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INTERNATIONAL A LEVEL MATHEMATICS • DECISION MATHEMATICS 1

Decision 1

Exam Questions by Topic

Pearson Edexcel International Advanced Level (WDM11/01) • Calculator allowed • Answer book pages included

Multi-step exam questions at the difficulty of the real WDM11 paper, with a full mark scheme

Chapters (tap a chapter to jump to it)		Qs	Marks	Page	My score
1	**Algorithms** Flow charts, bubble sort, quick sort, bin packing, binary search	10	80	2	
2	**Graphs and Networks** Vocabulary, types of graph, degrees, adjacency and distance matrices	10	80	28	
3	**Algorithms on Graphs** Kruskal, Prim (networks and matrices), Dijkstra	10	80	54	
4	**Route Inspection** Eulerian graphs, pairing odd vertices, start and finish points	10	80	80	
5	**The Travelling Salesman Problem** Classical and practical problems, upper and lower bounds	10	80	106	
6	**Critical Path Analysis** Activity networks, event times, floats, Gantt charts, scheduling	10	80	132	
7	**Linear Programming** Formulation, feasible regions, objective lines, integer solutions	10	80	158	

Full mark scheme Free courses More worksheets

Name ..... Class ..... Date .....

CHAPTER 1 • KEY SKILLS

1 Algorithms

Specification D1: Algorithms • 10 questions • 80 marks • about 96 minutes

1 Tracing algorithms

Work through the instructions in order, recording every variable in a table. Write the final answer on the output line.

2 Flow charts

Ovals start and stop, rectangles are instructions, diamonds are decisions. Follow each loop back to exactly the box it points to.

3 Bubble sort

Compare adjacent pairs, swapping when out of order. After each pass one more item is fixed; finish with a pass that makes no swaps.

4 Quick sort

Pivot is the middle item (middle-right for an even number). Keep items in their original order either side; give every sublist a pivot.

5 Order asked for

Sort into the order the question states (ascending or descending): the other order loses marks.

6 Binary search

Middle item at position $\left\lceil \frac{\text{first+last}}{2} \right\rceil$, rounded up. Discard the half that cannot contain the target, including the middle.

7 Bin packing

First fit: each item into the first bin it fits. First-fit decreasing: sort first. Full bin: look for combinations that fill bins.

8 Lower bound

Total ÷ bin size, rounded up. A packing that uses this many bins is optimal.

9 Deductions

Every item that went into (or did not fit) a bin gives an inequality for the bin size.

Skills map every skill above is tested at least once; interleaved skills come from GCSE or other chapters

Skill	Q1	Q2	Q3	Q4	Q5	Q6	Q7	Q8	Q9	Q10
Marks	6	6	6	6	6	10	10	10	10	10
1 Tracing algorithms				●				●		
2 Flow charts								●		
3 Bubble sort	●									●
4 Quick sort		●				●			●	
5 Order asked for	●	●				●			●	●
6 Binary search					●				●	
7 Bin packing			●			●	●			●
8 Lower bound			●			●	●			●
9 Deductions							●			
Interleaved: integer division and remainders (GCSE)				●				●		
Interleaved: inequalities (P1)							●			
Interleaved: alphabetical ordering					●				●	

Exam difficulty. Questions 1 to 5 check each skill on its own. Questions 6 to 10 are multi-step questions built like the last questions on a real WDM11 paper, where one part leads into the next.

Answer book. After each question is a copy of the page the real D1 answer book prints for it (lists, tables, answer lines), so you can practise using the answer book exactly as in the exam.

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1.

19 7 25 12 30 4 16 21

(a) Use a bubble sort, starting at the left-hand end of the list, to sort the numbers into **descending** order. You need only give the state of the list after each pass.

(4)

(b) State the number of comparisons and the number of swaps made in the first pass.

(1)

(c) State the maximum number of passes a bubble sort could need for a list of 8 numbers.

(1)

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Question 1 continued **D1 ANSWER BOOK**

In the exam this page is printed in the separate D1 answer book. Write your working here, on the copy provided.

19	7	25	12	30	4	16	21

19	7	25	12	30	4	16	21

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(Total for Question 1 is 6 marks)

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2.

23 8 41 15 32 11 27 6 19 36

(a) Use a quick sort to sort the list into **ascending** order. You must make your pivots clear.

(5)

(b) Explain how you know that your sort is complete.

(1)

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Question 2 continued **D1 ANSWER BOOK**

In the exam this page is printed in the separate D1 answer book. Write your working here, on the copy provided.

23	8	41	15	32	11	27	6	19	36

23	8	41	15	32	11	27	6	19	36

DO NOT WRITE IN THIS AREA

(Total for Question 2 is 6 marks)

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3.

8 15 4 12 9 6 11 7 5

The numbers in the list are the weights, in kg, of nine parcels. They are to be packed into crates that each hold up to 20 kg.

(a) Calculate a lower bound for the number of crates needed. You must make your method clear.

(2)

(b) Use the first-fit bin packing algorithm to allocate the parcels to crates, in the order given.

(3)

(c) Explain why your answer to part (b) is optimal.

(1)

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Question 3 continued **D1 ANSWER BOOK**

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(Total for Question 3 is 6 marks)

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4. An algorithm is described below.

1. Input positive integers A and B
2. Let $C = 0$
3. If B is odd, replace C with $C + A$
4. Replace A with $2A$ and replace B with INT $\left(\frac{B}{2}\right)$
5. If $B > 0$, go to step 3
6. Output C and stop

INT(x) means the integer part of x, so INT(5.5) = 5

(a) Implement the algorithm with $A = 37$ and $B = 22$, completing the table in the answer book.

(5)

(b) State the purpose of the algorithm.

(1)

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Question 4 continued **D1 ANSWER BOOK**

In the exam this page is printed in the separate D1 answer book. Write your working here, on the copy provided.

A	B	Is B odd?	C

Output: _____

(Total for Question 4 is 6 marks)

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5. The list below is in alphabetical order.

Adams, Begum, Chen, Diaz, Evans, Fofana, Garcia, Haddad, Ito, Kapoor, Lopez

(a) Use the binary search algorithm to locate the name Kapoor in the list. You must make your method clear.

(4)

(b) Use the binary search algorithm to show that the name Baker is not in the list.

(2)

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Question 5 continued D1 ANSWER BOOK

In the exam this page is printed in the separate D1 answer book. Write your working here, on the copy provided.

Position	Name
1	Adams
2	Begum
3	Chen
4	Diaz
5	Evans
6	Fofana
7	Garcia
8	Haddad
9	Ito
10	Kapoor
11	Lopez

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(Total for Question 5 is 6 marks)

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6.

18 31 12 26 9 22 35 14 27 7 20 16

The list shows the lengths, in minutes, of twelve video clips. They are to be stored on discs that each hold up to 50 minutes.

(a) Use a quick sort to sort the list into **descending** order. You must make your pivots clear.

(4)

(b) Apply the first-fit decreasing bin packing algorithm to your sorted list.

(3)

(c) Calculate a lower bound for the number of discs needed, and hence explain whether your packing in part (b) is optimal.

(2)

(d) Explain why first-fit decreasing usually gives a better packing than first fit.

(1)

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Question 6 continued D1 ANSWER BOOK

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18	31	12	26	9	22	35	14	27	7	20	16

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Question 6 continued

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(Total for Question 6 is 10 marks)

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7.

24 13 19 8 15 11 6

The list shows the sizes, in GB, of seven files. They are copied, in the order given, onto storage cards that each hold n GB, where n is an integer. The first-fit bin packing algorithm gives this allocation.

Card 1: 24 13 Card 2: 19 8 11 Card 3: 15 6

(a) Using the allocation, explain why $n \geqslant 38$

(2)

(b) Find the greatest possible value of n, giving reasons.

(3)

For the rest of this question, $n = 40$

(c) Apply the first-fit decreasing algorithm to the files.

(3)

(d) Calculate a lower bound for the number of cards, and comment on your answer to part (c).

(2)

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Question 7 continued **D1 ANSWER BOOK**

In the exam this page is printed in the separate D1 answer book. Write your working here, on the copy provided.

24	13	19	8	15	11	6

Possible values of n: _____

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Question 7 continued

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(Total for Question 7 is 10 marks)

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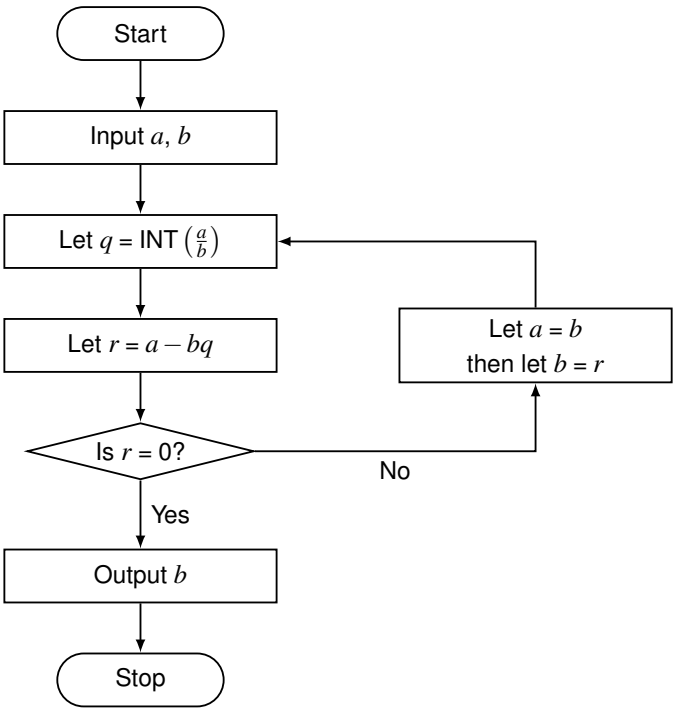

Figure 1

Figure 1 shows a flow chart. INT(x) means the integer part of x.

(a) Implement the algorithm with $a = 168$ and $b = 120$, completing the table in the answer book and stating the output.

(5)

(b) State the purpose of the algorithm.

(1)

(c) Implement the algorithm with $a = 120$ and $b = 168$, and comment on how this run differs from part (a).

(2)

(d) Without completing a new table, state the output when $a = 91$ and $b = 35$, showing the values of r that are found.

(2)

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Question 8 continued **D1 ANSWER BOOK**

In the exam this page is printed in the separate D1 answer book. Write your working here, on the copy provided.

a	b	q	r	Is $r = 0$?

Output: _____

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Question 8 continued

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(Total for Question 8 is 10 marks)

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9.

Okafor, Bell, Singh, Moreau, Tanaka, Diallo, Kowalski, Hughes, Rossi

(a) Use a quick sort to sort the names into alphabetical order. You must make your pivots clear.

(5)

(b) Use the binary search algorithm on your sorted list to locate the name Rossi.

(3)

(c) Use the binary search algorithm to show that the name Patel is not in the list.

(2)

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Question 9 continued D1 ANSWER BOOK

In the exam this page is printed in the separate D1 answer book. Write your working here, on the copy provided.

Okafor	Bell	Singh	Moreau	Tanaka	Diallo	Kowalski	Hughes	Rossi

Okafor	Bell	Singh	Moreau	Tanaka	Diallo	Kowalski	Hughes	Rossi

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Question 9 continued

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(Total for Question 9 is 10 marks)

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10.

14 9 21 6 17 11 25 8 19 4

The list shows the masses, in kg, of ten bags of sand.

(a) Use a bubble sort, starting at the left-hand end, to sort the masses into **descending** order. Give the list after each pass, and the number of comparisons and swaps in each pass, using the table in the answer book.

(5)

(b) Find the maximum number of comparisons that a bubble sort could need for a list of ten values.

(1)

The bags are to be loaded, **in the order given in the original list**, onto trolleys that each carry up to 30 kg.

(c) Use the first-fit bin packing algorithm to allocate the bags to trolleys.

(3)

(d) Calculate a lower bound for the number of trolleys, and comment on your answer to part (c).

(1)

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Question 10 continued D1 ANSWER BOOK

In the exam this page is printed in the separate D1 answer book. Write your working here, on the copy provided.

14	9	21	6	17	11	25	8	19	4

Pass	Comparisons	Swaps
1		
2		
3		
4		
5		
6		
7		
8		
9		

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Question 10 continued

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(Total for Question 10 is 10 marks)

TOTAL FOR CHAPTER 1 IS 80 MARKS

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CHAPTER 2 • KEY SKILLS

2 Graphs and Networks

Specification D1: Graphs and networks • 10 questions • 80 marks • about 96 minutes

1 Terminology

Walk: any route. Trail: no arc repeated. Path: no vertex repeated. Cycle: a closed path. Hamiltonian cycle: visits every vertex once.

2 Degree

The degree (valency) of a vertex is the number of arcs meeting it. A loop adds 2.

3 Handshaking lemma

Sum of degrees = 2× number of arcs, so there is always an even number of odd vertices.

4 Simple and connected

Simple: no loops, at most one arc per pair. Connected: there is a path between every pair of vertices.

5 Trees

A tree is connected with no cycles; with n vertices it has $n-1$ arcs. A spanning tree includes every vertex.

6 Special graphs

K_n has $\frac{1}{2}n(n-1)$ arcs. Bipartite: two sets, arcs only between sets. $K_{r,s}$ has rs arcs.

7 Matrices

Adjacency: number of arcs between each pair (2 for a double arc). Distance: weights, with – for no direct arc. A digraph's matrix need not be symmetric.

8 Isomorphic graphs

Same structure drawn differently: match degrees, then give a full vertex correspondence that maps every arc.

9 Give reasons

When asked "explain" or "give a reason", write a sentence using the precise term.

Skills map every skill above is tested at least once; interleaved skills come from GCSE or other chapters

Skill	Q1	Q2	Q3	Q4	Q5	Q6	Q7	Q8	Q9	Q10
Marks	6	6	6	6	6	10	10	10	10	10
1 Terminology	•			•				•	•	
2 Degree	•		•		•	•	•	•		•
3 Handshaking lemma	•		•			•	•	•		•
4 Simple and connected		•				•	•			
5 Trees			•				•		•	•
6 Special graphs			•			•			•	
7 Matrices		•		•				•		•
8 Isomorphic graphs					•					
9 Give reasons	•	•	•		•	•	•	•	•	•
Interleaved: combinations and factorials (P1, S1)			•			•			•	
Interleaved: proof by contradiction (P4)			•				•			
Interleaved: odd vertices and route inspection (Chapter 4)	•									•

Exam difficulty. Questions 1 to 5 check each skill on its own. Questions 6 to 10 are multi-step questions built like the last questions on a real WDM11 paper, where one part leads into the next.

Answer book. After each question is a copy of the page the real D1 answer book prints for it (reprinted matrices, vertex layouts to draw on, tables), so you can practise using the answer book exactly as in the exam.

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1.

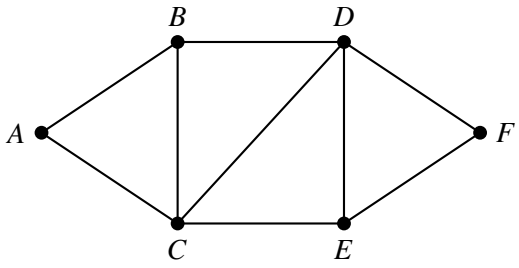

Figure 1

Figure 1 shows a graph G.

(a) Write down the degree of each vertex of G.

(2)

(b) Use your answer to part (a) to find the number of arcs in G, stating the result you use.

(2)

(c) Write down a path from A to F that uses exactly three arcs, and a Hamiltonian cycle starting at A.

(2)

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Question 1 continued D1 ANSWER BOOK

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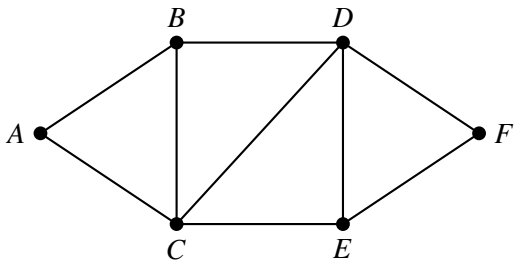

Diagram 1

Vertex	A	B	C	D	E	F
Degree						

(Total for Question 1 is 6 marks)

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2. The adjacency matrix below represents a graph H.

	P	Q	R	S	T
P	0	1	0	2	1
Q	1	0	1	0	0
R	0	1	0	1	1
S	2	0	1	0	0
T	1	0	1	0	0

(a) Draw H on the vertices given in the answer book.

(3)

(b) State, with a reason, whether H is simple, and whether H is connected.

(2)

(c) Write down the number of arcs in H.

(1)

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Question 2 continued D1 ANSWER BOOK

In the exam this page is printed in the separate D1 answer book. Write your working here, on the copy provided.

	P	Q	R	S	T
P	0	1	0	2	1
Q	1	0	1	0	0
R	0	1	0	1	1
S	2	0	1	0	0
T	1	0	1	0	0

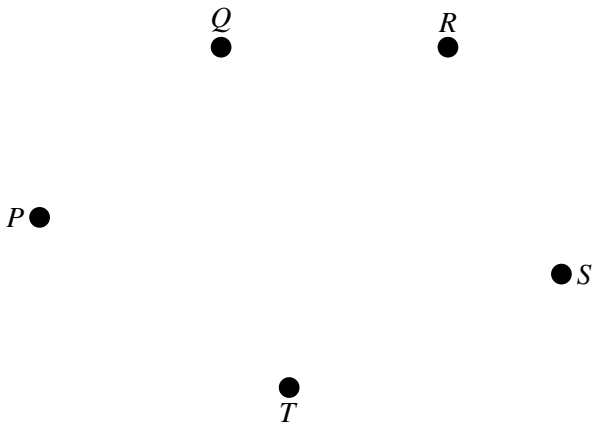

Diagram 2

(Total for Question 2 is 6 marks)

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3. **(a)** Write down the number of arcs in the complete graph K_7

(1)

(b) Draw the complete bipartite graph $K_{3,4}$, and state its number of arcs.

(2)

(c) Explain why there is no graph whose vertices have degrees 3, 3, 2, 2 and 1

(2)

(d) Write down the minimum number of arcs in a connected graph with 9 vertices.

(1)

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Question 3 continued D1 ANSWER BOOK

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Diagram 3

(Total for Question 3 is 6 marks)

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4. The table shows the lengths, in km, of the roads directly joining five villages. A dash means there is no direct road.

	A	B	C	D	E
A	–	7	–	5	9
B	7	–	4	–	6
C	–	4	–	8	–
D	5	–	8	–	3
E	9	6	–	3	–

(a) Draw the network on the vertices given in the answer book, labelling each arc with its length.

(3)

(b) Find the total length of the roads.

(1)

(c) List the two paths from A to C that use exactly two arcs, and state which is shorter.

(2)

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Question 4 continued D1 ANSWER BOOK

In the exam this page is printed in the separate D1 answer book. Write your working here, on the copy provided.

	A	B	C	D	E
A	–	7	–	5	9
B	7	–	4	–	6
C	–	4	–	8	–
D	5	–	8	–	3
E	9	6	–	3	–

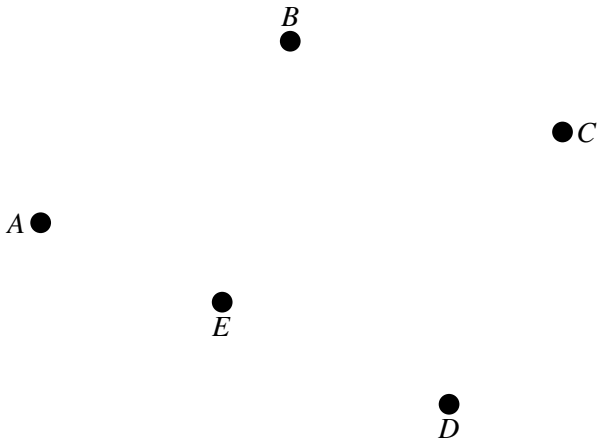

Diagram 4

(Total for Question 4 is 6 marks)

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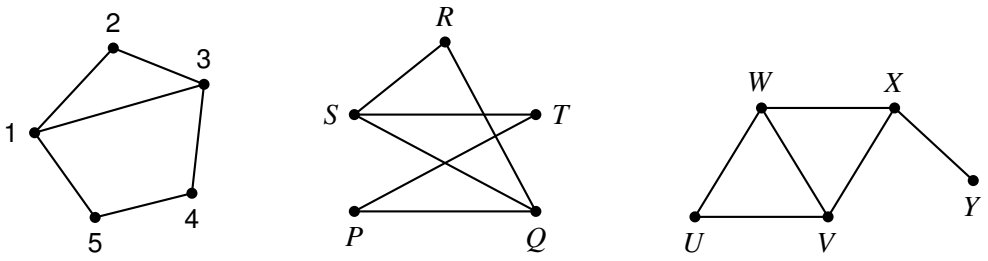
5.



Graph X Graph Y Graph Z

Figure 2

Figure 2 shows three graphs, X, Y and Z.

(a) Show that X and Y are isomorphic, by giving a correspondence between their vertices.

(4)

(b) Explain why Z is not isomorphic to X.

(2)

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Question 5 continued **D1 ANSWER BOOK**

In the exam this page is printed in the separate D1 answer book. Write your working here, on the copy provided.

Vertex of $X \rightarrow$ vertex of Y				
1	2	3	4	5

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(Total for Question 5 is 6 marks)

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6. Four volunteers, A, B, C and D, are to be given jobs from the list 1, 2, 3, 4, 5.

A can do jobs 1 and 3; B can do jobs 2, 3 and 5; C can do job 1 only; D can do jobs 1 and 4

(a) Draw a bipartite graph to model this information.

(2)

(b) Write down the number of arcs, and the sum of the degrees of the vertices.

(2)

(c) Explain why job 1 must be given to C if every volunteer is to have a different job, and hence find such an allocation.

(3)

(d) State, with a reason, whether your bipartite graph is connected.

(1)

(e) Find how many more arcs would be needed to make the graph $K_{4,5}$

(2)

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Question 6 continued D1 ANSWER BOOK

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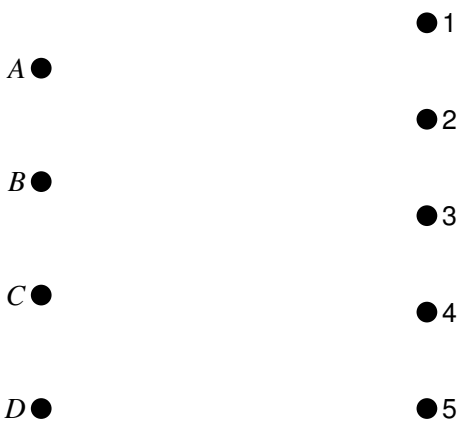

Diagram 5

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Question 6 continued

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(Total for Question 6 is 10 marks)

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7. **(a)** Prove that it is impossible for the five vertices of any graph to have degrees 1, 2, 3, 3 and 4

(2)

(b) Draw a simple graph with five vertices of degrees 2, 2, 3, 3 and 4, and state its number of arcs.

(3)

(c) Degrees 4, 4, 4, 1, 1 have an even total. Show that, even so, they cannot belong to a **simple** graph on five vertices.

(3)

(d) A tree has 6 vertices. Write down the number of arcs, and the sum of the degrees of its vertices.

(2)

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Question 7 continued D1 ANSWER BOOK

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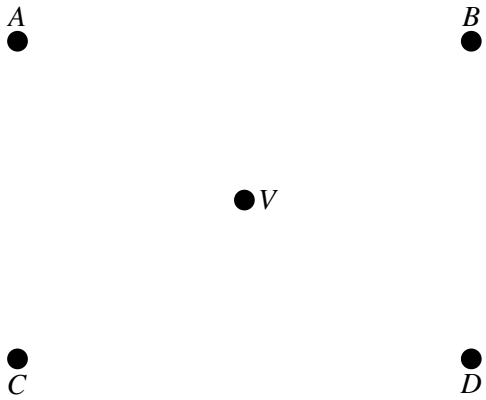

Diagram 6

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Question 7 continued

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(Total for Question 7 is 10 marks)

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8. Five junctions, A to E, are joined by one-way streets. An entry of 1 in the matrix means there is a street from the junction in that row to the junction in that column.

from \ to	A	B	C	D	E
A	0	1	1	0	0
B	0	0	1	1	0
C	0	0	0	1	1
D	1	0	0	0	1
E	0	1	0	0	0

(a) Draw the digraph on the vertices given in the answer book.

(3)

(b) Explain how the matrix shows that the streets are one-way.

(1)

(c) Complete the table of out-degrees and in-degrees, and use it to find the number of streets.

(3)

(d) List all the routes from A to E that use at most three streets and do not revisit a junction.

(3)

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Question 8 continued D1 ANSWER BOOK

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from \ to	A	B	C	D	E
A	0	1	1	0	0
B	0	0	1	1	0
C	0	0	0	1	1
D	1	0	0	0	1
E	0	1	0	0	0

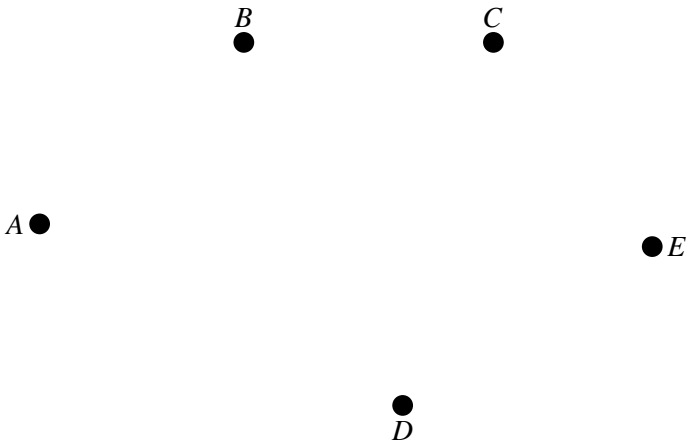

Diagram 7

Vertex	A	B	C	D	E
Out-degree					
In-degree					

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Question 8 continued

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(Total for Question 8 is 10 marks)

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9. **(a)** For the complete graph K_5, write down the number of arcs and the number of arcs in any spanning tree, and show that there are 12 different Hamiltonian cycles (a cycle and its reverse counted as the same).

(4)

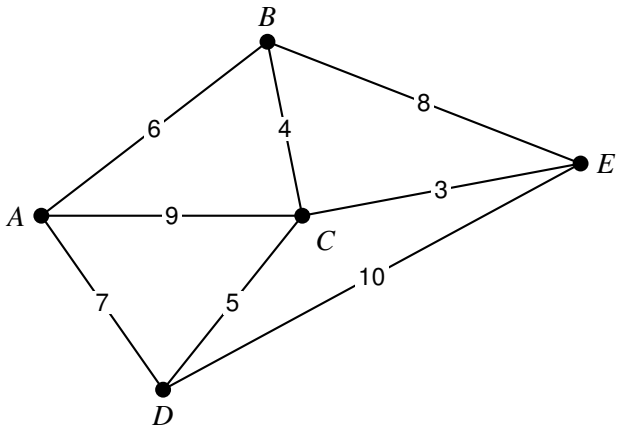

Figure 3

Figure 3 shows a network of paths between five huts. The number on each arc is the length of the path, in hundreds of metres.

(b) Find every Hamiltonian cycle starting and finishing at A, giving the length of each, and state the shortest.

(4)

(c) A new hut F is joined to the network by a single path, from E. Explain why the new network has no Hamiltonian cycle.

(2)

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Question 9 continued D1 ANSWER BOOK

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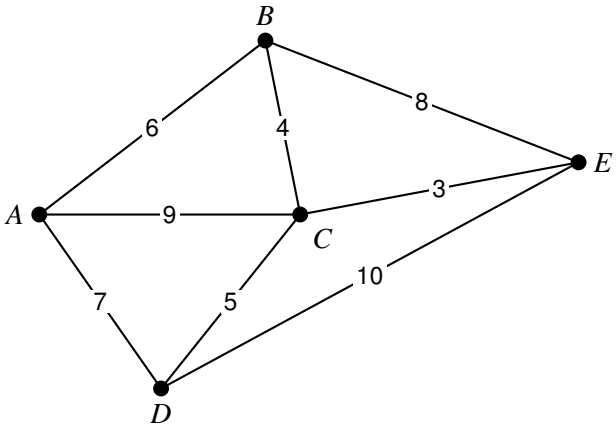

Diagram 8

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Question 9 continued

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(Total for Question 9 is 10 marks)

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10. The table gives the lengths, in metres, of the corridors in a building, joining six rooms A to F.

	A	B	C	D	E	F
A	–	5	8	–	–	6
B	5	–	3	7	–	–
C	8	3	–	4	9	–
D	–	7	4	–	2	5
E	–	–	9	2	–	6
F	6	–	–	5	6	–

(a) Draw the network on the vertices given in the answer book.

(3)

(b) Complete the table of degrees, and use the handshaking lemma to check the number of corridors.

(2)

(c) Explain why it is not possible to walk along every corridor exactly once and return to the start. State the least number of corridors that would need to be repeated, and suggest which.

(3)

(d) Find the total length of the corridors, and the number of corridors that must be removed to leave a spanning tree.

(2)

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Question 10 continued D1 ANSWER BOOK

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	A	B	C	D	E	F
A	–	5	8	–	–	6
B	5	–	3	7	–	–
C	8	3	–	4	9	–
D	–	7	4	–	2	5
E	–	–	9	2	–	6
F	6	–	–	5	6	–

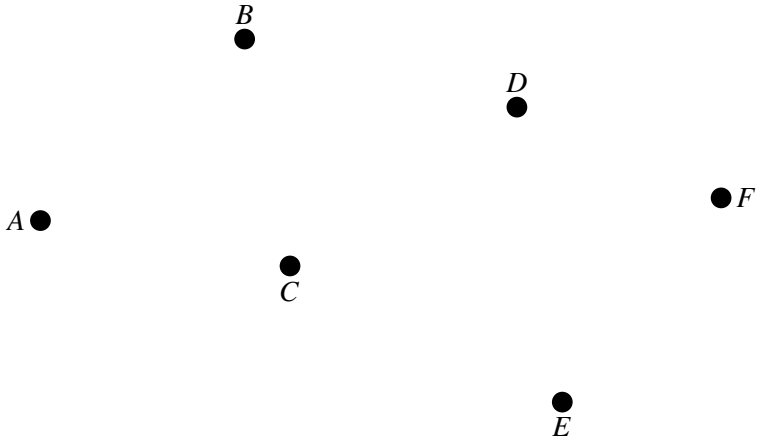

Diagram 9

Vertex	A	B	C	D	E	F
Degree						

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Question 10 continued

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(Total for Question 10 is 10 marks)

TOTAL FOR CHAPTER 2 IS 80 MARKS

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CHAPTER 3 • KEY SKILLS

3 Algorithms on Graphs

Specification D1: Algorithms on graphs • 10 questions • 80 marks • about 96 minutes

1 Kruskal

List arcs in increasing weight. Accept an arc unless it makes a cycle; state each one accepted or rejected. Stop at $n-1$ arcs.

2 Prim on a network

Start at the vertex given. Repeatedly add the shortest arc joining a new vertex to the tree.

3 Prim on a matrix

Cross out the start row, number its column, choose the least entry in the numbered columns, and list arcs in the order chosen.

4 Weight of the tree

Add only the accepted arcs. A tree on n vertices has $n-1$ arcs.

5 Equal weights

When weights are equal, different choices can give different minimum spanning trees with the same weight.

6 Dijkstra

Give the start order 1, final value 0. From each newly labelled vertex, update working values; label the smallest unlabelled value next.

7 Working values

Write every working value at a vertex in the order found, and only when it improves the current value.

8 Tracing the route

Work back from the end: an arc XY is on the route if final$(Y)-$final(X) = weight of XY

9 Using final values

Final values give shortest distances from the start to every vertex: use them for follow-up questions.

Skills map every skill above is tested at least once; interleaved skills come from GCSE or other chapters

Skill	Q1	Q2	Q3	Q4	Q5	Q6	Q7	Q8	Q9	Q10
Marks	6	6	6	6	6	10	10	10	10	10
1 Kruskal	•				•		•	•		•
2 Prim on a network		•								
3 Prim on a matrix			•				•			
4 Weight of the tree	•	•	•		•		•	•		•
5 Equal weights					•			•		
6 Dijkstra				•		•			•	•
7 Working values				•		•			•	•
8 Tracing the route				•		•			•	•
9 Using final values						•			•	•
Interleaved: graph terminology: trees and cycles (Chapter 2)	•	•			•			•		•
Interleaved: inequalities (P1)								•		
Interleaved: distance matrices (Chapter 2)			•				•			

Exam difficulty. Questions 1 to 5 check each skill on its own. Questions 6 to 10 are multi-step questions built like the last questions on a real WDM11 paper, where one part leads into the next.

Answer book. After each question is a copy of the page the real D1 answer book prints for it: networks with Dijkstra boxes and a key, reprinted matrices, and answer lines.

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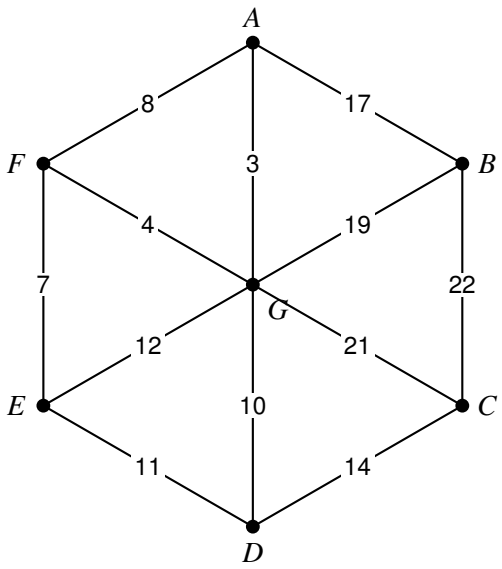

Figure 1

Figure 1 shows six villages, A to F, arranged around a central town G, and the possible cable routes between them. The number on each arc is the cost, in thousands of dollars, of laying that cable.

(a) Use Kruskal's algorithm to find a minimum spanning tree. List the arcs in the order you consider them, stating whether each is included or rejected.

(4)

(b) Draw the minimum spanning tree on the diagram in the answer book.

(1)

(c) State the minimum cost of connecting all seven places.

(1)

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Question 1 continued D1 ANSWER BOOK

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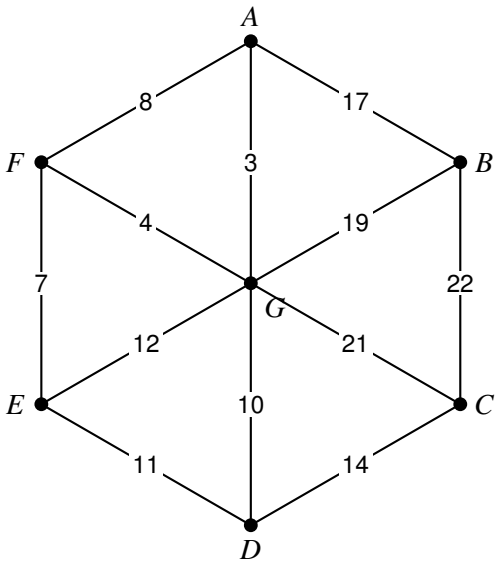

Diagram 1

Weight of the minimum spanning tree: _____

(Total for Question 1 is 6 marks)

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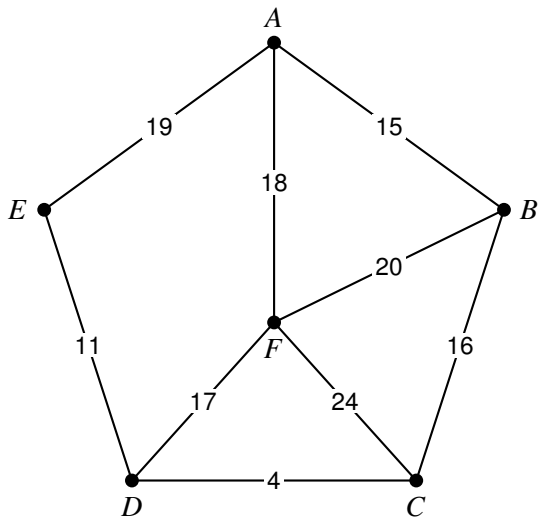

Figure 2

Figure 2 shows five farms around a lake, A to E, and an island jetty F. The weights are the lengths, in hundreds of metres, of possible water pipes.

(a) Use Prim's algorithm, starting at A, to find a minimum spanning tree. You must state the order in which you select the arcs.

(4)

(b) Find the weight of the minimum spanning tree.

(1)

(c) State one advantage of Prim's algorithm over Kruskal's algorithm.

(1)

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Question 2 continued D1 ANSWER BOOK

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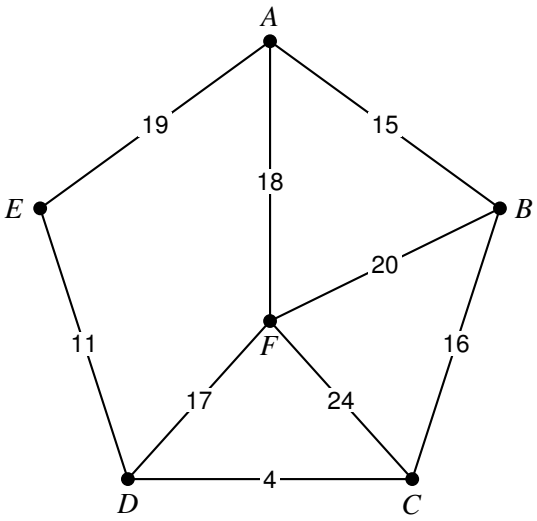

Diagram 2

Weight of the minimum spanning tree: _____

(Total for Question 2 is 6 marks)

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3. The table shows the distances, in km, between six towns.

	A	B	C	D	E	F
A	–	24	18	31	27	22
B	24	–	15	20	33	29
C	18	15	–	26	19	25
D	31	20	26	–	23	35
E	27	33	19	23	–	17
F	22	29	25	35	17	–

(a) Use Prim's algorithm, starting at A, to find a minimum spanning tree. You must list the arcs in the order that you select them.

(4)

(b) Draw the tree, and state its total length.

(2)

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Question 3 continued D1 ANSWER BOOK

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	A	B	C	D	E	F
A	–	24	18	31	27	22
B	24	–	15	20	33	29
C	18	15	–	26	19	25
D	31	20	26	–	23	35
E	27	33	19	23	–	17
F	22	29	25	35	17	–

	A	B	C	D	E	F
A	–	24	18	31	27	22
B	24	–	15	20	33	29
C	18	15	–	26	19	25
D	31	20	26	–	23	35
E	27	33	19	23	–	17
F	22	29	25	35	17	–

(Total for Question 3 is 6 marks)

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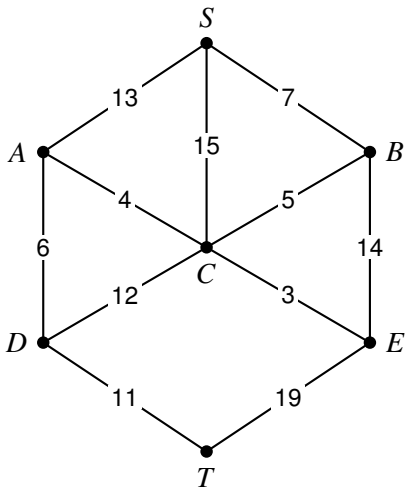

Figure 3

A walker plans to go from the summit S down to the town T. Figure 3 shows the paths, with the times in minutes.

(a) Use Dijkstra's algorithm to find the shortest time to travel from S to T.

(5)

(b) State the quickest route.

(1)

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Question 4 continued D1 ANSWER BOOK

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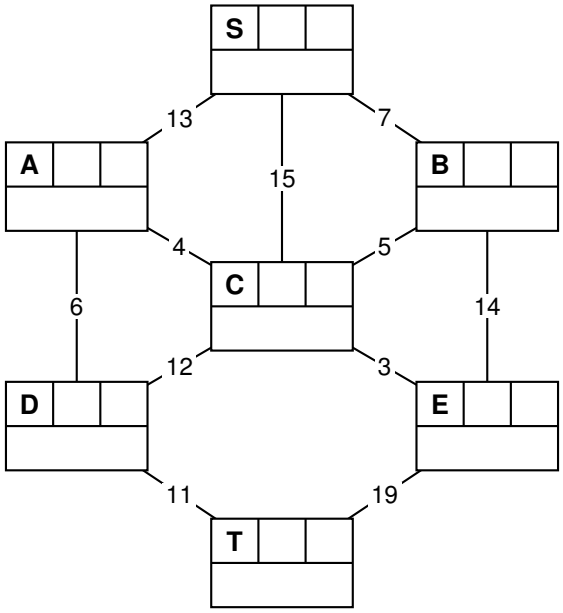

Diagram 3

Key:

Vertex	Order of labelling	Final value
Working values		

Shortest route from S to T: _____

Length of shortest route: _____

(Total for Question 4 is 6 marks)

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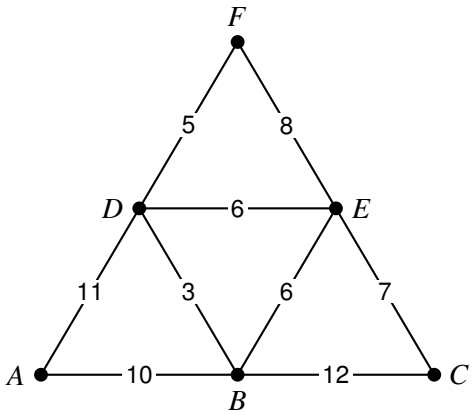

Figure 4

(a) Apply Kruskal's algorithm to the network in Figure 4, recording the decision taken for every arc examined.

(3)

(b) Some arcs have equal weights. Show that this allows a second, different minimum spanning tree, and draw it.

(2)

(c) Compare the total weights of your two trees.

(1)

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Question 5 continued D1 ANSWER BOOK

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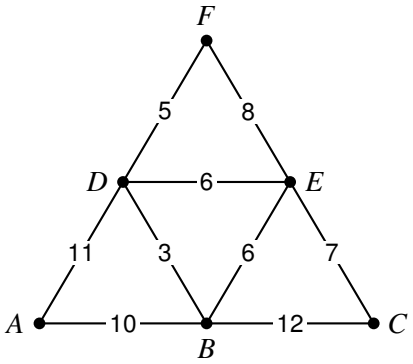

Diagram 4

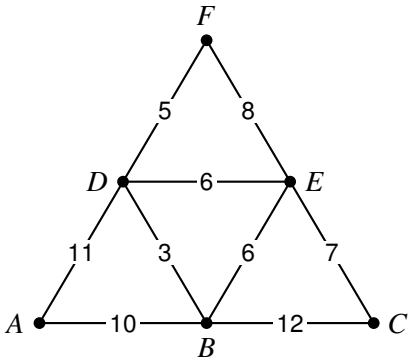

Diagram 5

(Total for Question 5 is 6 marks)

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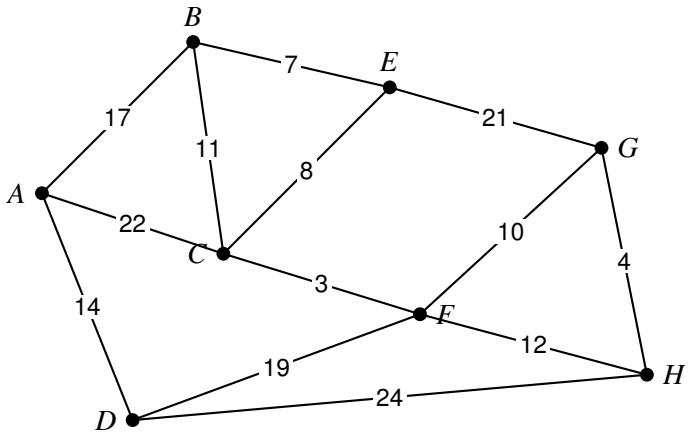

Figure 5

Figure 5 shows a map of footpaths between eight landmarks. The number on each arc is the length, in hundreds of metres, of the footpath.

(a) Use Dijkstra's algorithm to find the shortest distance from A to H.

(6)

(b) State the shortest route from A to H and its length.

(2)

The footpath FH is closed.

(c) Using your answer to part (a), find the new shortest route from A to H and its length, explaining your method.

(2)

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Question 6 continued D1 ANSWER BOOK

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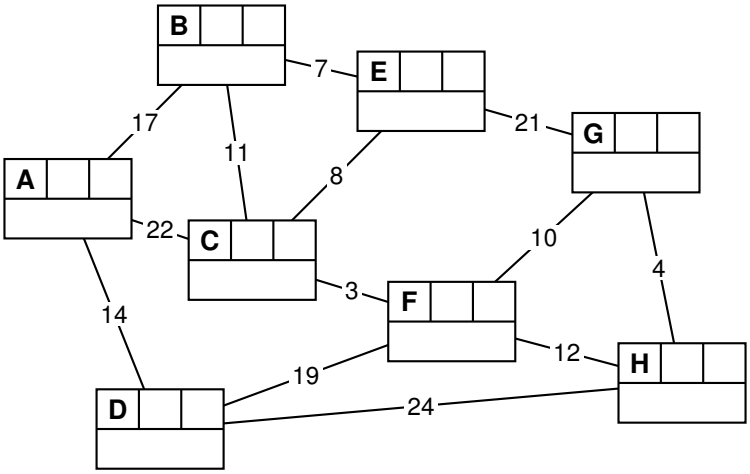

Diagram 6

Key:

Vertex	Order of labelling	Final value
Working values		

Shortest route from A to H: _____

Length of shortest route: _____

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Question 6 continued

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(Total for Question 6 is 10 marks)

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7. The table shows the distances, in metres, between seven cabins at a campsite. Water pipes are to be laid to connect all the cabins.

	A	B	C	D	E	F	G
A	–	38	27	45	33	51	29
B	38	–	42	24	36	47	40
C	27	42	–	39	21	35	44
D	45	24	39	–	30	26	48
E	33	36	21	30	–	32	37
F	51	47	35	26	32	–	41
G	29	40	44	48	37	41	–

(a) Use Prim's algorithm, starting at A, to find a minimum spanning tree. You must list the arcs in the order that you select them.

(4)

(b) State the minimum total length of pipe.

(1)

(c) Explain why Kruskal's algorithm is less convenient than Prim's algorithm for data given in this form.

(1)

A pipe must now be laid directly between A and F.

(d) Find the minimum total length of pipe needed if AF is included, explaining your method.

(4)

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Question 7 continued D1 ANSWER BOOK

In the exam this page is printed in the separate D1 answer book. Write your working here, on the copy provided.

	A	B	C	D	E	F	G
A	–	38	27	45	33	51	29
B	38	–	42	24	36	47	40
C	27	42	–	39	21	35	44
D	45	24	39	–	30	26	48
E	33	36	21	30	–	32	37
F	51	47	35	26	32	–	41
G	29	40	44	48	37	41	–

	A	B	C	D	E	F	G
A	–	38	27	45	33	51	29
B	38	–	42	24	36	47	40
C	27	42	–	39	21	35	44
D	45	24	39	–	30	26	48
E	33	36	21	30	–	32	37
F	51	47	35	26	32	–	41
G	29	40	44	48	37	41	–

Weight of the minimum spanning tree: _____

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Question 7 continued

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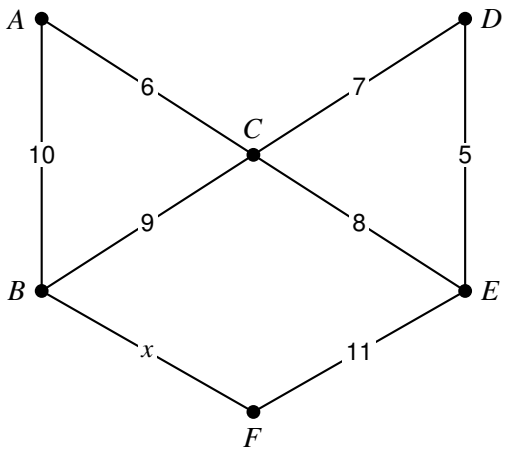

Figure 6

Figure 6 shows a network. The weight of arc BF is x, where x is a positive integer.

(a) Given that $x = 12$, use Kruskal's algorithm to find a minimum spanning tree, stating whether each arc considered is included or rejected, and find its weight.

(5)

(b) Find the range of values of x for which BF is in every minimum spanning tree, and the weight of the tree in terms of x in this case.

(3)

(c) Find the value of x for which there are two different minimum spanning trees, and explain why.

(2)

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Question 8 continued D1 ANSWER BOOK

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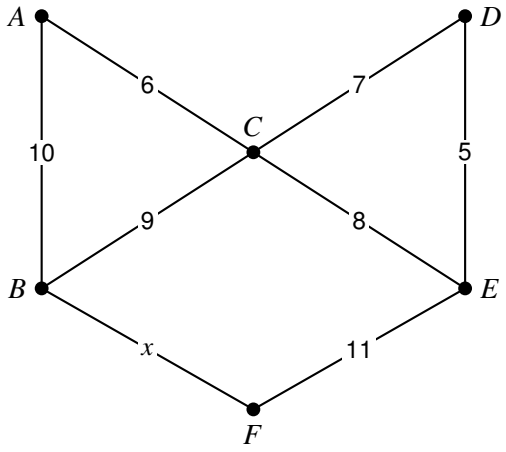

Diagram 7

Weight of the minimum spanning tree when $x = 12$: _____

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Question 8 continued

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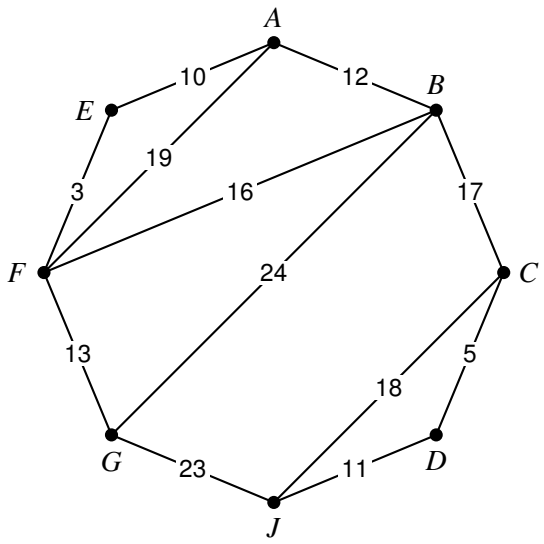

Figure 7

A courier company models eight depots and the lanes between them as the network in Figure 7; weights are lane lengths in km.

(a) Apply Dijkstra's algorithm, starting from depot A, to determine the minimum distance to J, then state the route that achieves it.

(7)

One van must collect a parcel at depot E before finishing at J.

(b) Using the final values from part (a) where possible, determine the van's minimum distance and route.

(3)

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Question 9 continued D1 ANSWER BOOK

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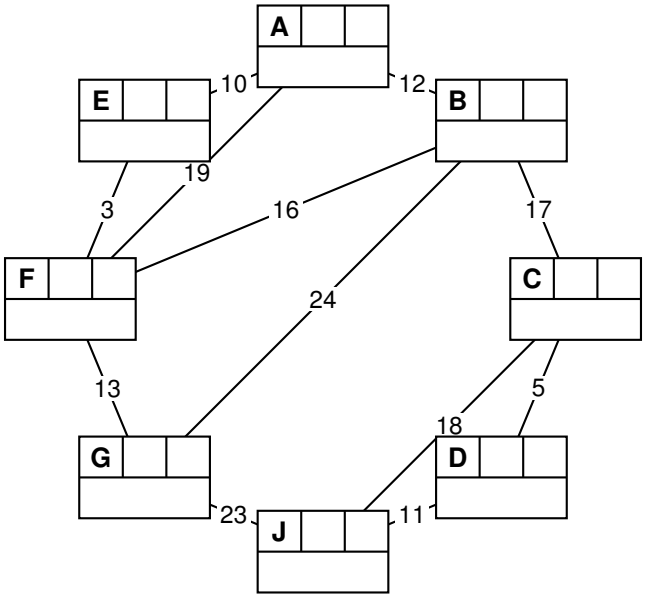

Diagram 8

Key:

Vertex	Order of labelling	Final value
Working values		

Shortest route from A to J: _____

Length of shortest route: _____

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Question 9 continued

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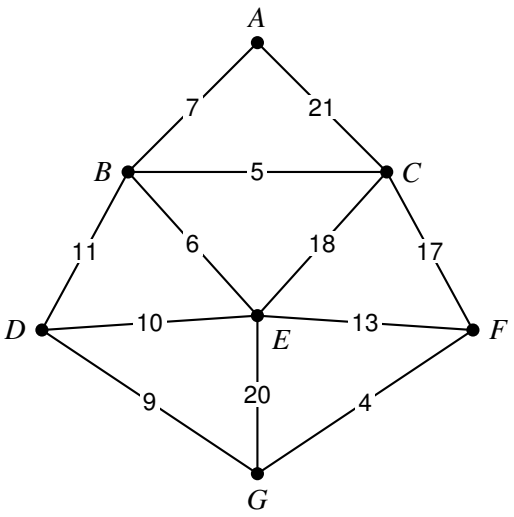

Figure 8

Figure 8 shows a network of roads. The number on each arc is the length of the road, in km.

(a) Use Kruskal's algorithm to find a minimum spanning tree, and state its weight.

(4)

(b) Use Dijkstra's algorithm to find the shortest route from A to G, and its length.

(5)

(c) Explain why the route in part (b) is not contained in the minimum spanning tree.

(1)

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Question 10 continued D1 ANSWER BOOK

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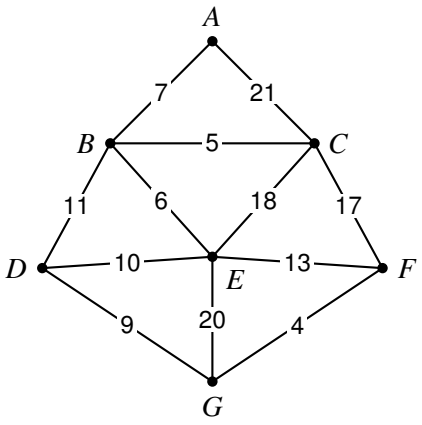

Diagram 9

Weight of the minimum spanning tree: _____

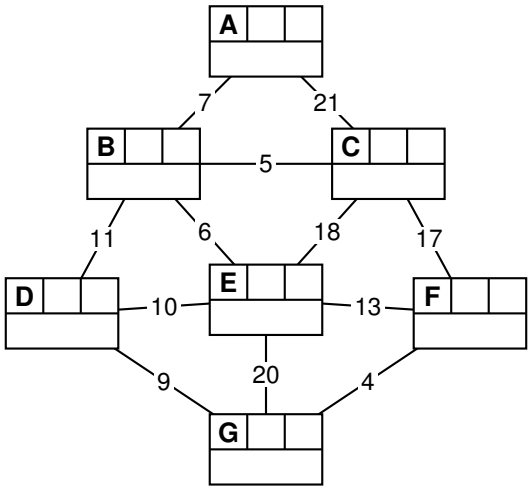

Diagram 10

Key:

Vertex	Order of labelling	Final value
Working values		

Shortest route from A to G: _____

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Question 10 continued

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(Total for Question 10 is 10 marks)

TOTAL FOR CHAPTER 3 IS 80 MARKS

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CHAPTER 4 • KEY SKILLS

4 Route Inspection

Specification D1: Route inspection • 10 questions • 80 marks • about 96 minutes

1 Eulerian

All vertices even: a closed route can use every arc exactly once. Exactly two odd: semi-Eulerian, start at one odd vertex and finish at the other.

2 Odd vertices

List the odd vertices first: there are always an even number of them.

3 Pairings

Four odd vertices P, Q, R, S give three pairings: $PQ + RS$, $PR + QS$, $PS + QR$. Find every total.

4 Shortest paths

The repeated route between two odd vertices is the shortest path, which may use several arcs.

5 Route length

Total weight of the network plus the least pairing total.

6 Different start and finish

Leave two odd vertices unpaired as the start and finish; repeat the shortest path between the other two. Check every choice.

7 Writing a route

Any order that uses every arc once (and repeated arcs twice), starting and finishing where required.

8 Passing through

In a closed route a vertex of degree d appears $\frac{d}{2}$ times (count repeated arcs in the degree).

9 Changes

Adding or removing an arc changes the degrees: recount the odd vertices before pairing again.

Skills map every skill above is tested at least once; interleaved skills come from GCSE or other chapters

Skill	Q1	Q2	Q3	Q4	Q5	Q6	Q7	Q8	Q9	Q10
Marks	6	6	6	6	6	10	10	10	10	10
1 Eulerian	•		•	•						
2 Odd vertices	•	•	•		•	•	•	•	•	•
3 Pairings		•			•	•	•	•	•	•
4 Shortest paths	•	•				•	•	•	•	•
5 Route length	•	•	•	•	•	•	•	•	•	•
6 Different start and finish			•		•	•				•
7 Writing a route	•			•						
8 Passing through				•						•
9 Changes							•	•		
Interleaved: Dijkstra's algorithm (Chapter 3)									•	
Interleaved: degrees and the handshaking lemma (Chapter 2)	•	•	•	•			•			
Interleaved: linear inequalities in an unknown (P1)								•		

Exam difficulty. Questions 1 to 5 check each skill on its own. Questions 6 to 10 are multi-step questions built like the last questions on a real WDM11 paper, where one part leads into the next.

Answer book. After each question is a copy of the page the real D1 answer book prints for it: the network reprinted (with Dijkstra boxes where needed) and answer lines.

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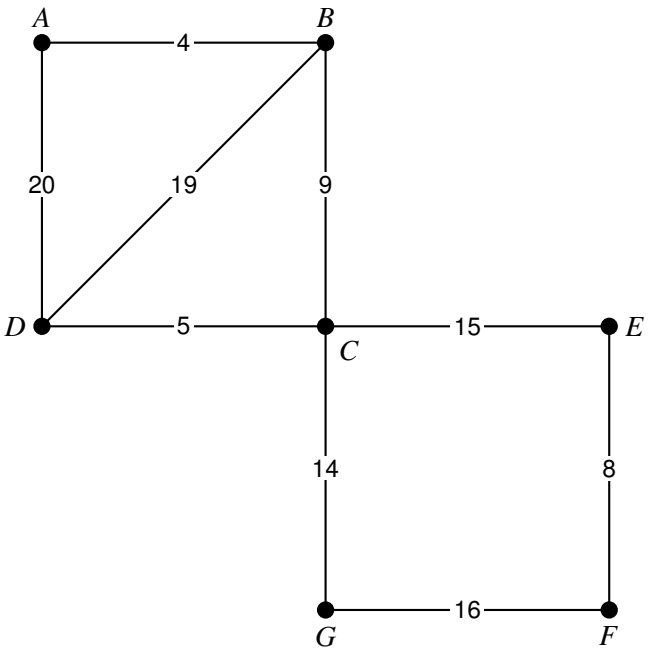

Figure 1

Figure 1 shows a network of paths in a park. The number on each arc is the length of the path, in hundreds of metres. A warden must walk along every path, starting and finishing at A.

(a) Explain why the warden must repeat some paths, and state which paths should be repeated.

(3)

(b) Find the length of the shortest route.

(1)

(c) Write down a possible shortest route.

(2)

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Question 1 continued D1 ANSWER BOOK

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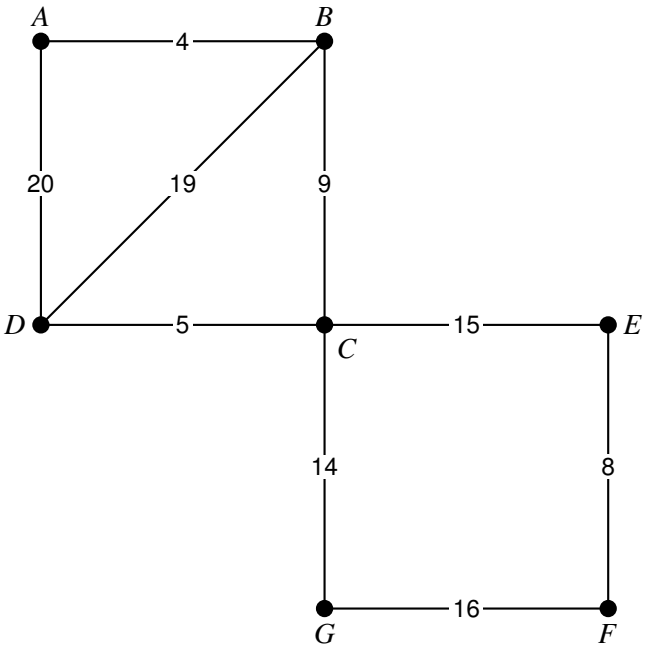

Diagram 1

Paths repeated: _____

Length of route: _____

Route: _____

(Total for Question 1 is 6 marks)

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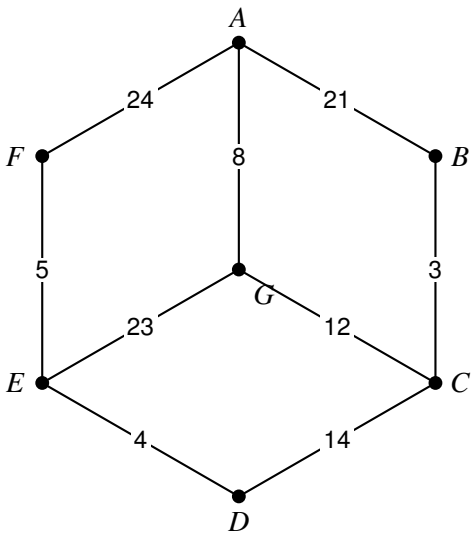

Figure 2

Figure 2 shows a network of roads. The number on each arc is the length of the road, in km. The total length of the roads is 114 km. A road gritter must travel along every road, starting and finishing at the depot B.

(a) Use the route inspection algorithm to find the roads that must be traversed twice. You must make your method clear, considering all possible pairings.

(5)

(b) Find the length of the shortest route.

(1)

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Question 2 continued D1 ANSWER BOOK

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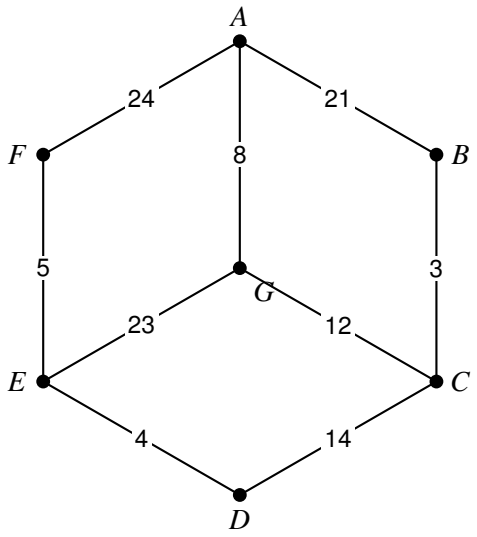

Diagram 2

Roads traversed twice: _____

Length of route: _____

(Total for Question 2 is 6 marks)

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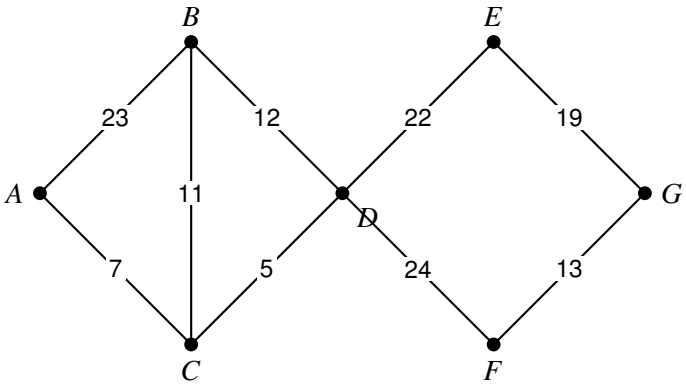

Figure 3

The corridors of a museum are modelled by the network in Figure 3, with weights giving corridor lengths in metres.

(a) A visitor wishes to use each corridor once only. Show that this can be done, and identify the two rooms where the visit must begin and end.

(2)

(b) Find the length of this walk.

(1)

(c) A guard must walk along every corridor, starting and finishing at A. Find the length of the shortest route, stating which corridor or corridors are repeated.

(3)

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Question 3 continued D1 ANSWER BOOK

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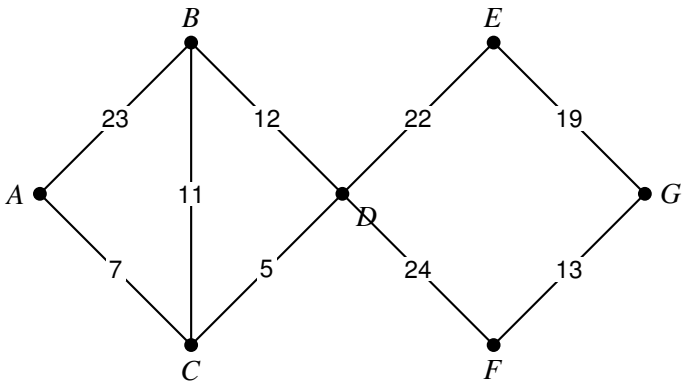

Diagram 3

Start and finish: _____

Corridors repeated: _____

Length of route: _____

(Total for Question 3 is 6 marks)

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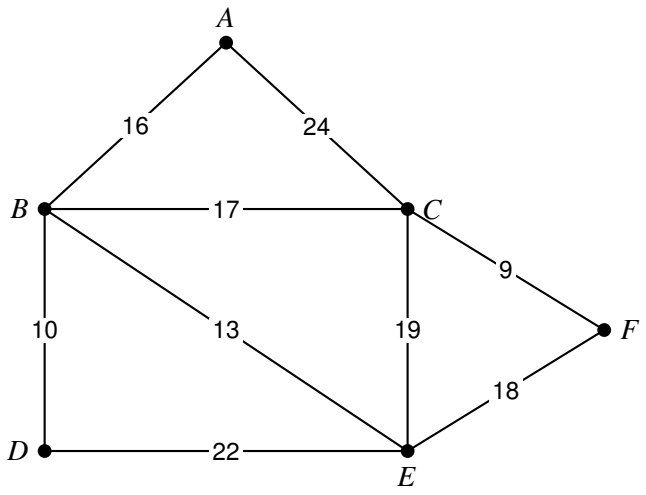

Figure 4

A postal worker's streets form the network in Figure 4; arc weights are street lengths in hundreds of metres.

(a) Explain why the postal worker can walk along every street exactly once and return to the start.

(1)

(b) Write down a suitable route starting and finishing at A, and state its length.

(3)

(c) State the number of times the route passes through B, giving a reason.

(2)

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Question 4 continued D1 ANSWER BOOK

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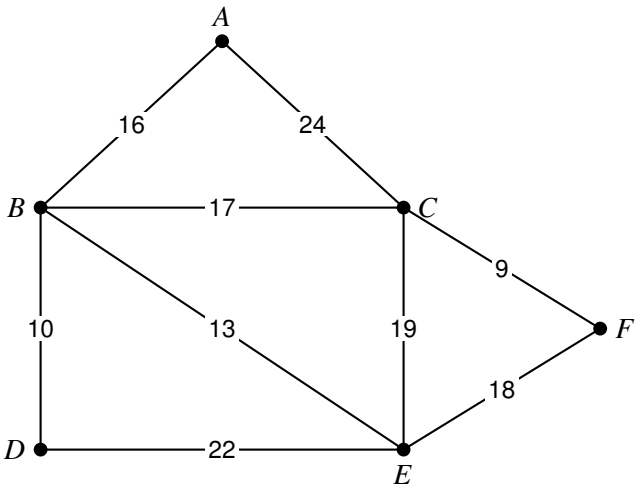

Diagram 4

Route: _____

Length of route: _____

(Total for Question 4 is 6 marks)

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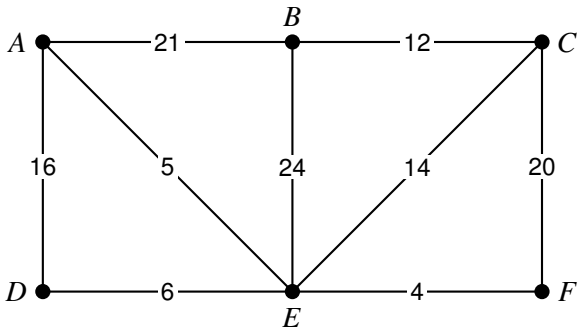

Figure 5

Cycle tracks with lengths in km form the network in Figure 5 (total 122 km). Every track is to be checked by one rider.

(a) Find the length of the shortest route that covers every track and starts and finishes at D, considering all pairings of the odd vertices.

(3)

(b) The route must instead start at E and may finish at any vertex. Find where it should finish and its length, explaining your choice.

(3)

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Question 5 continued D1 ANSWER BOOK

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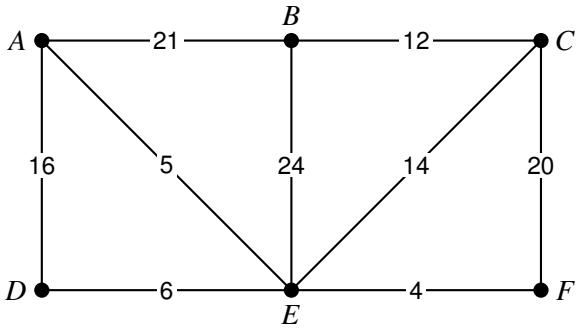

Diagram 5

Length of closed route: _____

Finish point and length of route from E: _____

(Total for Question 5 is 6 marks)

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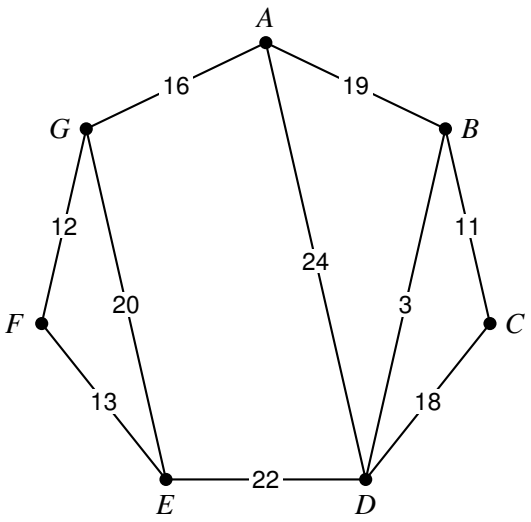

Figure 6

An engineer must inspect each road in Figure 6. Weights are lengths in km, and together the roads total 158 km.

(a) Use the route inspection algorithm to find the shortest route that starts and finishes at the same vertex. You must state which roads are repeated, show your consideration of all pairings, and give the length of the route.

(5)

(b) The route may instead start and finish at different vertices. Find the starting and finishing vertices that give the shortest route, and its length, justifying your choice.

(3)

(c) The inspector must start at G. State where the route should now finish, and its length.

(2)

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Question 6 continued D1 ANSWER BOOK

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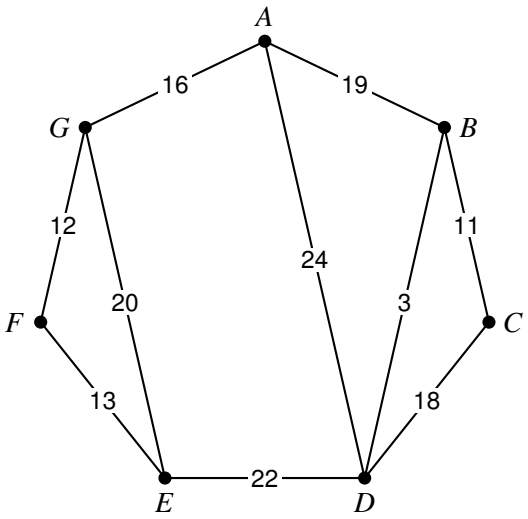

Diagram 6

Roads repeated: _____

Length of route: _____

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Question 6 continued

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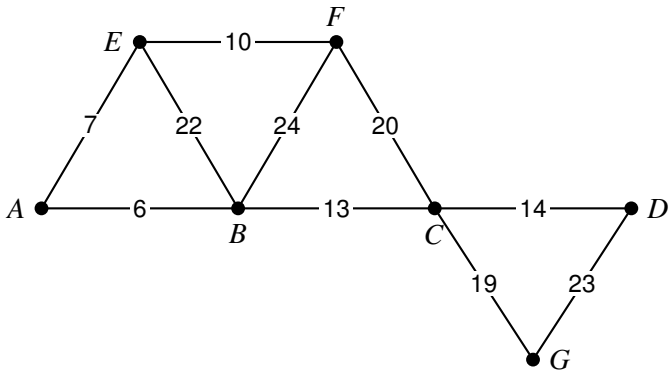

Figure 7

Figure 7 shows the network of footpaths in a nature reserve. The number on each arc is the length of the path, in hundreds of metres. The total length of the paths is 158.

(a) A ranger must walk every path, starting and finishing at A. Find the length of the shortest route, stating which paths are repeated.

(3)

A new path BG, of length 17, is built. It must also be walked.

(b) Explain the effect of the new path on the odd vertices, and use the route inspection algorithm to find the new shortest route length, showing all pairings.

(6)

(c) Comment on how the new route length compares with your answer to part (a).

(1)

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Question 7 continued D1 ANSWER BOOK

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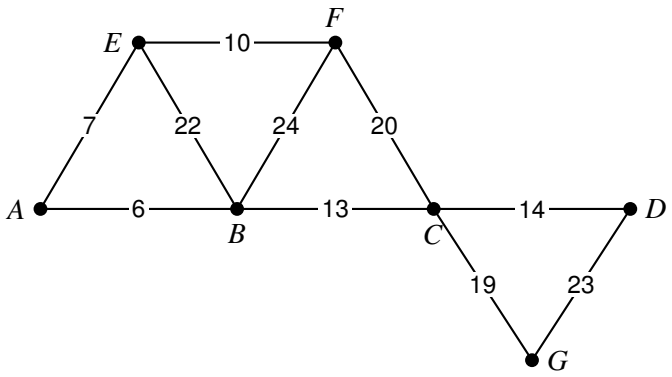

Diagram 7

Length of route in part (a): _____

Length of route in part (b): _____

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Question 7 continued

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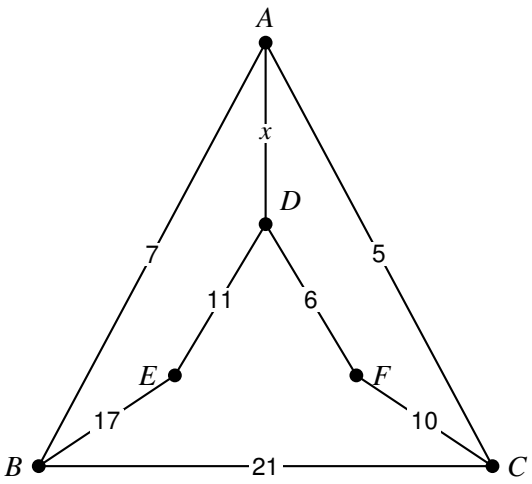

Figure 8

Figure 8 shows a network of pipes. The number on each arc is the length of the pipe, in metres. The length of AD is x metres, where $x > 0$. Every pipe must be inspected by a route that starts and finishes at A.

(a) Find, in terms of x, the total length of the pipes, and the total of each of the three pairings of the odd vertices.

(4)

(b) Find the range of values of x for which AD should be repeated, and write down the length of the route in each case.

(3)

(c) The shortest route has length 116 m. Find x, showing that the other case is impossible.

(3)

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Question 8 continued D1 ANSWER BOOK

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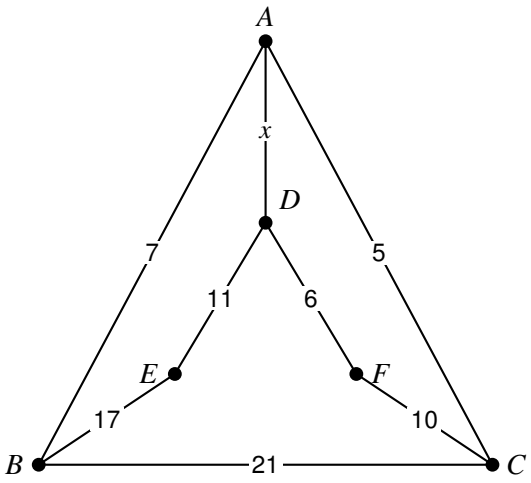

Diagram 8

Length of route in terms of x: _____

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Question 8 continued

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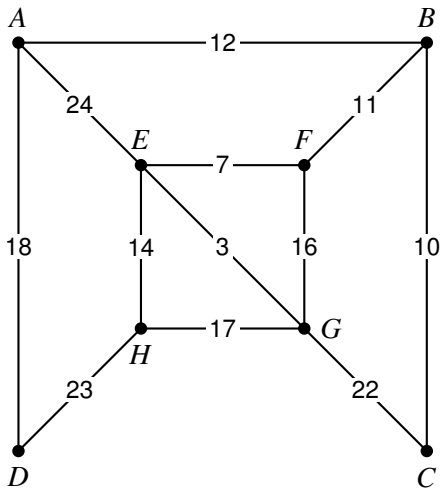

Figure 9

In Figure 9, each arc is a village road marked with its length in hundreds of metres; together the roads total 177.

(a) Use Dijkstra's algorithm to find the shortest distances from A to each of the other vertices.

(5)

A road sweeper must travel along every road, starting and finishing at A.

(b) Using your answer to part (a) where possible, find the roads that must be traversed twice, showing all pairings.

(4)

(c) Find the length of the shortest route.

(1)

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Question 9 continued D1 ANSWER BOOK

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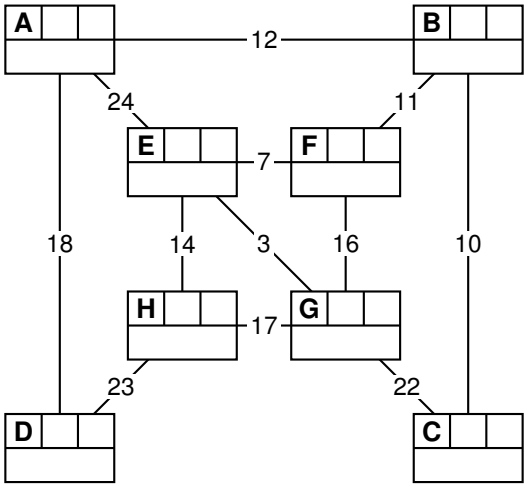

Diagram 9

Key:

Vertex	Order of labelling	Final value
Working values		

Roads traversed twice: _____

Length of route: _____

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Question 9 continued

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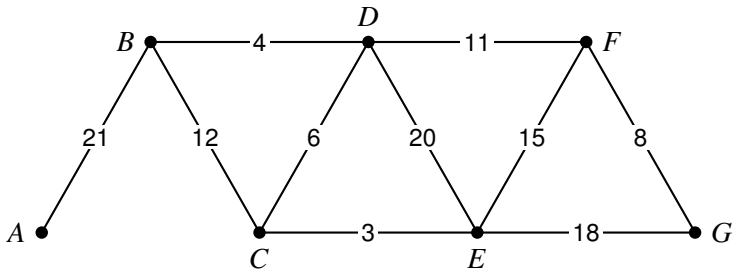

Figure 10

Figure 10 shows a network of streets to be cleaned. The number on each arc is the length of the street, in hundreds of metres. The total length of the streets is 118.

(a) Use the route inspection algorithm to find the length of the shortest route that starts and finishes at A. You must make your method clear.

(5)

(b) The cleaning vehicle must start at A but may finish at any vertex. Determine where it should finish, the streets it repeats and the length of the route, justifying your answer.

(4)

(c) State the number of times the route in part (b) passes through D.

(1)

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Question 10 continued D1 ANSWER BOOK

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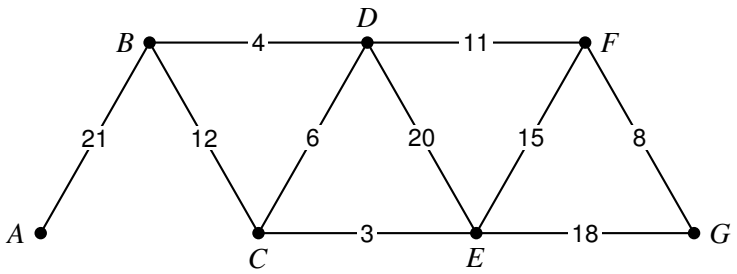

Diagram 10

Length of closed route: _____

Finishing point: _____

Length of route: _____

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Question 10 continued

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(Total for Question 10 is 10 marks)

TOTAL FOR CHAPTER 4 IS 80 MARKS

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CHAPTER 5 • KEY SKILLS

5 The Travelling Salesman Problem

Specification D1: The travelling salesman problem • 10 questions • 80 marks • about 96 minutes

1 Two problems

Classical: visit every vertex exactly once and return to the start. Practical: visit every vertex at least once and return to the start.

2 Least distances

Replace each entry by the shortest distance between the vertices (maybe via others); the table then satisfies the triangle inequality.

3 Upper bound from a tree

Twice the weight of a minimum spanning tree is an upper bound.

4 Shortcuts

Replace a repeated section of the doubled-tree walk by a direct arc; the saving lowers the upper bound. State each shortcut.

5 Nearest neighbour

From the start, go to the nearest unvisited vertex each time, then return to the start. Give the route and its length.

6 Lower bound

Delete a vertex, find a minimum spanning tree of the rest, then add the two least arcs from the deleted vertex.

7 Best bounds

The best upper bound is the least; the best lower bound is the greatest.

8 The interval

Lower bound $\leqslant$ optimal $\leqslant$ upper bound, with non-strict inequalities.

9 Optimal?

A tour is known to be optimal only if its length equals a lower bound.

Skills map every skill above is tested at least once; interleaved skills come from GCSE or other chapters

Skill	Q1	Q2	Q3	Q4	Q5	Q6	Q7	Q8	Q9	Q10
Marks	6	6	6	6	6	10	10	10	10	10
1 Two problems	•					•	•			
2 Least distances	•						•			
3 Upper bound from a tree			•			•				•
4 Shortcuts			•							•
5 Nearest neighbour		•				•	•	•	•	•
6 Lower bound				•		•	•	•	•	•
7 Best bounds		•		•	•	•		•		
8 The interval					•	•	•	•	•	•
9 Optimal?					•			•		
Interleaved: Prim's algorithm on a matrix (Chapter 3)			•	•		•	•	•	•	•
Interleaved: Dijkstra and shortest paths (Chapter 3)	•						•			
Interleaved: inequalities in an unknown (P1)									•	

Exam difficulty. Questions 1 to 5 check each skill on its own. Questions 6 to 10 are multi-step questions built like the last questions on a real WDM11 paper, where one part leads into the next.

Answer book. After each question is a copy of the page the real D1 answer book prints for it: the table (with a spare copy), the tree for the shortcut method, and answer lines.

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1.

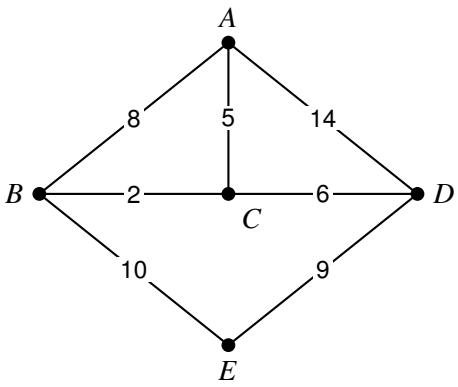

Figure 1

Figure 1 shows the roads between five towns, with lengths in km.

(a) Complete the table of least distances between the towns in the answer book.

(4)

(b) Explain the difference between the classical and the practical travelling salesman problems.

(2)

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Question 1 continued **D1 ANSWER BOOK**

In the exam this page is printed in the separate D1 answer book. Write your working here, on the copy provided.

	A	B	C	D	E
A	–	7	5	11	17
B	7	–			
C	5		–		
D	11			–	
E	17				–

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(Total for Question 1 is 6 marks)

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2. The table shows the travelling times, in minutes, between six offices.

	A	B	C	D	E	F
A	–	40	18	35	41	31
B	40	–	24	23	67	16
C	18	24	–	29	55	22
D	35	23	29	–	48	9
E	41	67	55	48	–	52
F	31	16	22	9	52	–

(a) Use the nearest neighbour algorithm, starting at A, to find an upper bound for the length of a tour. You must state your route.

(3)

(b) Repeat part (a) starting at D.

(2)

(c) State which of your upper bounds is better, giving a reason.

(1)

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Question 2 continued D1 ANSWER BOOK

In the exam this page is printed in the separate D1 answer book. Write your working here, on the copy provided.

	A	B	C	D	E	F
A	–	40	18	35	41	31
B	40	–	24	23	67	16
C	18	24	–	29	55	22
D	35	23	29	–	48	9
E	41	67	55	48	–	52
F	31	16	22	9	52	–

	A	B	C	D	E	F
A	–	40	18	35	41	31
B	40	–	24	23	67	16
C	18	24	–	29	55	22
D	35	23	29	–	48	9
E	41	67	55	48	–	52
F	31	16	22	9	52	–

Route from A and its length: _____

Route from D and its length: _____

(Total for Question 2 is 6 marks)

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3. The table shows the distances, in km, between six depots.

	A	B	C	D	E	F
A	–	25	14	9	13	24
B	25	–	39	23	29	49
C	14	39	–	20	18	11
D	9	23	20	–	6	26
E	13	29	18	6	–	22
F	24	49	11	26	22	–

A minimum spanning tree for the depots is AD, DE, AC, CF, DB; it is drawn in the answer book.

(a) Use the minimum spanning tree to find an initial upper bound for the length of a tour.

(2)

(b) Starting with the minimum spanning tree in the answer book, use the shortcut method to find an upper bound below 120 km. You must state the shortcut(s) you use and the length of your upper bound.

(4)

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Question 3 continued D1 ANSWER BOOK

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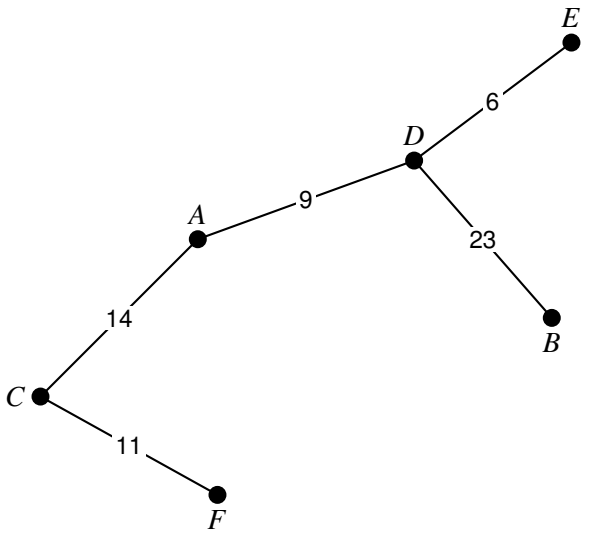

Diagram 1

	A	B	C	D	E	F
A	–	25	14	9	13	24
B	25	–	39	23	29	49
C	14	39	–	20	18	11
D	9	23	20	–	6	26
E	13	29	18	6	–	22
F	24	49	11	26	22	–

Upper bound: _____

(Total for Question 3 is 6 marks)

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4. The table shows the distances, in km, between six farms.

	A	B	C	D	E	F
A	–	11	16	44	40	13
B	11	–	27	45	43	21
C	16	27	–	38	32	20
D	44	45	38	–	7	55
E	40	43	32	7	–	50
F	13	21	20	55	50	–

(a) By deleting A, and all of its arcs, find a lower bound for the length of a tour. You must make your method clear.

(3)

(b) By deleting B instead, find a second lower bound.

(2)

(c) State which lower bound is better, giving a reason.

(1)

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Question 4 continued D1 ANSWER BOOK

In the exam this page is printed in the separate D1 answer book. Write your working here, on the copy provided.

	A	B	C	D	E	F
A	–	11	16	44	40	13
B	11	–	27	45	43	21
C	16	27	–	38	32	20
D	44	45	38	–	7	55
E	40	43	32	7	–	50
F	13	21	20	55	50	–

	A	B	C	D	E	F
A	–	11	16	44	40	13
B	11	–	27	45	43	21
C	16	27	–	38	32	20
D	44	45	38	–	7	55
E	40	43	32	7	–	50
F	13	21	20	55	50	–

Lower bound deleting A: _____

Lower bound deleting B: _____

(Total for Question 4 is 6 marks)

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5. For a travelling salesman problem on six cities, the nearest neighbour algorithm gave tours of lengths 135, 128 and 143 km, starting at A, B and C respectively. Deleting A, B and D in turn gave lower bounds of 104, 100 and 117 km.

(a) Write down the best interval within which the length, T km, of an optimal tour must lie.

(2)

(b) Explain why the lower bounds are not all the same, and why the greatest one is used.

(2)

(c) A tour of length 120 km is suggested. State, with a reason, whether it could be optimal, and whether the information shows that it is.

(2)

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Question 5 continued **D1 ANSWER BOOK**

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Best interval: _____

(Total for Question 5 is 6 marks)

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6. The table shows the distances, in km, between six warehouses. A driver must visit each warehouse, starting and finishing at A, and wishes to minimise the distance travelled.

	A	B	C	D	E	F
A	–	45	47	38	40	36
B	45	–	19	46	21	31
C	47	19	–	33	7	18
D	38	46	33	–	27	16
E	40	21	7	27	–	11
F	36	31	18	16	11	–

(a) Explain why the classical and practical problems are the same here.

(1)

(b) Use Prim's algorithm, starting at A, to find a minimum spanning tree, listing arcs in the order selected, and hence find an initial upper bound.

(4)

(c) Use the nearest neighbour algorithm, starting at A, to find a second upper bound. You must state your route.

(2)

(d) By deleting A, find a lower bound.

(2)

(e) Write down the best interval for the length, T km, of an optimal route.

(1)

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Question 6 continued D1 ANSWER BOOK

In the exam this page is printed in the separate D1 answer book. Write your working here, on the copy provided.

	A	B	C	D	E	F
A	–	45	47	38	40	36
B	45	–	19	46	21	31
C	47	19	–	33	7	18
D	38	46	33	–	27	16
E	40	21	7	27	–	11
F	36	31	18	16	11	–

	A	B	C	D	E	F
A	–	45	47	38	40	36
B	45	–	19	46	21	31
C	47	19	–	33	7	18
D	38	46	33	–	27	16
E	40	21	7	27	–	11
F	36	31	18	16	11	–

Upper bound from the tree: _____

Nearest neighbour route and length: _____

Lower bound: _____

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Question 6 continued

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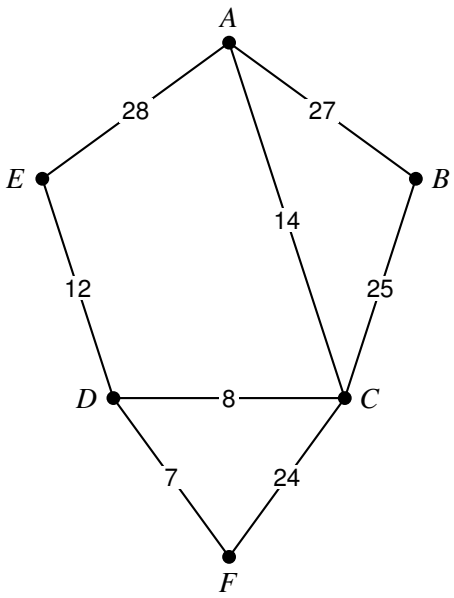

Figure 2

Figure 2 shows the roads between six villages, with lengths in km. A health visitor must visit every village, starting and finishing at A.

(a) Complete the table of least distances in the answer book.

(3)

(b) Use the nearest neighbour algorithm on your table, starting at A, to find an upper bound for the length of the route, and write down the corresponding route through the actual roads.

(4)

(c) By deleting A, find a lower bound for the length of the route.

(3)

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Question 7 continued D1 ANSWER BOOK

In the exam this page is printed in the separate D1 answer book. Write your working here, on the copy provided.

	A	B	C	D	E	F
A	–					
B		–				
C			–			
D				–		
E					–	
F						–

Route through the roads: _____

Upper bound: _____

Lower bound: _____

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Question 7 continued

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(Total for Question 7 is 10 marks)

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8. The table shows the times, in minutes, to travel between six sites.

	A	B	C	D	E	F
A	–	35	46	43	57	61
B	35	–	37	11	52	49
C	46	37	–	48	15	16
D	43	11	48	–	62	58
E	57	52	15	62	–	12
F	61	49	16	58	12	–

(a) Use the nearest neighbour algorithm, starting at A and then starting at B, to find two upper bounds. State your routes.

(4)

(b) By deleting A and then deleting B, find two lower bounds.

(4)

(c) Hence write down the best interval for the time, T minutes, of an optimal tour, and state whether either nearest neighbour route is known to be optimal.

(2)

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Question 8 continued D1 ANSWER BOOK

In the exam this page is printed in the separate D1 answer book. Write your working here, on the copy provided.

	A	B	C	D	E	F
A	–	35	46	43	57	61
B	35	–	37	11	52	49
C	46	37	–	48	15	16
D	43	11	48	–	62	58
E	57	52	15	62	–	12
F	61	49	16	58	12	–

	A	B	C	D	E	F
A	–	35	46	43	57	61
B	35	–	37	11	52	49
C	46	37	–	48	15	16
D	43	11	48	–	62	58
E	57	52	15	62	–	12
F	61	49	16	58	12	–

Upper bounds: _____

Lower bounds: _____

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Question 8 continued

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9. The table shows the distances, in km, between six towns. The distance between A and D is x km, where $36 < x < 45$

	A	B	C	D	E	F
A	–	45	47	x	40	36
B	45	–	19	46	21	31
C	47	19	–	33	7	18
D	x	46	33	–	27	16
E	40	21	7	27	–	11
F	36	31	18	16	11	–

(a) By deleting A, show that a lower bound for the length of a tour is $89 + \min(x, 40)$ km, explaining each step.

(4)

(b) Explain why the nearest neighbour algorithm starting at A first travels to F, and show that it gives the route A–F–E–C–B–D–A with length $(119 + x)$ km.

(3)

(c) The upper bound in part (b) is 157 km. Find x, and hence the best interval for the length, T km, of an optimal tour that these two methods give.

(3)

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Question 9 continued D1 ANSWER BOOK

In the exam this page is printed in the separate D1 answer book. Write your working here, on the copy provided.

	A	B	C	D	E	F
A	–	45	47	x	40	36
B	45	–	19	46	21	31
C	47	19	–	33	7	18
D	x	46	33	–	27	16
E	40	21	7	27	–	11
F	36	31	18	16	11	–

	A	B	C	D	E	F
A	–	45	47	x	40	36
B	45	–	19	46	21	31
C	47	19	–	33	7	18
D	x	46	33	–	27	16
E	40	21	7	27	–	11
F	36	31	18	16	11	–

Lower bound in terms of x: _____

Interval: _____

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Question 9 continued

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(Total for Question 9 is 10 marks)

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10. The table shows the distances, in hundreds of metres, between seven stalls at a fair. An inspector must visit every stall, returning to the start.

	A	B	C	D	E	F	G
A	–	41	30	26	12	19	22
B	41	–	11	63	47	37	31
C	30	11	–	52	36	27	21
D	26	63	52	–	17	28	35
E	12	47	36	17	–	14	20
F	19	37	27	28	14	–	7
G	22	31	21	35	20	7	–

(a) Use Prim's algorithm, starting at A, to find a minimum spanning tree. List the arcs in the order selected, and state its weight.

(3)

(b) Starting from twice the weight of your tree, use the shortcut method to find an upper bound below 160. State the shortcut you use.

(3)

(c) Use the nearest neighbour algorithm, starting at D, to find another upper bound, stating the route.

(2)

(d) By deleting D, find a lower bound, and hence state the best interval for the length, T, of an optimal tour.

(2)

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Question 10 continued D1 ANSWER BOOK

In the exam this page is printed in the separate D1 answer book. Write your working here, on the copy provided.

	A	B	C	D	E	F	G
A	–	41	30	26	12	19	22
B	41	–	11	63	47	37	31
C	30	11	–	52	36	27	21
D	26	63	52	–	17	28	35
E	12	47	36	17	–	14	20
F	19	37	27	28	14	–	7
G	22	31	21	35	20	7	–

	A	B	C	D	E	F	G
A	–	41	30	26	12	19	22
B	41	–	11	63	47	37	31
C	30	11	–	52	36	27	21
D	26	63	52	–	17	28	35
E	12	47	36	17	–	14	20
F	19	37	27	28	14	–	7
G	22	31	21	35	20	7	–

Upper bounds: _____

Lower bound and interval: _____

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Question 10 continued

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(Total for Question 10 is 10 marks)

TOTAL FOR CHAPTER 5 IS 80 MARKS

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CHAPTER 6 • KEY SKILLS

6 Critical Path Analysis

Specification D1: Critical path analysis • 10 questions • 80 marks • about 96 minutes

1 Drawing networks

Activities are arcs with arrows in the middle; one start event and one end event; draw large, with a ruler.

2 Dummies

A dotted arc of zero duration: for logic (one activity depends on another that another does not) or for uniqueness (two activities with the same start and end).

3 Early event times

Forward pass: the largest of (earlier event time + duration) over arcs coming in. Top boxes increase along every path.

4 Late event times

Backward pass: the smallest of (later event time − duration) over arcs going out. Bottom boxes decrease going back.

5 Critical activities

Early = late at both ends, and late end − early start = duration. They form the critical path(s).

6 Total float

Float = late finish of the end event − early start − duration.

7 Cascade charts

Critical activities in one row; each other activity as a bar from its earliest start, with its float shown and shaded.

8 At a given time

An activity must be in progress during $(t, t+1)$ if its latest start $\leqslant t$ and earliest finish $\geqslant t+1$

9 Workers

Lower bound = total of all durations ÷ critical time, rounded up. Schedule by giving each free worker the available activity with the earliest late finish.

Skills map every skill above is tested at least once; interleaved skills come from GCSE or other chapters

Skill	Q1	Q2	Q3	Q4	Q5	Q6	Q7	Q8	Q9	Q10
Marks	6	6	6	6	6	10	10	10	10	10
1 Drawing networks	•					•				
2 Dummies	•				•	•	•			
3 Early event times		•				•	•		•	•
4 Late event times		•				•	•		•	•
5 Critical activities		•	•			•	•	•		•
6 Total float			•	•			•	•		•
7 Cascade charts				•				•		
8 At a given time				•				•		
9 Workers							•	•	•	
Interleaved: precedence and digraphs (Chapter 2)	•				•	•				
Interleaved: inequalities and max/min reasoning (P1)				•				•		•
Interleaved: algebra with an unknown (P1)										•

Exam difficulty. Questions 1 to 5 check each skill on its own. Questions 6 to 10 are multi-step questions built like the last questions on a real WDM11 paper, where one part leads into the next.

Answer book. After each question is a copy of the page the real D1 answer book prints for it: the precedence table and space to draw, networks with early and late event boxes and a key, and time grids for cascade charts and schedules.

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1. A project is modelled by the precedence table below.

Activity	Immediately preceding activities
A	–
B	–
C	A
D	A, B
E	C
F	C, D
G	E, F

Draw the activity network for the project, using activity on arc. Your network must contain the minimum number of dummies.

(6)

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Question 1 continued D1 ANSWER BOOK

In the exam this page is printed in the separate D1 answer book. Write your working here, on the copy provided.

Activity	Immediately preceding activities
A	–
B	–
C	A
D	A, B
E	C
F	C, D
G	E, F

Draw your activity network here.

(Total for Question 1 is 6 marks)

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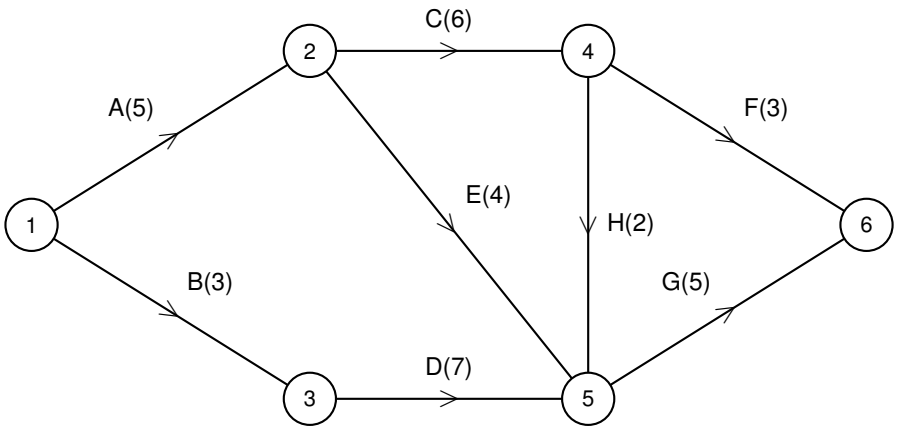

Figure 1

Figure 1 shows an activity network for a project. Each activity is shown with its duration in days.

(a) Complete Diagram 1 in the answer book to show the early event times and the late event times.

(4)

(b) State the critical activities and the minimum completion time.

(2)

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Question 2 continued D1 ANSWER BOOK

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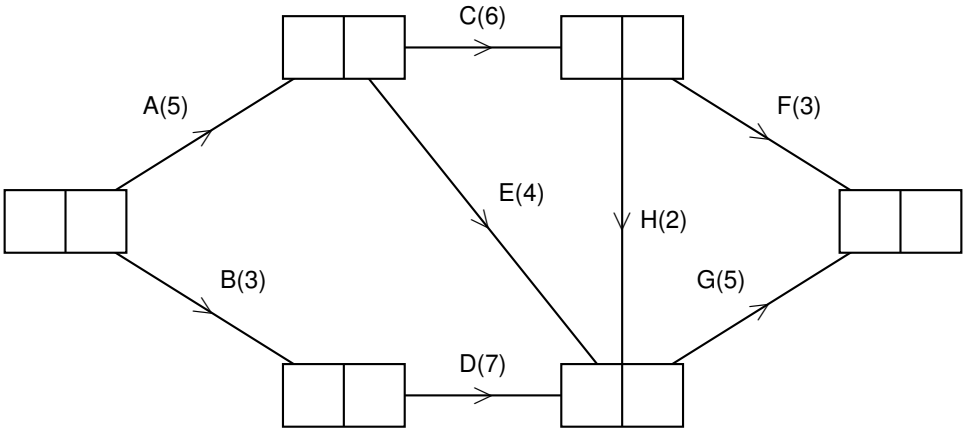

Diagram 1

Key:

Early event time	Late event time

(Total for Question 2 is 6 marks)

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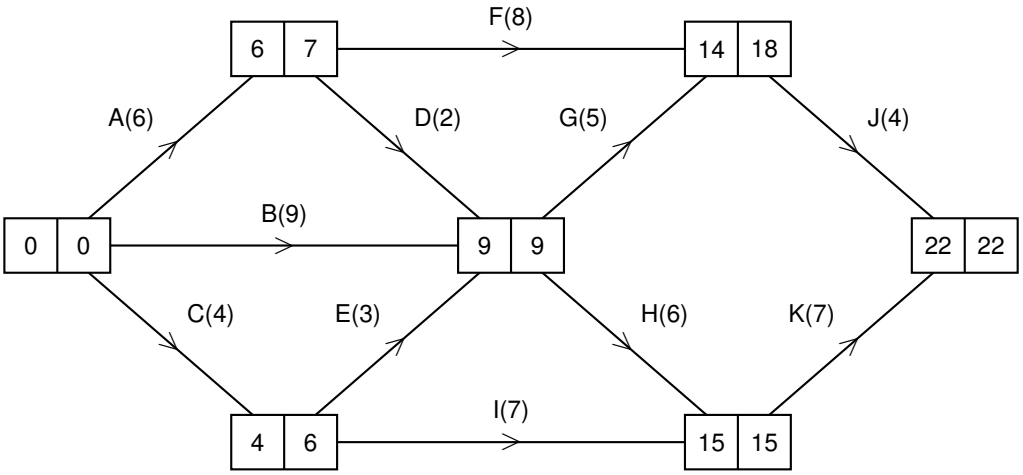

Figure 2

Figure 2 shows an activity network, with durations in hours. The early and late event times are shown at each event (early on the left, late on the right).

(a) Calculate the total float of each of the activities C, F and I, showing your working.

(3)

(b) State the critical path.

(1)

(c) Activity C is delayed by 3 hours. Explain the effect on the project.

(2)

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Question 3 continued **D1 ANSWER BOOK**

In the exam this page is printed in the separate D1 answer book. Write your working here, on the copy provided.

Activity	Early start	Late finish	Duration	Total float
C				
F				
I				

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(Total for Question 3 is 6 marks)

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4. The table gives the duration, earliest start time and total float, in days, of each activity in a project. The minimum completion time is 18 days.

Activity	Duration	Earliest start	Total float
A	4	0	0
B	6	0	1
C	5	4	0
D	2	6	1
E	8	4	3
F	4	9	2
G	3	9	0
H	5	6	1
I	3	13	2
J	6	12	0

(a) Draw a cascade (Gantt) chart for the project on the grid in the answer book.

(4)

(b) Use your chart to state the activities that must be happening at time 7.5 days.

(2)

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Question 4 continued **D1 ANSWER BOOK**

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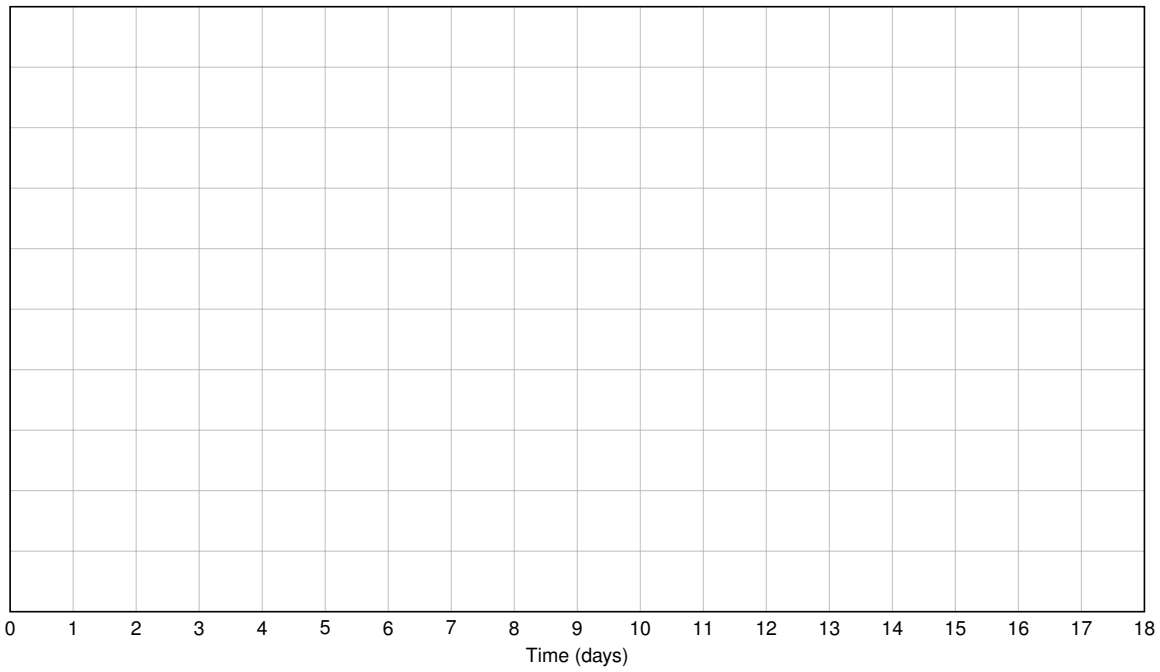

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(Total for Question 4 is 6 marks)

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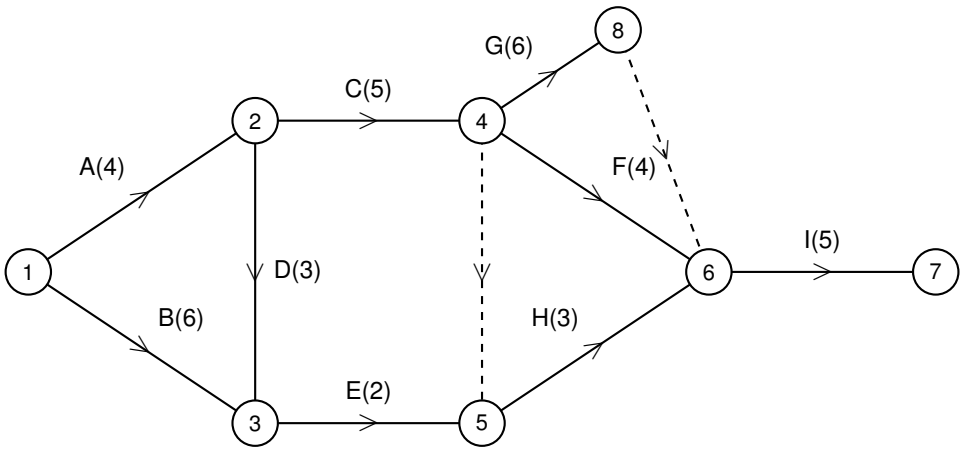

Figure 3

Figure 3 shows an activity network for a project. The dotted arcs are dummies.

(a) Complete the precedence table in the answer book.

(4)

(b) Explain why each of the two dummies is needed.

(2)

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Question 5 continued **D1 ANSWER BOOK**

In the exam this page is printed in the separate D1 answer book. Write your working here, on the copy provided.

Activity	Immediately preceding activities
A	
B	
C	
D	
E	
F	
G	
H	
I	

(Total for Question 5 is 6 marks)

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6. A project is modelled by the precedence table below. Each activity takes 1 or 2 days, as shown.

Activity	Immediately preceding activities
A	–
B	–
C	–
D	A
E	A, B
F	B, C
G	D, E
H	D, E
I	F

Durations: A 2, B 1, C 2, D 1, E 2, F 1, G 2, H 1, I 2

(a) Draw the activity network for the project, using activity on arc and exactly five dummies.

(6)

(b) Find the early and late event times on your network, and hence state the critical activities and the minimum completion time.

(4)

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Question 6 continued D1 ANSWER BOOK

In the exam this page is printed in the separate D1 answer book. Write your working here, on the copy provided.

Activity	Immediately preceding activities
A	–
B	–
C	–
D	A
E	A, B
F	B, C
G	D, E
H	D, E
I	F

Draw your activity network here.

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Question 6 continued

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(Total for Question 6 is 10 marks)

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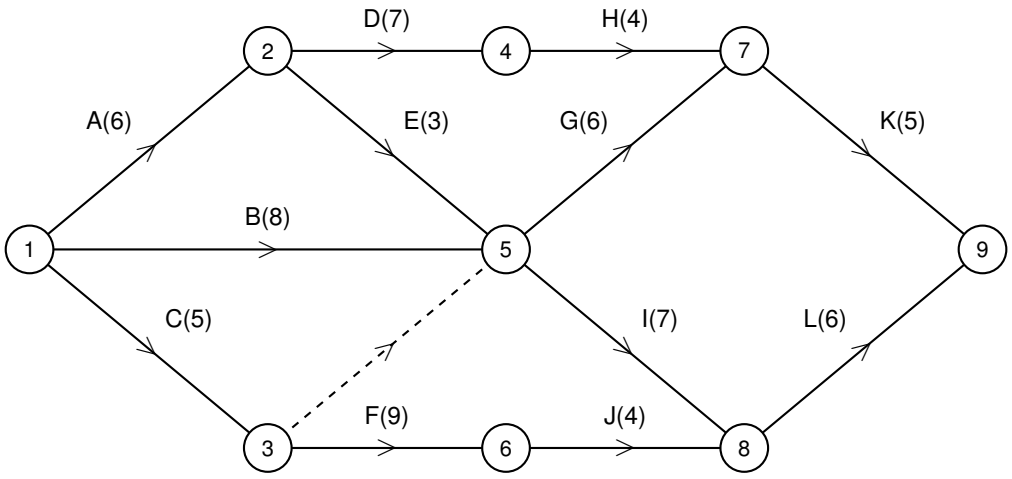

Figure 4

Figure 4 shows an activity network for a building project, with durations in days.

(a) Explain why the dummy is needed.

(2)

(b) Complete Diagram 1 in the answer book to show the early and late event times.

(4)

(c) State the critical path.

(1)

(d) Calculate the total float on activity H.

(1)

(e) Calculate a lower bound for the number of workers needed to complete the project in the minimum time.

(2)

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Question 7 continued D1 ANSWER BOOK

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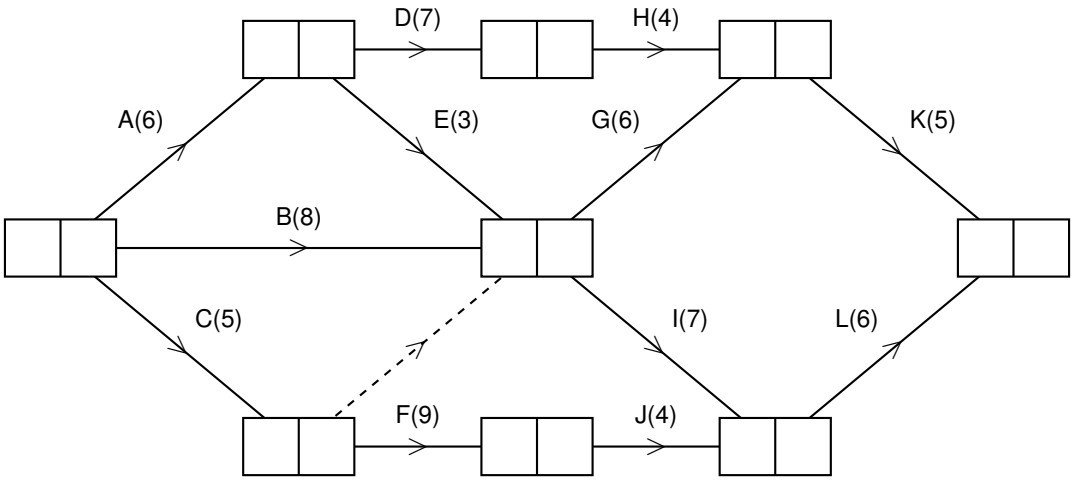

Diagram 2

Key:

Early event time	Late event time

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Question 7 continued

(Total for Question 7 is 10 marks)

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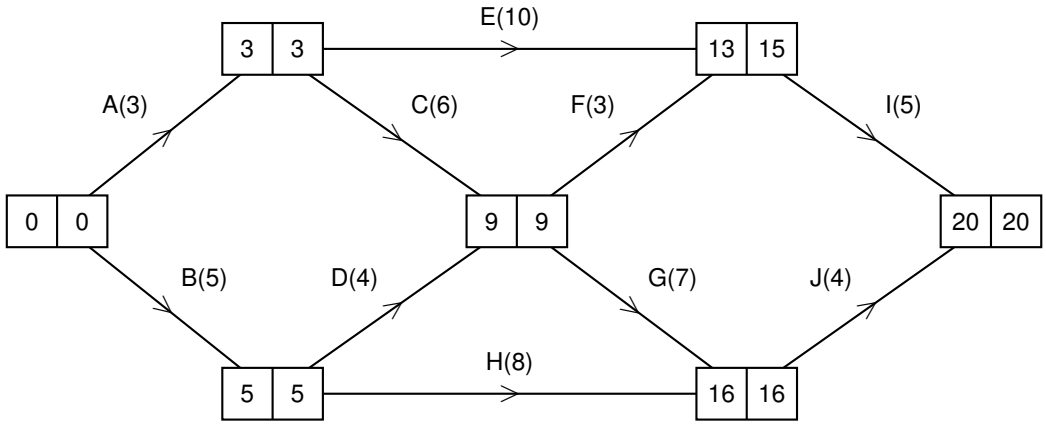

Figure 5

Figure 5 shows an activity network for a project, with durations in days and the early and late event times.

(a) Find the total float on each of the activities E, F, H and I, and explain why there are two critical paths.

(3)

(b) Draw a cascade chart for the project on the grid in the answer book.

(4)

(c) Using your cascade chart, state which activities must be happening during day 13, and hence the minimum number of workers needed at that time. Give the time interval precisely.

(3)

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Question 8 continued

D1 ANSWER BOOK

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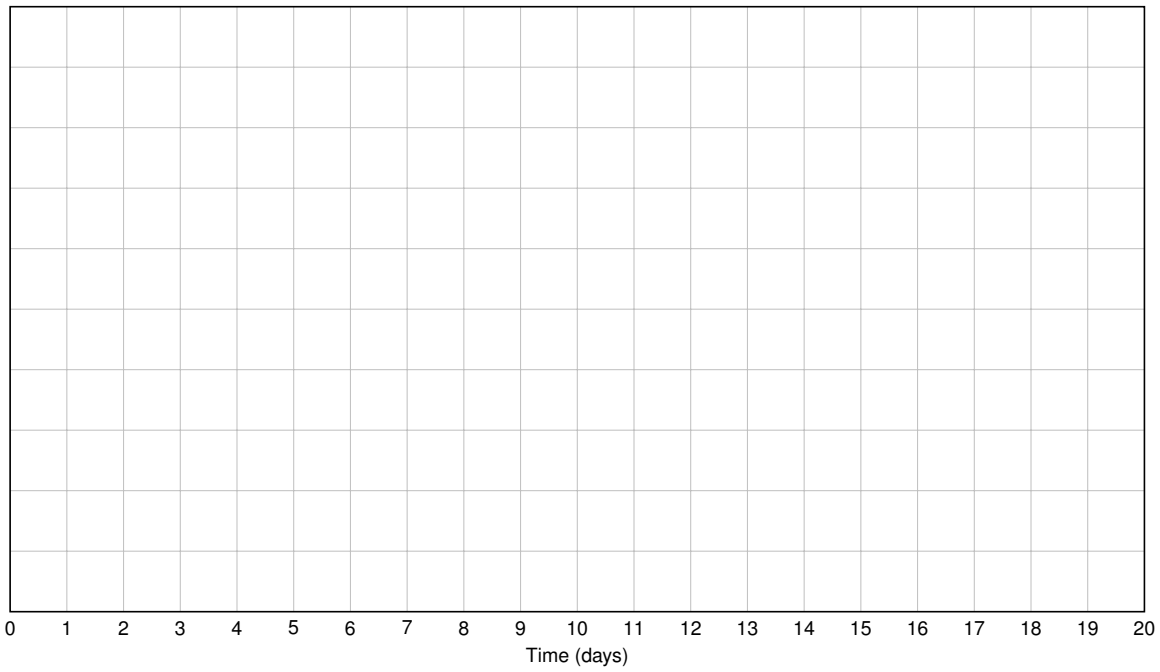

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Question 8 continued

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(Total for Question 8 is 10 marks)

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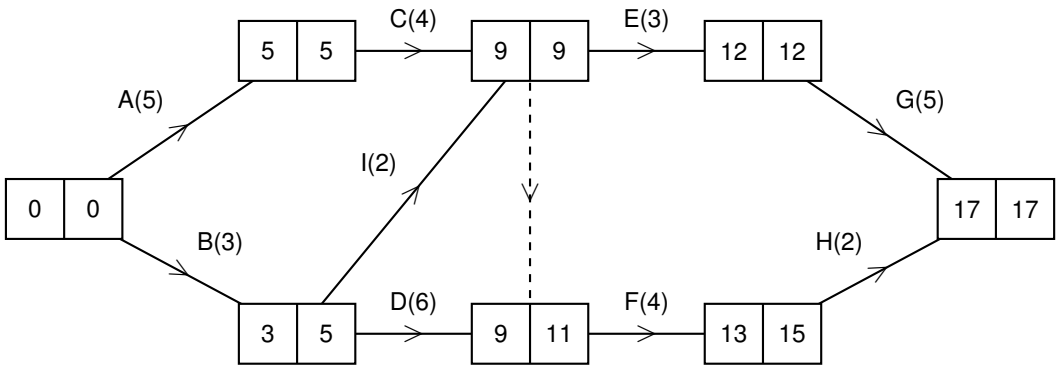

Figure 6

Figure 6 shows an activity network for a project, with durations in days. The minimum completion time is 17 days.

(a) Calculate a lower bound for the number of workers needed to complete the project in 17 days.

(2)

(b) On the grid in the answer book, draw a scheduling diagram to show how this number of workers can complete the project in 17 days. Each activity needs one worker.

(5)

(c) Activity D takes 3 days longer than planned. State, with a reason, the effect on the minimum completion time.

(3)

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Question 9 continued

D1 ANSWER BOOK

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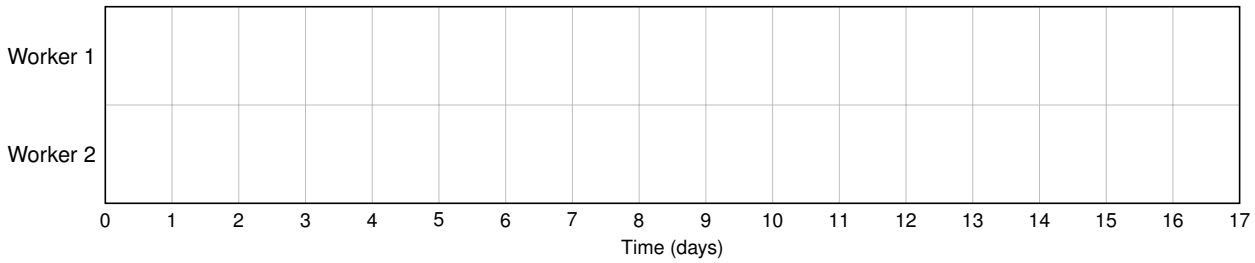

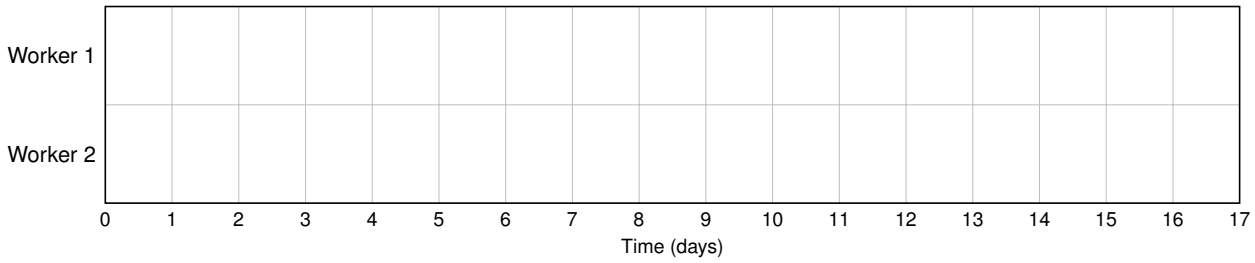

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Question 9 continued

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(Total for Question 9 is 10 marks)

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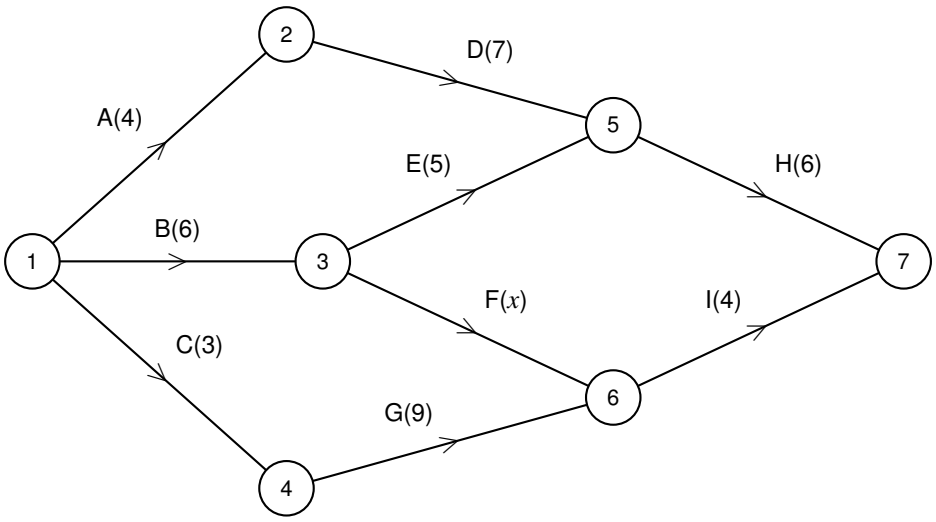

Figure 7

Figure 7 shows an activity network, with durations in hours. The duration of activity F is x hours.

(a) Given that $x = 4$, complete the early and late event times in the answer book, and state the critical activities and the minimum completion time.

(5)

(b) Find the range of values of x for which F is a critical activity, and the minimum completion time, in terms of x, in this case.

(3)

(c) Find the total float on F in terms of x when F is not critical.

(2)

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Question 10 continued D1 ANSWER BOOK

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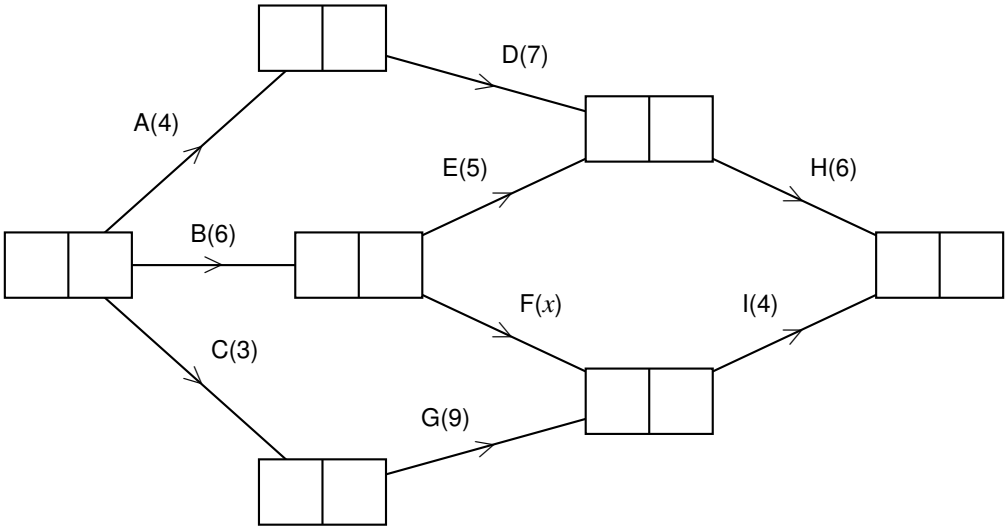

Diagram 3

Key:

Early event time	Late event time

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Question 10 continued

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(Total for Question 10 is 10 marks)

TOTAL FOR CHAPTER 6 IS 80 MARKS

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CHAPTER 7 • KEY SKILLS

7 Linear Programming

Specification D1: Linear programming • 10 questions • 80 marks • about 96 minutes

1 Decision variables

Define each variable precisely, with units: "x is the number of trays of muffins made each day".

2 Objective

Write "Maximise $P = \ldots$" or "Minimise $C = \ldots$" in full; do not rescale it.

3 Constraints

One inequality per resource or condition; percentages and ratios as inequalities; simplify to integer coefficients.

4 Fractions of a resource

If a machine makes 400 of A or 300 of B in a day: $\frac{x}{400} + \frac{y}{300} \leqslant 1$

5 Drawing

Draw each boundary line accurately, shade the unwanted side (or the region) and label the feasible region R

6 Objective line

Draw $ax + by = k$ for a convenient k; slide it parallel to reach the last point of R (first point for a minimum).

7 Vertex method

Find every vertex exactly by simultaneous equations and evaluate the objective at each one.

8 Integer solutions

Test integer points near the optimal vertex that lie inside R (for a minimum, on the feasible side of every line).

9 Changing the objective

A vertex stays optimal while the objective's gradient lies between the gradients of the two lines meeting there.

Skills map every skill above is tested at least once; interleaved skills come from GCSE or other chapters

Skill	Q1	Q2	Q3	Q4	Q5	Q6	Q7	Q8	Q9	Q10
Marks	6	6	6	6	6	10	10	10	10	10
1 Decision variables	●					●			●	
2 Objective	●					●			●	
3 Constraints	●					●	●		●	
4 Fractions of a resource						●				
5 Drawing		●		●				●	●	●
6 Objective line			●						●	●
7 Vertex method		●	●	●	●		●	●		●
8 Integer solutions					●		●		●	●
9 Changing the objective								●		●
Interleaved: straight lines and gradients (P1)		●	●	●			●	●	●	●
Interleaved: simultaneous equations (P1)		●	●	●	●		●	●	●	●
Interleaved: inequalities and percentages (P1, GCSE)	●					●	●		●	

Exam difficulty. Questions 1 to 5 check each skill on its own. Questions 6 to 10 are multi-step questions built like the last questions on a real WDM11 paper, where one part leads into the next.

Answer book. After each question is a copy of the page the real D1 answer book prints for it: a labelled grid (Diagram 1) or the figure reprinted with the region R.

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1. A bakery makes trays of muffins and trays of scones. Each tray of muffins needs 20 minutes of oven time and 3 kg of flour; each tray of scones needs 15 minutes of oven time and 2 kg of flour. Each day the oven is available for 6 hours and 50 kg of flour is available. The number of trays of muffins must be no more than three times the number of trays of scones. The profit is £12 on a tray of muffins and £9 on a tray of scones. The bakery wishes to maximise its daily profit.

(a) Define the decision variables.

(1)

(b) Write down the objective.

(1)

(c) Write down the constraints, simplifying each one so that it has integer coefficients.

(4)

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Question 1 continued **D1 ANSWER BOOK**

In the exam this page is printed in the separate D1 answer book. Write your working here, on the copy provided.

Decision variables: _____

Objective: _____

Subject to: _____

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(Total for Question 1 is 6 marks)

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2. A linear programming problem has the constraints

$$x + 2y \leqslant 16, \qquad 3x + 2y \leqslant 24, \qquad x \leqslant 2y, \qquad x,\ y \geqslant 0$$

(a) Represent these constraints on Diagram 1 in the answer book. Make your shading clear and label the feasible region R.

(4)

(b) Write down the coordinates of the vertices of R.

(2)

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Question 2 continued **D1 ANSWER BOOK**

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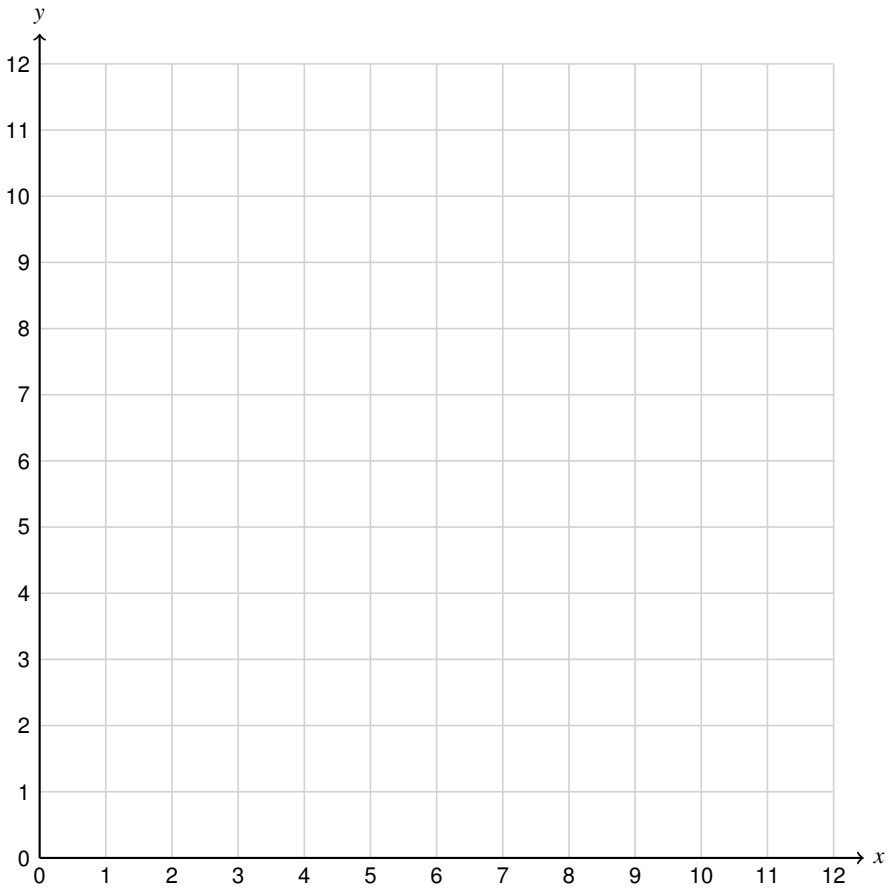

Diagram 1

(Total for Question 2 is 6 marks)

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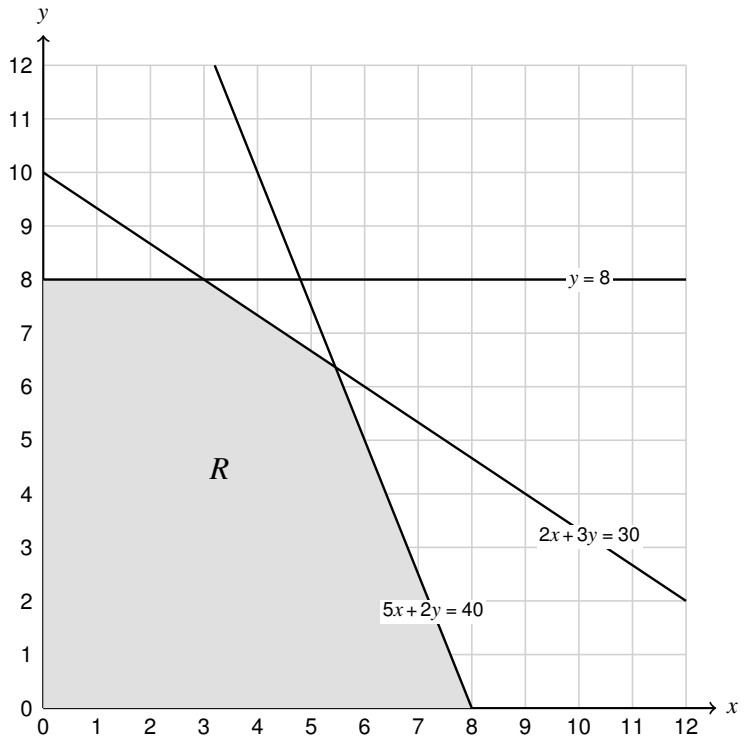

Figure 1

Figure 1 shows the feasible region R for a linear programming problem. The objective is to maximise $P = 3x + 4y$

(a) Use an objective line to identify the optimal vertex of R. Make your objective line clear.

(3)

(b) Find the exact coordinates of this vertex.

(2)

(c) Find the exact maximum value of P.

(1)

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Question 3 continued **D1 ANSWER BOOK**

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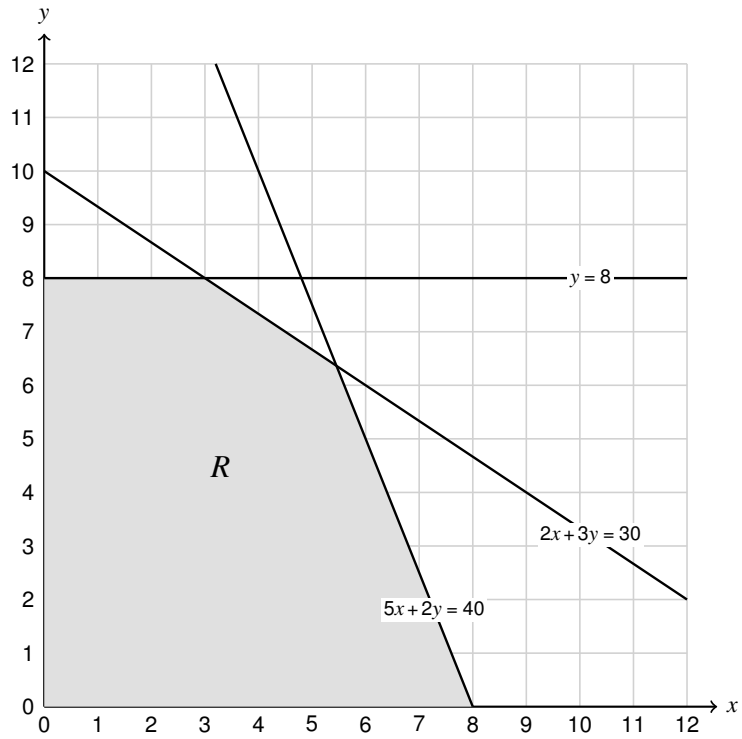

Diagram 2

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4. Minimise $C = 5x + 2y$ subject to

$$x + y \geqslant 8, \qquad 3x + y \geqslant 12, \qquad x + 3y \geqslant 12, \qquad x,\ y \geqslant 0$$

(a) Represent the constraints on Diagram 1 in the answer book, labelling the feasible region R.

(2)

(b) Use the vertex method to find the minimum value of C and the values of x and y at which it occurs.

(4)

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Question 4 continued **D1 ANSWER BOOK**

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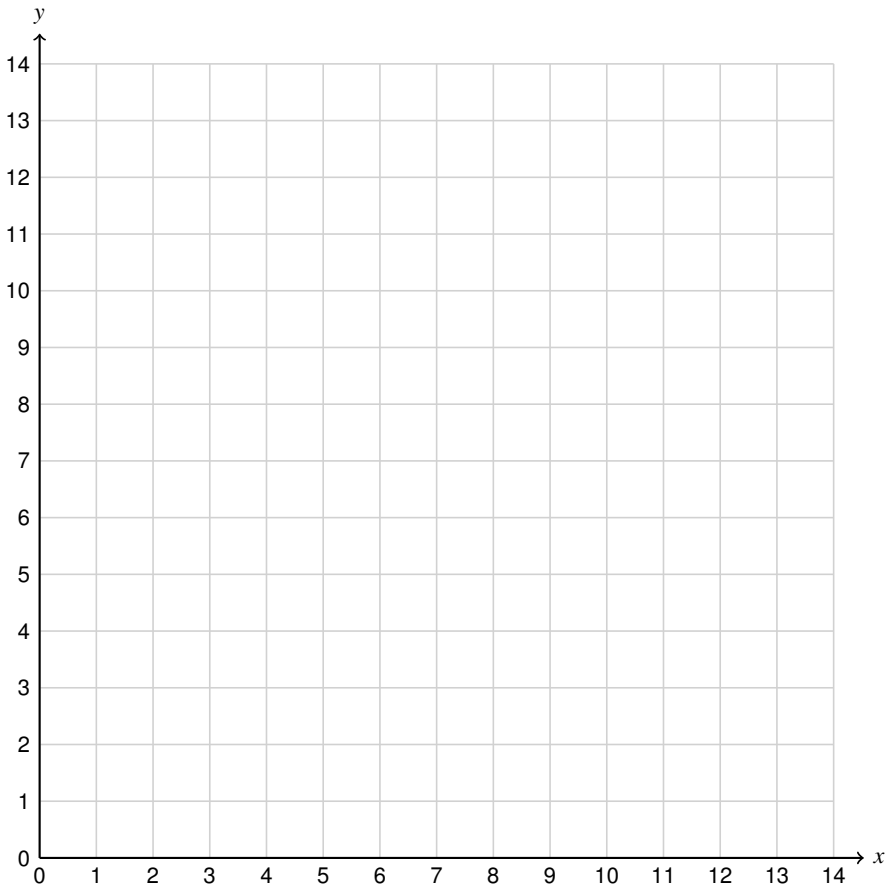

Diagram 3

(Total for Question 4 is 6 marks)

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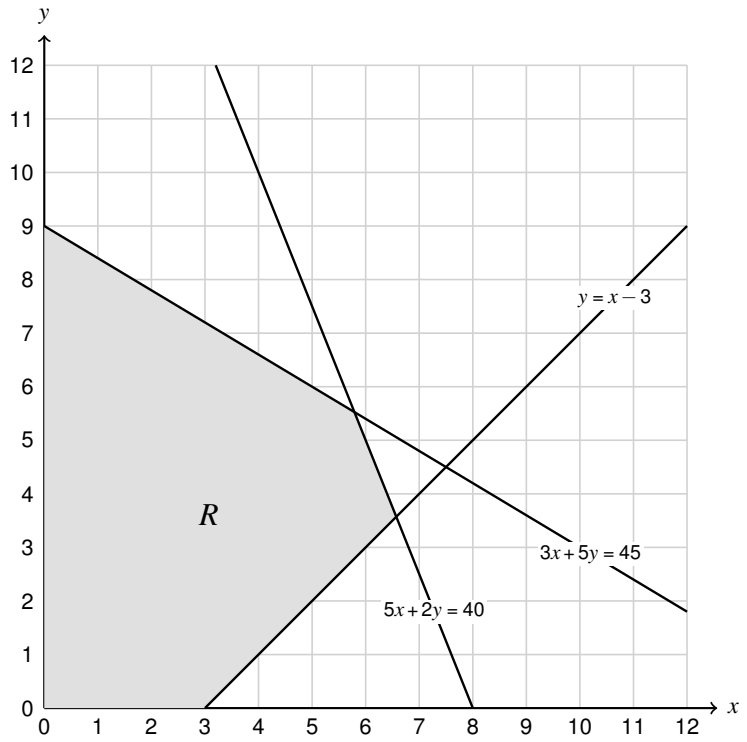

Figure 2

Figure 2 shows the feasible region R for the problem: maximise $P = 4x + 5y$ subject to $3x + 5y \leqslant 45$, $5x + 2y \leqslant 40$, $x - y \leqslant 3$, $x,\ y \geqslant 0$. The variables x and y must be integers.

(a) Find the exact coordinates of the vertex of R at which P is greatest if x and y need not be integers.

(2)

(b) Find the optimal integer solution, and the corresponding value of P. You must make your method clear.

(4)

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Question 5 continued **D1 ANSWER BOOK**

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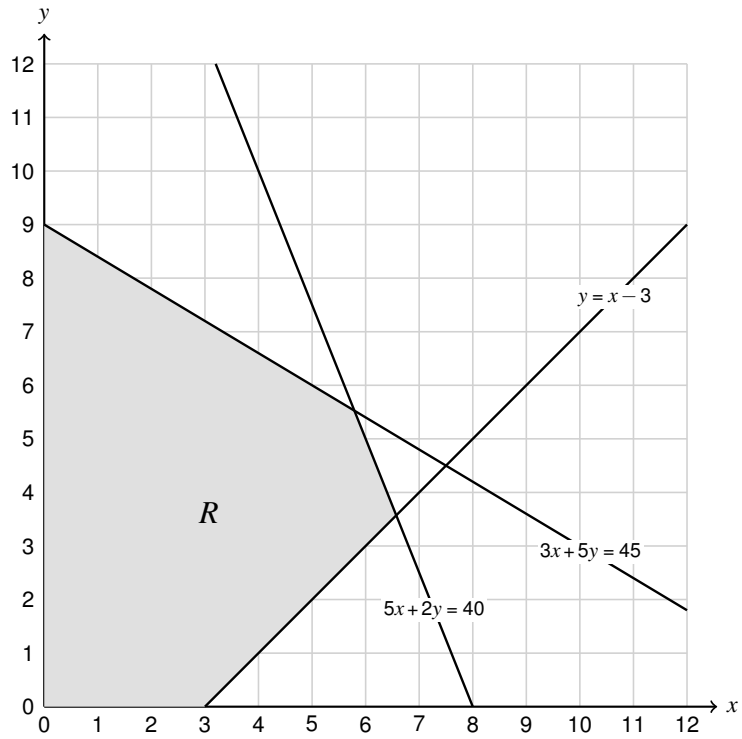

Diagram 4

(Total for Question 5 is 6 marks)

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6. A factory makes two drinks, A and B. In one day the bottling machine can fill either 400 litres of A or 300 litres of B, or any combination in proportion. At least 30% of the total output must be drink B, and at least 120 litres of A must be made each day. The profit is £0.80 per litre of A and £1.20 per litre of B. Let x and y be the numbers of litres of A and B made each day.

(a) Write down the objective.

(2)

(b) Show that the bottling machine gives the constraint $3x + 4y \leqslant 1200$

(3)

(c) Write down, simplifying where possible, two further constraints on x and y.

(3)

(d) Explain why the problem does not require integer solutions, and state one assumption of the model.

(2)

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Question 6 continued D1 ANSWER BOOK

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Objective: _____

Constraints: _____

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Question 6 continued

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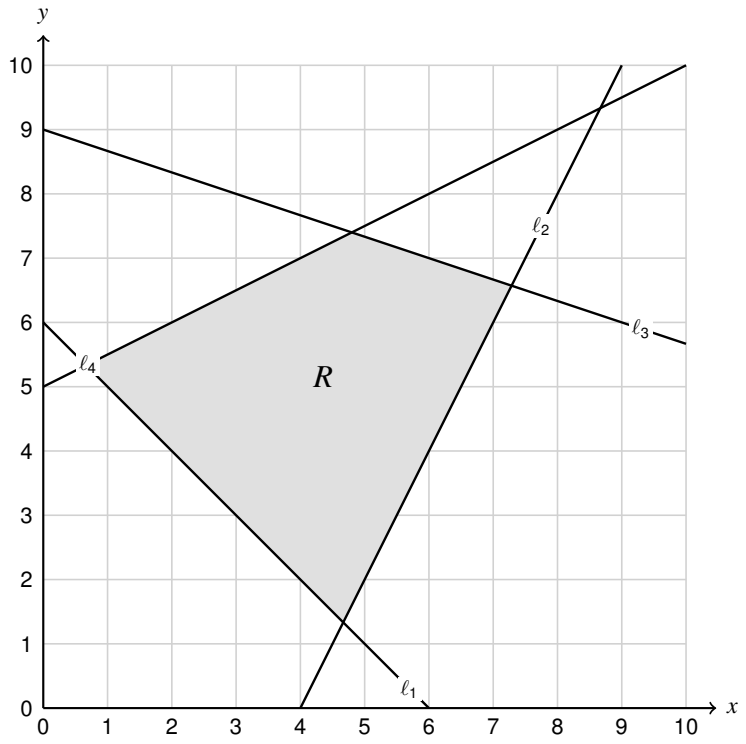

Figure 3

Figure 3 shows the lines ℓ_1: $x+y=6$, ℓ_2: $2x-y=8$, ℓ_3: $x+3y=27$ and ℓ_4: $2y-x=10$, and the feasible region R

(a) Write down the four inequalities that define R.

(2)

(b) The objective is to find the maximum and the minimum values of $P=2x+3y$ on R. Find each value exactly, and the vertex at which it occurs.

(5)

(c) Given that x and y must be integers, find the minimum value of P, and the point at which it occurs.

(3)

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Question 7 continued D1 ANSWER BOOK

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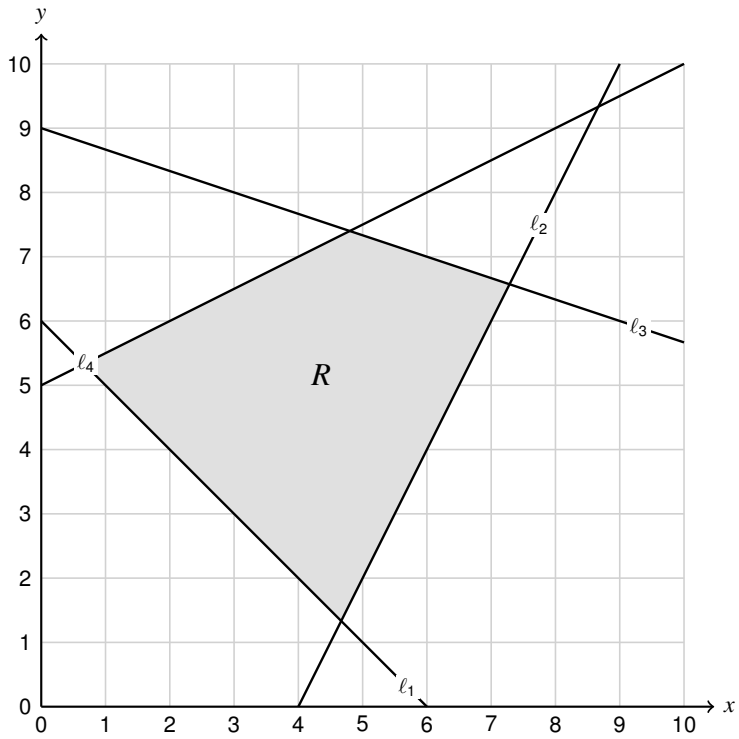

Diagram 5

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Question 7 continued

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8. A linear programming problem is: maximise $P = kx + y$, where k is a positive constant, subject to

$$x + 2y \leqslant 20, \qquad 3x + 2y \leqslant 36, \qquad y \leqslant 8, \qquad x \leqslant 10, \qquad x,\ y \geqslant 0$$

(a) Represent the constraints on Diagram 1 in the answer book, and label the feasible region R.

(3)

(b) Find the coordinates of the vertices of R.

(2)

(c) Given that the optimal point is the vertex $(8, 6)$, find the range of possible values of k.

(3)

(d) Given that $k = 1$, find the maximum value of P.

(2)

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Question 8 continued **D1 ANSWER BOOK**

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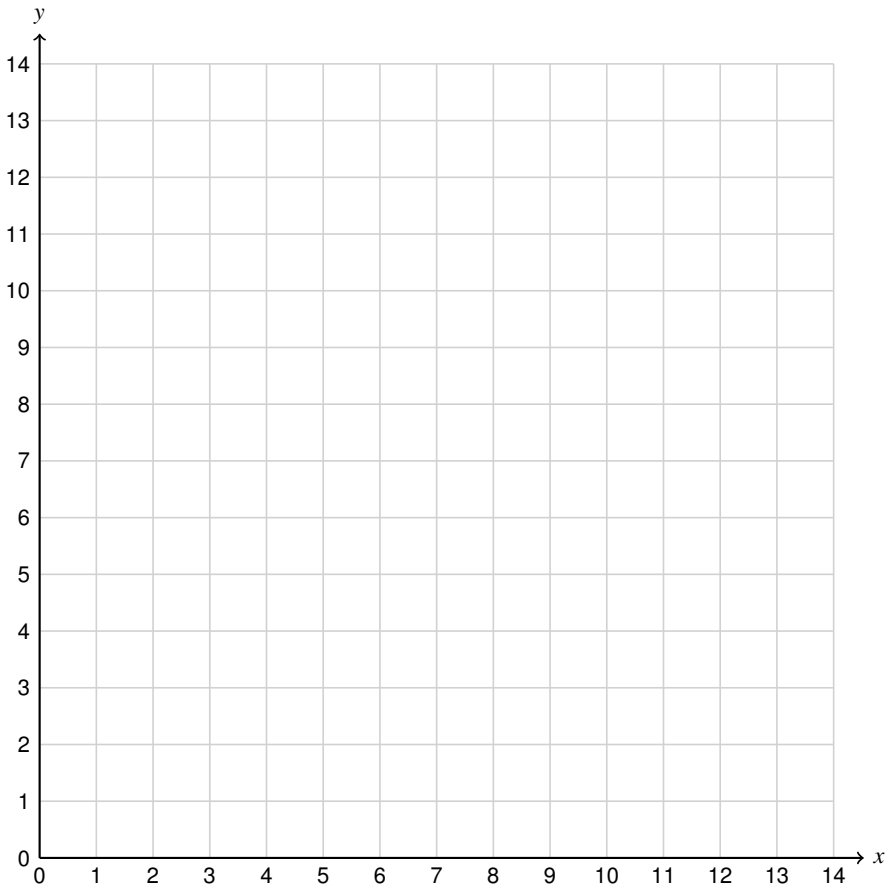

Diagram 6

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Question 8 continued

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(Total for Question 8 is 10 marks)

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9. A craftsperson makes chairs and tables. A chair takes 3 hours and uses 4 units of wood; a table takes 5 hours and uses 3 units of wood. Each week there are 45 hours and 48 units of wood available. At least twice as many chairs as tables must be made. The profit is £30 on a chair and £45 on a table.

(a) Formulate this as a linear programming problem, defining your variables.

(3)

(b) Represent the constraints on Diagram 1 in the answer book, and label the feasible region R.

(3)

(c) Using an objective line, find the numbers of chairs and tables that should be made each week to maximise the profit, and the maximum profit.

(4)

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Question 9 continued D1 ANSWER BOOK

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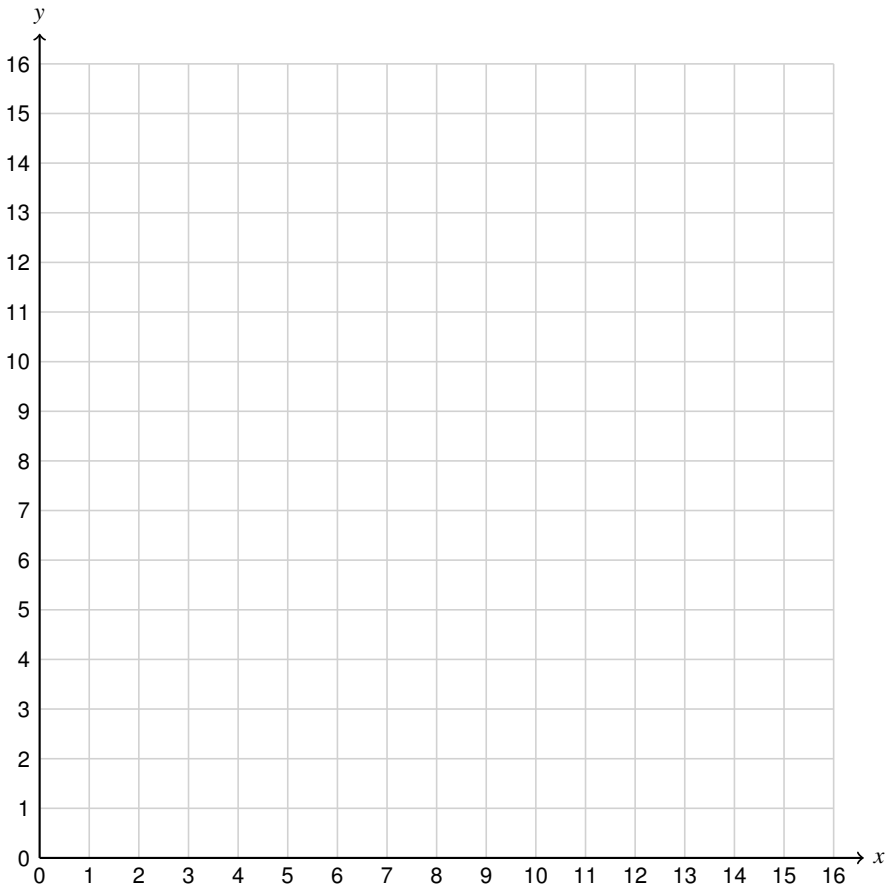

Diagram 7

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Question 9 continued

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10. An animal feed is made by mixing x kg of feed A and y kg of feed B. The mix must satisfy

$$6x + 2y \geqslant 25, \qquad 2x + 4y \geqslant 21, \qquad x + y \geqslant 8, \qquad x,\ y \geqslant 0$$

Feed A costs £5 per kg and feed B costs £4 per kg. The cost is to be minimised.

(a) Represent the constraints on Diagram 1 in the answer book, and label the feasible region R.

(3)

(b) Find the exact optimal point and the minimum cost.

(3)

(c) Feed is sold in whole kilograms only. Find the cheapest mix and its cost.

(2)

(d) The cost of feed A changes to £c per kg, while feed B still costs £4 per kg. Find the range of values of c for which your answer to part (b) is still optimal.

(2)

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Question 10 continued D1 ANSWER BOOK

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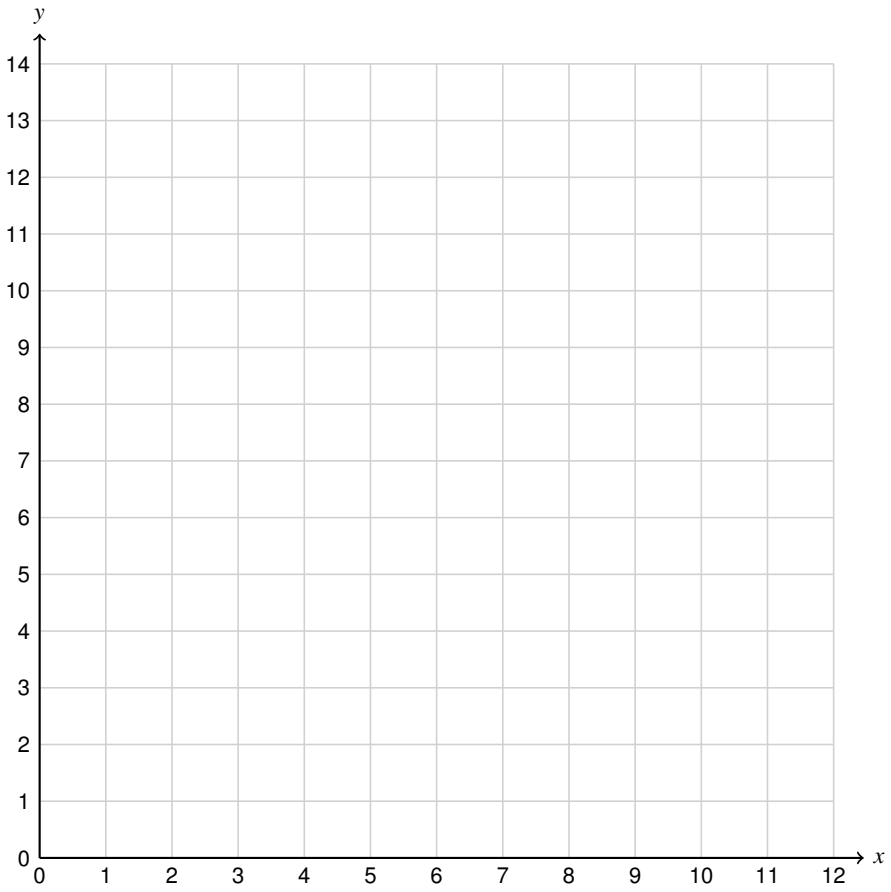

Diagram 8

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Question 10 continued

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TOTAL FOR CHAPTER 7 IS 80 MARKS

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