alpha + \beta)$

(2)

Given that $\alpha^2 + \beta^2 = -1$

(b) determine the value of p

(2)

(c) explain why the roots of $f(x) = 0$ cannot be real.

(1)

Without solving the equation $f(x) = 0$

(d) find a quadratic equation which has roots $(\alpha^3 + 1)$ and $(\beta^3 + 1)$, giving your answer in the form $ax^2 + bx + c = 0$, where a, b and c are integers to be determined.

(4)

(Total for Question E3 is 9 marks)

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