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INTERNATIONAL A LEVEL MATHEMATICS • MECHANICS 1

# Mechanics 1

Exam Questions by Topic

Pearson Edexcel International Advanced Level (WME01/01) • Calculator allowed • Take  $g = 9.8 \text{ m s}^{-2}$

Multi-step exam questions at the difficulty of the real WME01 paper, with a full mark scheme

| Chapters (tap a chapter to jump to it)                                                                                  | Qs | Marks | Page           | My score |
|-------------------------------------------------------------------------------------------------------------------------|----|-------|----------------|----------|
| <b>1 Mathematical Models in Mechanics</b><br>Not examined on its own: modelling ideas are tested within Chapters 2 to 8 | —  | —     | Not applicable |          |
| <b>2 Constant Acceleration</b><br>Motion graphs, the suvat formulae, vertical motion under gravity                      | 10 | 82    | 2              |          |
| <b>3 Vectors in Mechanics</b><br>Resultants, velocity and position vectors, $\mathbf{r} = \mathbf{r}_0 + \mathbf{v}t$   | 10 | 81    | 28             |          |
| <b>4 Dynamics of a Particle Moving in a Straight Line</b><br>$F = ma$ , connected particles, pulleys, lifts             | 10 | 82    | 54             |          |
| <b>5 Forces and Friction</b><br>Resolving forces, friction, inclined planes                                             | 10 | 86    | 80             |          |
| <b>6 Momentum and Impulse</b><br>Conservation of momentum, impulse                                                      | 10 | 81    | 106            |          |
| <b>7 Statics of a Particle</b><br>Equilibrium, limiting friction                                                        | 10 | 82    | 132            |          |
| <b>8 Moments</b><br>Beams and rods, tilting, non-uniform rods                                                           | 10 | 82    | 158            |          |

[Full mark scheme](#)

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## CHAPTER 2 • KEY SKILLS

## 2

## Constant Acceleration

Specification M1: Kinematics • 10 questions • 82 marks • about 98 minutes

## 1 Units

Work in metres and seconds:

$$1 \text{ km} = 1000 \text{ m}, 36 \text{ km h}^{-1} = 10 \text{ m s}^{-1}.$$

Give units with every answer.

## 2 The suvat formulae

$$v = u + at \quad s = \frac{1}{2}(u + v)t$$

$$s = ut + \frac{1}{2}at^2 \quad s = vt - \frac{1}{2}at^2$$

$$v^2 = u^2 + 2as \quad (\text{list } s, u, v, a, t \text{ first})$$

## 3 Direction and signs

Choose a positive direction. A

deceleration is a negative  $a$ ; give its size as a positive answer.

Displacement is not distance.

## 4 Velocity–time graphs

Gradient = acceleration; area =

distance. Straight lines only, and no solid vertical lines. Label the values on both axes.

## 5 Multi-stage motion

The final velocity of one stage is the initial velocity of the next. Keep a separate time for each stage.

## 6 Vertical motion

With up positive,  $a = -9.8 \text{ m s}^{-2}$ throughout the flight. At the top  $v = 0$ .After using  $g = 9.8$ , give 2 or 3 s.f.

## 7 Two objects

Write each displacement from the same point in terms of the same  $t$ , then equate: they meet or draw level.

## 8 Quadratics

Solve fully, then reject a root that is negative or outside the stage, giving a reason in context.

## 9 Other motion graphs

Displacement–time: gradient =

velocity. Acceleration–time: horizontal

lines for constant  $a$ . Average speed

$$= \frac{\text{total distance}}{\text{total time}}$$

**Skills map** every skill above is tested at least once; interleaved skills come from GCSE or other chapters

| Skill                                                                       | Q1 | Q2 | Q3 | Q4 | Q5 | Q6 | Q7 | Q8 | Q9 | Q10 |
|-----------------------------------------------------------------------------|----|----|----|----|----|----|----|----|----|-----|
| Marks                                                                       | 6  | 6  | 7  | 6  | 6  | 10 | 10 | 10 | 10 | 11  |
| 1 Units                                                                     |    |    | •  | •  |    |    |    | •  |    |     |
| 2 The suvat formulae                                                        | •  | •  | •  |    | •  | •  | •  | •  | •  | •   |
| 3 Direction and signs                                                       |    | •  |    | •  | •  | •  | •  |    | •  |     |
| 4 Velocity–time graphs                                                      |    |    | •  | •  |    | •  | •  | •  | •  | •   |
| 5 Multi-stage motion                                                        |    |    | •  |    |    | •  | •  | •  |    | •   |
| 6 Vertical motion                                                           |    | •  |    |    | •  |    | •  | •  | •  |     |
| 7 Two objects                                                               |    |    |    |    |    |    |    |    | •  | •   |
| 8 Quadratics                                                                |    | •  |    |    | •  | •  |    | •  |    | •   |
| 9 Other motion graphs                                                       |    |    |    | •  |    |    |    |    |    | •   |
| Interleaved: quadratics, completing the square, simultaneous equations (P1) | •  | •  |    |    | •  | •  |    | •  | •  | •   |
| Interleaved: geometric series (P2)                                          |    |    |    |    |    |    | •  |    |    |     |
| Interleaved: modelling assumptions (Chapter 1 ideas)                        |    |    |    |    |    |    | •  | •  |    | •   |

**Exam difficulty.** Questions 1 to 5 check each skill on its own. Questions 6 to 10 are multi-step questions built like the last questions on a real WME01 paper, where one part leads into the next and skills from different topics are combined.

**Must learn.** Take  $g = 9.8 \text{ m s}^{-2}$  (not 9.81) and give answers to 2 or 3 significant figures. Draw a sketch, choose a positive direction and keep to it. When a quadratic gives two roots, say why one is rejected.



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1. A car moves along a straight horizontal road with constant acceleration. The car passes the point  $P$  with speed  $u \text{ m s}^{-1}$  and, 6 seconds later, it passes the point  $Q$  with speed  $2.5u \text{ m s}^{-1}$ . The distance  $PQ$  is 84 m.

(a) Find the value of  $u$ .

(3)

(b) Find the acceleration of the car.

(1)

(c) Find the distance of the car from  $P$  at the instant when its speed is  $14 \text{ m s}^{-1}$

(2)

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## Question 1 continued

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(Total for Question 1 is 6 marks)



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2. A ball is projected vertically upwards with speed  $14.7 \text{ ms}^{-1}$  from a point  $A$  which is  $19.6 \text{ m}$  above horizontal ground. The ball moves freely under gravity until it hits the ground.

Find

- (a) the speed of the ball at the instant before it hits the ground, (3)

- (b) the time between the instant the ball is projected and the instant it hits the ground. (3)

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**Question 2 continued**

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## Question 3 continued

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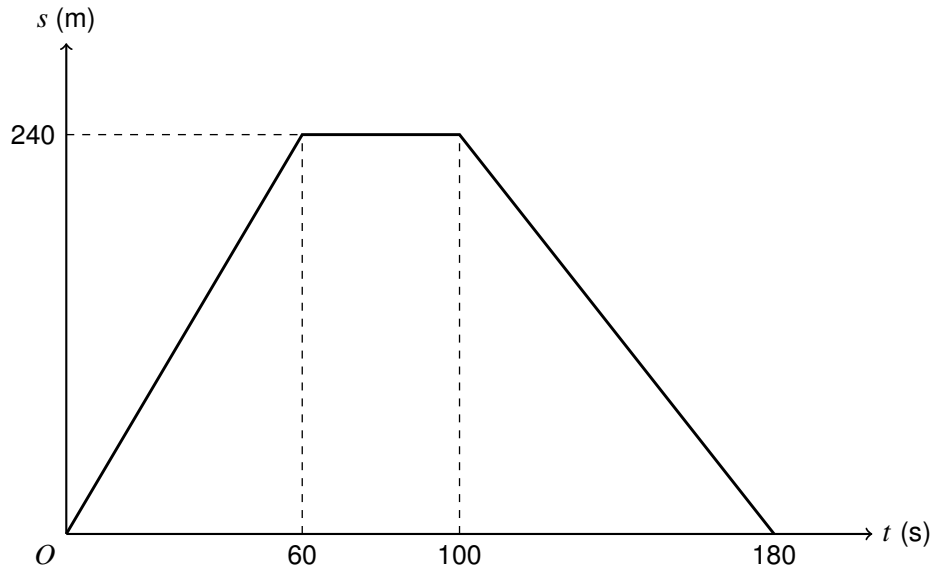


Figure 1

A cyclist rides along a straight path. Figure 1 shows the displacement–time graph for the motion of the cyclist, where  $s$  metres is the displacement of the cyclist from her starting point  $O$ ,  $t$  seconds after she leaves  $O$ .

- (a) Find the velocity of the cyclist for  $0 < t < 60$  and for  $100 < t < 180$  (2)
- (b) Find the average speed and the average velocity of the cyclist for the whole 180 seconds. (2)
- (c) Sketch a velocity–time graph for the motion of the cyclist for  $0 < t < 180$  (2)



**Question 4 continued**

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5. A ball is projected vertically upwards with speed  $24.5 \text{ ms}^{-1}$  from a point  $O$ . The ball moves freely under gravity.

(a) Find the greatest height above  $O$  reached by the ball.

(2)

(b) Find the length of time for which the ball is at least  $19.6 \text{ m}$  above  $O$ .

(4)

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## Question 5 continued

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(Total for Question 5 is 6 marks)



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- (a)** Sketch a speed–time graph for the motion of the cyclist from *A* to *C*.

**(b)** Show that  $5V^2 - 63V - 180 = 0$

**(c)** Hence find the value of  $V$ , explaining why there is only one possible value.

**(d)** Find the deceleration of the cyclist between  $B$  and  $C$ .

**(2)**



## Question 6 continued

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## Question 6 continued

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7. A small ball is released from rest at a point 10 m above horizontal ground. The ball falls freely under gravity. Each time the ball hits the ground it rebounds vertically upwards with half of the speed with which it hit the ground.
- (a) Find the speed of the ball at the instant before it first hits the ground. (2)
- (b) Find the greatest height reached by the ball after its first bounce. (2)
- (c) Show that the greatest heights after successive bounces form a geometric sequence and hence find the total distance travelled by the ball before it comes to rest. (3)
- (d) Taking downwards as positive, sketch a velocity–time graph for the motion of the ball from the instant it is released until it reaches its greatest height after the first bounce. Show the values of the velocity at the bounce. (2)
- (e) Give one reason why the model is unlikely to predict the total distance accurately. (1)



## Question 7 continued

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## Question 7 continued

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## Question 8 continued

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## Question 8 continued

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(Total for Question 8 is 10 marks)



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9. A ball  $A$  is projected vertically upwards with speed  $19.6 \text{ m s}^{-1}$  from a point  $O$  on horizontal ground. At the same instant, a ball  $B$  is released from rest at a point  $29.4 \text{ m}$  vertically above  $O$ . The balls move freely under gravity and they collide.
- (a) Show that the balls collide  $1.5 \text{ s}$  after they start to move. (3)
- (b) Find the height above  $O$  at which they collide. (2)
- (c) Find the velocity of each ball at the instant before they collide, and state whether  $A$  is moving up or down. (2)
- (d) Taking upwards as positive, sketch on the same axes the velocity–time graphs for the two balls from  $t = 0$  to  $t = 1.5$  (2)
- (e) Explain why the two lines in your sketch are parallel. (1)



## Question 9 continued

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## Question 9 continued

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(Total for Question 9 is 10 marks)



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A velocity-time graph showing the motion of two particles, P and Q. The vertical axis is velocity  $v$  in  $\text{ms}^{-1}$ , with markings at 16 and 24. The horizontal axis is time  $t$  in seconds, with a marking at 12. Particle Q moves with a constant velocity of  $16 \text{ ms}^{-1}$ , represented by a horizontal line. Particle P starts from rest and accelerates uniformly to  $24 \text{ ms}^{-1}$  at  $t = 12 \text{ s}$ , then continues at that constant velocity. The origin is labeled  $O$ .

Two cars  $P$  and  $Q$  move in the same direction along parallel straight lanes of a road. At time  $t = 0$ , car  $P$  starts from rest at the point  $O$  and car  $Q$  passes  $O$ . Figure 2 shows the velocity–time graphs for the two cars for  $0 \leq t \leq 24$

- (a) Show that, at  $t = 12$ , car  $Q$  is 48 m ahead of car  $P$ . (2)
- (b) Find the greatest distance between the two cars before car  $P$  draws level with car  $Q$ . (3)
- (c) Find the time at which car  $P$  draws level with car  $Q$ . (3)
- (d) Sketch an acceleration–time graph for car  $P$  for  $0 \leq t \leq 24$  (2)
- (e) Give one criticism of the model for the motion of car  $P$ . (1)

## Question 10 continued

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## Question 10 continued

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**TOTAL FOR CHAPTER 2 IS 82 MARKS**

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## CHAPTER 3 • KEY SKILLS

## 3

## Vectors in Mechanics

Specification M1: Vectors • 10 questions • 81 marks • about 97 minutes

**1 Magnitude and direction**

$|a\mathbf{i} + b\mathbf{j}| = \sqrt{a^2 + b^2}$ . For the angle, draw the vector and use  $\tan \theta = \frac{b}{a}$  on the right-angled triangle.

**2 Bearings**

Measured clockwise from north ( $\mathbf{j}$ ), three figures.  $4\mathbf{i} - 3\mathbf{j}$  points south of east:  $90^\circ + 36.9^\circ = 127^\circ$

**3 Adding vectors**

Add the  $\mathbf{i}$  and  $\mathbf{j}$  parts, or draw a triangle with the vectors top to tail and use the sine and cosine rules.

**4 Parallel vectors**

Parallel to  $2\mathbf{i} - \mathbf{j}$  means a multiple of it:  $x\mathbf{i} + y\mathbf{j} = \lambda(2\mathbf{i} - \mathbf{j})$ , so  $\frac{x}{2} = \frac{y}{-1}$

**5 Speed and velocity**

Speed =  $|\mathbf{v}|$ . Speed 26 in the direction  $5\mathbf{i} - 12\mathbf{j}$ :  $\mathbf{v} = \frac{26}{13}(5\mathbf{i} - 12\mathbf{j})$

**6 Position at time  $t$** 

$\mathbf{r} = \mathbf{r}_0 + \mathbf{v}t$ , where  $\mathbf{r}_0$  is the position when  $t = 0$ . From  $t = 1$  to  $t = 9$  is 8 seconds, not 9.

**7 Relative position**

$\vec{AB} = \mathbf{r}_B - \mathbf{r}_A$  and  $AB = |\vec{AB}|$ . Particles collide only if *both* components agree at the same  $t$ .

**8 Direction conditions**

Due north of  $O$ :  $\mathbf{i}$  part = 0. North-east of  $O$ : the  $\mathbf{i}$  and  $\mathbf{j}$  parts are equal and positive.

**9 Least distance**

$AB^2$  is a quadratic in  $t$ : complete the square (or differentiate) to find the least value and when it happens.

**Skills map** every skill above is tested at least once; interleaved skills come from GCSE or other chapters

| Skill                                                                       | Q1 | Q2 | Q3 | Q4 | Q5 | Q6 | Q7 | Q8 | Q9 | Q10 |
|-----------------------------------------------------------------------------|----|----|----|----|----|----|----|----|----|-----|
| Marks                                                                       | 6  | 6  | 6  | 6  | 6  | 10 | 10 | 10 | 10 | 11  |
| 1 Magnitude and direction                                                   | •  |    | •  |    |    | •  | •  | •  | •  |     |
| 2 Bearings                                                                  | •  | •  |    |    |    | •  | •  | •  | •  | •   |
| 3 Adding vectors                                                            |    | •  | •  |    |    |    |    |    | •  |     |
| 4 Parallel vectors                                                          |    |    | •  |    |    |    |    |    |    |     |
| 5 Speed and velocity                                                        |    |    |    | •  |    |    |    | •  | •  |     |
| 6 Position at time $t$                                                      | •  |    |    | •  | •  | •  | •  | •  |    | •   |
| 7 Relative position                                                         |    |    |    |    | •  | •  | •  |    |    | •   |
| 8 Direction conditions                                                      | •  |    |    | •  |    |    | •  | •  | •  |     |
| 9 Least distance                                                            |    |    |    |    |    |    |    |    |    | •   |
| Interleaved: trigonometry: sine and cosine rules (P1)                       |    | •  |    |    |    |    |    |    | •  |     |
| Interleaved: quadratics, completing the square, simultaneous equations (P1) |    |    |    |    | •  | •  |    |    |    | •   |
| Interleaved: speed, distance and time (Chapter 2)                           |    |    |    | •  |    | •  | •  |    | •  |     |

**Exam difficulty.** Questions 1 to 5 check each skill on its own. Questions 6 to 10 are multi-step questions built like the last questions on a real WME01 paper, where one part leads into the next and skills from different topics are combined.

**Must learn.** Draw a diagram with north marked. Give velocities in  $\mathbf{i}, \mathbf{j}$  form and speeds as magnitudes, bearings as three figures, and times as clock times when the question uses them.



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1. [In this question,  $\mathbf{i}$  and  $\mathbf{j}$  are horizontal unit vectors directed due east and due north respectively, and position vectors are given relative to a fixed origin  $O$ .]

A particle  $P$  moves with constant velocity  $(4\mathbf{i} - 3\mathbf{j}) \text{ m s}^{-1}$ . At time  $t = 0$ ,  $P$  passes through the point with position vector  $(-10\mathbf{i} + 18\mathbf{j}) \text{ m}$ .

- (a) Find the direction of motion of  $P$ , giving your answer as a bearing to the nearest degree.

(2)

- (b) Write down the position vector of  $P$  at time  $t$  seconds.

(1)

- (c) Find the value of  $t$  when  $P$  is due east of  $O$ , and the distance of  $P$  from  $O$  at this time.

(3)

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## Question 1 continued

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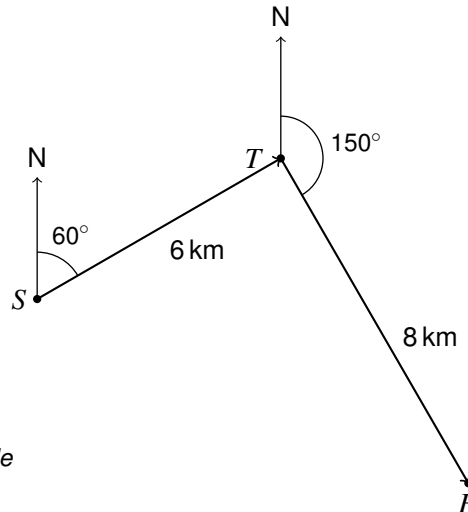
(Total for Question 1 is 6 marks)



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2.

**Figure 1**

A hiker walks 6 km in a straight line from her start point  $S$  on a bearing of  $060^\circ$  to the point  $T$ . She then walks 8 km in a straight line on a bearing of  $150^\circ$  to the point  $F$ , as shown in Figure 1.

(a) Find the distance  $SF$ . (3)

(b) Find the bearing of  $F$  from  $S$ , giving your answer to the nearest degree. (3)



**Question 2 continued**

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3. A small robot moves across a horizontal floor. It first moves with displacement  $\mathbf{a} = (3\mathbf{i} + 4\mathbf{j})$  m and then with displacement  $\mathbf{b} = (p\mathbf{i} - 6\mathbf{j})$  m, where  $p$  is a constant.
- (a) Find the angle between  $\mathbf{a}$  and the vector  $\mathbf{i}$ , giving your answer in degrees to one decimal place. (2)

The resultant displacement  $\mathbf{a} + \mathbf{b}$  is parallel to the vector  $(2\mathbf{i} - \mathbf{j})$ .

- (b) Find the value of  $p$ . (3)
- (c) Find the magnitude of the resultant displacement. (1)

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**(Total for Question 3 is 6 marks)****Join students already acing their exams.**Book your FREE lesson at [www.eclassroom.org.uk](http://www.eclassroom.org.uk)

4. [In this question,  $\mathbf{i}$  and  $\mathbf{j}$  are horizontal unit vectors directed due east and due north respectively, and position vectors are given relative to a fixed origin  $O$ .]

A ship moves with constant speed  $26 \text{ km h}^{-1}$  in the direction of the vector  $(5\mathbf{i} - 12\mathbf{j})$ . At 09:00 the ship is at the point with position vector  $(-20\mathbf{i} + 60\mathbf{j}) \text{ km}$ .

- (a) Find the velocity of the ship in the form  $(x\mathbf{i} + y\mathbf{j}) \text{ km h}^{-1}$  (2)
- (b) Find the position vector of the ship  $t$  hours after 09:00 (1)
- (c) Find the time of day at which the ship is due east of  $O$ , and its distance from  $O$  at this time. (3)

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(Total for Question 4 is 6 marks)



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5. [In this question,  $\mathbf{i}$  and  $\mathbf{j}$  are perpendicular horizontal unit vectors, and position vectors are given relative to a fixed origin  $O$ .]

Two particles  $A$  and  $B$  move on a smooth horizontal plane with constant velocities  $(2\mathbf{i} + \mathbf{j}) \text{ m s}^{-1}$  and  $(-\mathbf{i} + 4\mathbf{j}) \text{ m s}^{-1}$  respectively. At time  $t = 0$ ,  $A$  is at the point with position vector  $(\mathbf{i} + 2\mathbf{j}) \text{ m}$  and  $B$  is at the point with position vector  $(7\mathbf{i} - 4\mathbf{j}) \text{ m}$ .

- (a) Write down the position vectors of  $A$  and  $B$  at time  $t$  seconds.

(2)

- (b) Show that  $A$  and  $B$  collide, and find the position vector of the point where they collide.

(4)

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## Question 5 continued

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(Total for Question 5 is 6 marks)



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6. [In this question,  $\mathbf{i}$  and  $\mathbf{j}$  are horizontal unit vectors directed due east and due north respectively, and position vectors are given relative to a fixed origin  $O$ .]

Two ships  $A$  and  $B$  are moving with constant velocities. The velocity of  $A$  is  $(4\mathbf{i} + 3\mathbf{j}) \text{ km h}^{-1}$  and the velocity of  $B$  is  $(p\mathbf{i} + q\mathbf{j}) \text{ km h}^{-1}$ . At 14:00,  $A$  is at the point with position vector  $(2\mathbf{i} - 3\mathbf{j}) \text{ km}$  and  $B$  is at the point with position vector  $(12\mathbf{i} + 7\mathbf{j}) \text{ km}$ .

- (a) Find the speed of  $A$ .

(1)

At time  $t$  hours after 14:00,  $\vec{AB} = [(10 - 6t)\mathbf{i} + (10 - 8t)\mathbf{j}] \text{ km}$

- (b) Find the value of  $p$  and the value of  $q$ .

(4)

- (c) Find the two times of day at which the ships are  $2\sqrt{10} \text{ km}$  apart.

(3)

- (d) Find the bearing of  $B$  from  $A$  at the later of these two times, giving your answer to the nearest degree.

(2)



## Question 6 continued

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## Question 6 continued

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7. [In this question,  $\mathbf{i}$  and  $\mathbf{j}$  are horizontal unit vectors directed due east and due north respectively, and position vectors are given relative to a fixed origin  $O$ .]

A boat leaves a harbour at  $O$  at 12:00 and moves with constant velocity  $(6\mathbf{i} + 8\mathbf{j}) \text{ km h}^{-1}$ . At 13:30 the boat changes direction and then moves with constant velocity  $(-4\mathbf{i} + 3\mathbf{j}) \text{ km h}^{-1}$ . A lighthouse  $L$  is at the point with position vector  $(5\mathbf{i} + 18\mathbf{j}) \text{ km}$ .

- (a) Find the speed of the boat before 13:30 (1)
- (b) Find the position vector of the boat at 13:30 (2)
- (c) Find the position vector of the boat  $s$  hours after 13:30 (2)
- (d) Find the time of day at which the boat is due west of  $L$ , and its distance from  $L$  at that time. (3)
- (e) Find the bearing of  $L$  from the boat at 13:30, giving your answer to the nearest degree. (2)



## Question 7 continued

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8. [In this question,  $\mathbf{i}$  and  $\mathbf{j}$  are horizontal unit vectors directed due east and due north respectively, and position vectors are given relative to a fixed origin  $O$ .]

A puck moves in a straight line with constant velocity across a smooth horizontal ice rink. At time  $t = 2$  seconds the puck is at the point with position vector  $(-3\mathbf{i} + 5\mathbf{j})$  m and at time  $t = 5$  seconds it is at the point with position vector  $(12\mathbf{i} + 11\mathbf{j})$  m.

- (a) Find the velocity of the puck. (2)
- (b) Find the speed of the puck and its direction of motion as a bearing, to the nearest degree. (3)
- (c) Find the position vector of the puck at time  $t$  seconds. (2)
- (d) Find the time at which the puck is north-east of  $O$ . (2)
- (e) Find the distance of the puck from  $O$  at this time. (1)



## Question 8 continued

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## Question 8 continued

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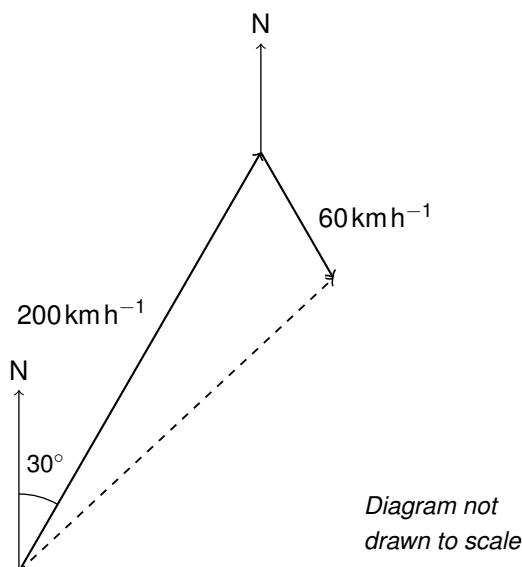


Figure 2

[In this question,  $\mathbf{i}$  and  $\mathbf{j}$  are horizontal unit vectors directed due east and due north respectively.]

An aircraft is flying with a speed of  $200 \text{ km h}^{-1}$  relative to the air, heading on a bearing of  $030^\circ$ . A wind is blowing at  $60 \text{ km h}^{-1}$  towards the bearing  $150^\circ$ . The velocity of the aircraft over the ground is the sum of these two velocities, shown by the dashed line in Figure 2.

- (a) Find the speed of the aircraft over the ground.

(3)

- (b) Find the bearing of the direction in which the aircraft is moving over the ground, to the nearest degree.

(3)

The pilot now changes heading, keeping a speed of  $200 \text{ km h}^{-1}$  relative to the air, so that the aircraft moves due east over the ground. The wind is unchanged.

- (c) Find the new heading of the aircraft as a bearing, to the nearest degree, and its new speed over the ground.

(4)



## Question 9 continued

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## Question 9 continued

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10. [In this question,  $\mathbf{i}$  and  $\mathbf{j}$  are horizontal unit vectors directed due east and due north respectively, and position vectors are given relative to a fixed origin  $O$ .]

Two ships  $A$  and  $B$  move with constant velocities  $(\mathbf{i} + 3\mathbf{j}) \text{ km h}^{-1}$  and  $(4\mathbf{i} - 3\mathbf{j}) \text{ km h}^{-1}$  respectively. At 12:00,  $A$  is at the point with position vector  $(5\mathbf{i} + 2\mathbf{j}) \text{ km}$  and  $B$  is at the point with position vector  $(-13\mathbf{i} + 23\mathbf{j}) \text{ km}$ .

- (a) Show that both ships pass through the point  $X$  with position vector  $(7\mathbf{i} + 8\mathbf{j}) \text{ km}$ , and explain why they do not collide. (3)
- (b) Show that,  $t$  hours after 12:00,  $AB^2 = 45(t^2 - 8t + 17)$  (3)
- (c) Find the least distance between the ships and the time of day at which it occurs. (3)
- (d) Find the bearing of  $B$  from  $A$  at this time, to the nearest degree. (2)



## Question 10 continued

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## Question 10 continued

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**TOTAL FOR CHAPTER 3 IS 81 MARKS**

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## CHAPTER 4 • KEY SKILLS

## 4

## Dynamics of a Particle Moving in a Straight Line

Specification M1: Dynamics • 10 questions • 82 marks • about 98 minutes

**1 Force diagrams**

Mark every force: weight  $mg$ , normal reaction  $R$ , tension  $T$ , thrust, resistance. Mass in kg, weight in newtons.

**2 Newton's second law**

Resultant force in the direction of the acceleration  $= ma$ . Put  $g$  in the weight, never in the  $ma$  term.

**3 Vertical motion and lifts**

Upwards:  $T - mg = ma$ . Decelerating upwards means  $a$  acts downwards. The floor pushes up on a passenger with force  $R$ .

**4 Forces as vectors**

$\mathbf{F}_1 + \mathbf{F}_2 + \dots = m\mathbf{a}$ . Then

$|\mathbf{a}| = \sqrt{a_1^2 + a_2^2}$ : give a magnitude when asked, not a vector.

**5 Connected particles**

Whole system for  $a$  (internal forces cancel), then one part for the tension or thrust. A towbar can push (thrust).

**6 Pulleys**

Light inextensible string and smooth pulley: same tension both sides, same acceleration. Force on the pulley: add the two tensions as vectors.

**7 Newton's third law**

$B$  pushes on  $A$  with the same size force as  $A$  pushes on  $B$ , in the opposite direction.

**8 When the motion changes**

If a string breaks or goes slack, find the new acceleration. The final speed of one stage starts the next.

**9 Modelling**

Light: same tension throughout the string. Inextensible: same acceleration. Smooth pulley: no friction at the pulley.

**Skills map** every skill above is tested at least once; interleaved skills come from GCSE or other chapters

| Skill                                                              | Q1 | Q2 | Q3 | Q4 | Q5 | Q6 | Q7 | Q8 | Q9 | Q10 |
|--------------------------------------------------------------------|----|----|----|----|----|----|----|----|----|-----|
| Marks                                                              | 6  | 6  | 6  | 6  | 6  | 10 | 10 | 10 | 10 | 12  |
| 1 Force diagrams                                                   |    |    | •  | •  | •  | •  | •  |    |    | •   |
| 2 Newton's second law                                              | •  | •  | •  | •  | •  | •  | •  | •  | •  | •   |
| 3 Vertical motion and lifts                                        |    |    |    | •  | •  | •  | •  |    |    |     |
| 4 Forces as vectors                                                | •  |    |    |    |    |    |    |    | •  |     |
| 5 Connected particles                                              |    |    | •  | •  |    |    | •  | •  |    |     |
| 6 Pulleys                                                          |    |    |    |    |    | •  |    |    |    | •   |
| 7 Newton's third law                                               |    |    | •  |    |    |    | •  |    |    |     |
| 8 When the motion changes                                          |    |    |    |    | •  | •  | •  | •  |    | •   |
| 9 Modelling                                                        |    |    |    |    |    | •  |    |    |    | •   |
| Interleaved: constant acceleration and vertical motion (Chapter 2) |    | •  | •  |    | •  | •  |    | •  |    | •   |
| Interleaved: vectors, magnitude, direction (Chapter 3)             | •  |    |    |    |    |    |    |    | •  | •   |
| Interleaved: simultaneous equations (P1)                           | •  |    |    |    |    | •  |    |    |    | •   |

**Exam difficulty.** Questions 1 to 5 check each skill on its own. Questions 6 to 10 are multi-step questions built like the last questions on a real WME01 paper, where one part leads into the next and skills from different topics are combined.

**Must learn.** Draw a separate force diagram for each particle. Write each equation of motion for one named particle (or the whole system) with its own mass in the  $ma$  term. After using  $g = 9.8$ , give answers to 2 or 3 s.f.



1. [In this question,  $\mathbf{i}$  and  $\mathbf{j}$  are perpendicular horizontal unit vectors.]

Three forces  $(3\mathbf{i} - 2\mathbf{j})$  N,  $(-\mathbf{i} + 5\mathbf{j})$  N and  $(p\mathbf{i} + q\mathbf{j})$  N act on a particle  $P$  of mass 0.5 kg. The acceleration of  $P$  is  $(8\mathbf{i} - 6\mathbf{j}) \text{ m s}^{-2}$

- (a) Find the value of  $p$  and the value of  $q$ .

(3)

- (b) Find the magnitude of the acceleration of  $P$ .

(1)

- (c) Find the angle between the acceleration of  $P$  and the vector  $\mathbf{i}$

(2)

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## Question 1 continued

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(Total for Question 1 is 6 marks)



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2. A car of mass 900 kg moves along a straight horizontal road. The engine of the car produces a constant driving force and the resistance to the motion of the car is modelled as a constant force of magnitude 450 N. The speed of the car increases from  $10 \text{ m s}^{-1}$  to  $25 \text{ m s}^{-1}$  while it travels a distance of 150 m.

**(a)** Find the acceleration of the car.

(2)

**(b)** Find the magnitude of the driving force.

(2)

When the car reaches  $25 \text{ m s}^{-1}$  the engine is switched off. The resistance is unchanged.

**(c)** Find the time taken for the car to come to rest.

(2)

## Question 2 continued

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3. A car of mass 1200 kg is towing a trailer of mass 400 kg along a straight horizontal road using a light rigid horizontal towbar. The resistances to motion of the car and of the trailer are modelled as constant forces of magnitude 300 N and 100 N respectively. When the car and trailer are moving at  $20 \text{ ms}^{-1}$ , the driver applies the brakes, which produce a constant braking force of 3000 N on the car only.

(a) Find the deceleration of the car and trailer.

(2)

(b) Find the magnitude of the force in the towbar, stating whether it is a tension or a thrust.

(3)

(c) Find the distance travelled from the instant the brakes are applied until the car stops.

(1)

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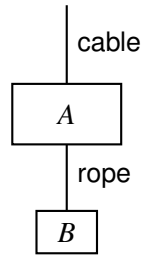
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4.

**Figure 1**

A crane lifts two crates vertically. Crate *A*, of mass 200 kg, is attached to the crane's cable. Crate *B*, of mass 50 kg, hangs below *A*, attached to it by a light inextensible rope, as shown in Figure 1. The crates are modelled as particles and air resistance is ignored. The crates are moving upwards and the tension in the cable is 2800 N.

(a) Find the acceleration of the crates. (2)

(b) Find the tension in the rope. (2)

(c) Find the tension in the cable when the crates move upwards at constant speed. (1)

Later, the crates are moving downwards and slowing down with deceleration  $0.6 \text{ ms}^{-2}$

(d) Find the tension in the rope. (1)



## Question 4 continued

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5. A ball of mass  $0.5 \text{ kg}$  is projected vertically upwards with speed  $18 \text{ m s}^{-1}$ . The air resistance acting on the ball is modelled as a constant force of magnitude  $1.1 \text{ N}$ .

(a) Find the deceleration of the ball as it moves upwards. (2)

(b) Find the greatest height above the point of projection reached by the ball. (2)

(c) Find the speed of the ball when it returns to the point of projection. (2)

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## Question 5 continued

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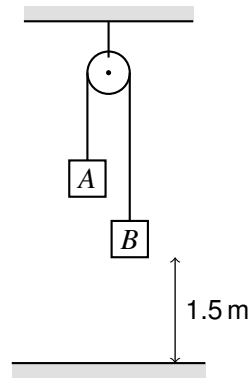
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6.

**Figure 2**

Two particles  $A$  and  $B$ , of mass  $2\text{ kg}$  and  $3\text{ kg}$  respectively, are attached to the ends of a light inextensible string. The string passes over a small smooth pulley which is fixed in a horizontal ceiling. The particles hang with the string taut and the hanging parts of the string vertical.  $B$  is  $1.5\text{ m}$  above horizontal ground, as shown in Figure 2. The system is released from rest. In the subsequent motion,  $A$  does not reach the pulley.

- (a) Find the acceleration of  $B$  and the tension in the string while both particles are moving. (5)
- (b) Find the speed of  $B$  at the instant it reaches the ground. (2)
- $B$  does not rebound from the ground.
- (c) Find the greatest height of  $A$  above its initial position. (2)
- (d) State how you have used the fact that the string is inextensible. (1)



## Question 6 continued

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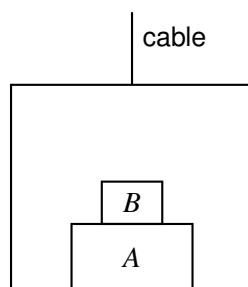
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**7.**



### Figure 3

A lift of mass 300 kg is raised and lowered by a vertical cable attached to the top of the lift. A crate  $A$  of mass 40 kg rests on the horizontal floor of the lift, and a box  $B$  of mass 10 kg rests on top of  $A$ , as shown in Figure 3. The cable is modelled as light and inextensible, and  $A$  and  $B$  as particles. The lift is moving upwards with constant acceleration and the tension in the cable is 3850 N.

- (a) Show that the acceleration of the lift is  $1.2 \text{ m s}^{-2}$  (3)
- (b) Find the magnitude of the force exerted by  $B$  on  $A$ . (2)
- (c) Find the magnitude of the force exerted by the floor of the lift on  $A$ . (2)

Later, the lift is moving upwards and decelerating at a constant rate, and the force exerted by the floor of the lift on  $A$  is  $440\text{ N}$ .

- (d)** Find the deceleration of the lift and the tension in the cable. **(3)**

[illegible]

## Question 7 continued

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8. An engine of mass 20 000 kg pulls a carriage of mass 10 000 kg along a straight horizontal track. The carriage is attached to the engine by a light horizontal coupling. The engine produces a constant driving force of 30 000 N. The resistances to motion of the engine and of the carriage are constant forces of magnitude 5000 N and 2500 N respectively. The train starts from rest.

**(a)** Find the acceleration of the train.

**(2)**

**(b)** Find the tension in the coupling.

**(2)**

**(c)** Find the speed of the train 40 s after it starts.

(1)

At this instant the coupling breaks. The driving force and the resistances are unchanged.

(d) Find the distance travelled by the carriage from the instant the coupling breaks until it comes to rest.

**(3)**

(e) Find the distance between the engine and the carriage at the instant the carriage comes to rest.

**(2)**

## Question 8 continued

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## Question 8 continued

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9. [In this question,  $\mathbf{i}$  and  $\mathbf{j}$  are horizontal unit vectors directed due east and due north respectively.]

A particle  $P$  of mass  $2\text{ kg}$  moves on a smooth horizontal surface under the action of two forces  $(4\mathbf{i} - 2\mathbf{j})\text{ N}$  and  $(p\mathbf{i} + 6\mathbf{j})\text{ N}$ , where  $p$  is a constant. The resultant of the two forces is parallel to the vector  $(\mathbf{i} + 2\mathbf{j})$ .

- (a) Find the value of  $p$ . (3)

- (b) Find the magnitude of the acceleration of  $P$ . (2)

At time  $t = 0$ ,  $P$  is moving with velocity  $(3\mathbf{i} - 4\mathbf{j})\text{ m s}^{-1}$

- (c) Find the time at which  $P$  is moving due east, and its speed at this time. (3)

- (d) Find the distance travelled by  $P$  from  $t = 0$  to this time. (2)



## Question 9 continued

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## Question 9 continued

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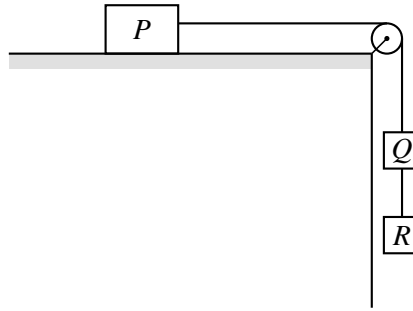
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(Total for Question 9 is 10 marks)



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## Question 10 continued

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## Question 10 continued

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(Total for Question 10 is 12 marks)

**TOTAL FOR CHAPTER 4 IS 82 MARKS**

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## CHAPTER 5 • KEY SKILLS

## 5

## Forces and Friction

Specification M1: Forces and friction • 10 questions • 86 marks • about 103 minutes

**1 Resolving a force**

A force  $P$  at  $\theta$  to a direction has components  $P \cos \theta$  along it and  $P \sin \theta$  perpendicular to it.

**2 Inclined planes**

Weight  $mg$ :  $mg \sin \alpha$  down the plane and  $mg \cos \alpha$  into the plane. Resolve parallel and perpendicular to the plane.

**3 Normal reaction**

Resolve perpendicular to the surface:  $R$  is not always  $mg$  or  $mg \cos \alpha$ . A pull upwards reduces  $R$ ; a push downwards increases it.

**4 Friction**

$F \leq \mu R$ . When sliding,  $F = \mu R$  and it opposes the motion, so it reverses when the motion reverses.

**5 Equations of motion**

Along the plane: resultant  $= ma$ .  
Perpendicular to the plane: resultant  $= 0$ . Constant speed means  $a = 0$ .

**6 Exact trigonometry**

$\tan \alpha = \frac{3}{4}$  gives  $\sin \alpha = \frac{3}{5}$ ,  $\cos \alpha = \frac{4}{5}$ ;  
 $\tan \alpha = \frac{5}{12}$  gives  $\frac{5}{13}$ ,  $\frac{12}{13}$ . Keep these exact.

**7 Will it move?**

Compare the force trying to move the particle with the greatest possible friction  $\mu R$ . On a plane alone: slides if  $\tan \alpha > \mu$

**8 Pulleys on planes**

One equation for each particle. Force on the pulley: add the two tensions, using the angle between the two parts of the string.

**9 When forces change**

Removing a force changes  $R$ , so friction and the acceleration change. Show an inequality when asked whether something is reached.

**Skills map** every skill above is tested at least once; interleaved skills come from GCSE or other chapters

| Skill                                                            | Q1 | Q2 | Q3 | Q4 | Q5 | Q6 | Q7 | Q8 | Q9 | Q10 |
|------------------------------------------------------------------|----|----|----|----|----|----|----|----|----|-----|
| Marks                                                            | 6  | 6  | 6  | 6  | 6  | 12 | 11 | 11 | 10 | 12  |
| 1 Resolving a force                                              | •  | •  |    | •  |    | •  |    |    |    |     |
| 2 Inclined planes                                                |    | •  |    |    | •  | •  | •  |    | •  | •   |
| 3 Normal reaction                                                | •  | •  |    | •  |    | •  |    |    |    |     |
| 4 Friction                                                       | •  |    | •  | •  | •  | •  | •  | •  |    | •   |
| 5 Equations of motion                                            | •  | •  | •  | •  | •  | •  | •  | •  | •  | •   |
| 6 Exact trigonometry                                             |    |    |    |    | •  |    | •  |    |    | •   |
| 7 Will it move?                                                  |    |    |    |    | •  | •  |    |    |    | •   |
| 8 Pulleys on planes                                              |    |    |    |    |    |    | •  | •  |    |     |
| 9 When forces change                                             |    |    |    | •  |    | •  | •  | •  | •  |     |
| Interleaved: constant acceleration and motion graphs (Chapter 2) |    |    | •  |    |    | •  | •  | •  | •  | •   |
| Interleaved: connected particles and pulleys (Chapter 4)         |    |    |    |    |    |    | •  | •  | •  |     |
| Interleaved: trigonometry and inequalities (P1)                  |    | •  |    |    | •  |    | •  | •  |    |     |

**Exam difficulty.** Questions 1 to 5 check each skill on its own. Questions 6 to 10 are multi-step questions built like the last questions on a real WME01 paper, where one part leads into the next and skills from different topics are combined.

**Must learn.** Draw a force diagram with friction opposing the motion. Resolve perpendicular to the surface first to find  $R$ , including every force with a perpendicular component. Use exact values of  $\sin \alpha$  and  $\cos \alpha$  when  $\tan \alpha$  is given.



A diagram showing a rectangular block on a horizontal surface. A force vector  $P$  is applied to the top-right corner of the block, pointing upwards and to the right. The force vector is labeled  $P$  at its tip. The angle between the force vector and a horizontal dashed line extending from the point of application is  $30^\circ$ .

## Question 1 continued

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(Total for Question 1 is 6 marks)



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**Question 2 continued**

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**(Total for Question 2 is 6 marks)****Join students already acing their exams.**Book your FREE lesson at [www.eclassroom.org.uk](http://www.eclassroom.org.uk)

3. A small stone slides in a straight line across a rough horizontal sheet of ice. The stone passes a point  $A$  with speed  $8 \text{ m s}^{-1}$  and comes to rest  $10 \text{ m}$  from  $A$ . The stone is modelled as a particle.
- (a) Find the deceleration of the stone. (2)
- (b) Find the coefficient of friction between the stone and the ice. (2)
- (c) Find the time taken for the stone to come to rest after passing  $A$ . (1)
- (d) Explain why your answer to part (b) does not depend on the mass of the stone. (1)

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## Question 3 continued

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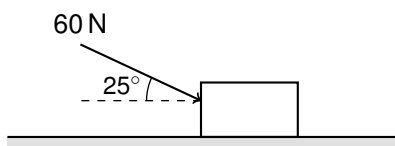
(Total for Question 3 is 6 marks)



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**4.**



### Figure 3

A trolley of mass 15 kg is pushed along a rough horizontal floor by a force of magnitude 60 N acting at  $25^\circ$  below the horizontal, as shown in Figure 3. The trolley moves in a straight line at constant speed. The trolley is modelled as a particle.

- (a)** Find the magnitude of the normal reaction on the trolley. **(2)**

- (b)** Find the coefficient of friction between the trolley and the floor. **(2)**

The 60 N force is now applied horizontally.

- (c)** Find the acceleration of the trolley. **(2)**

## Question 4 continued

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5. A particle is placed at rest on a rough plane inclined at an angle  $\alpha$  to the horizontal, where  $\tan \alpha = \frac{5}{12}$ . The coefficient of friction between the particle and the plane is 0.3. The particle is released.

(a) Show that the particle slides down the plane.

(3)

(b) Find the acceleration of the particle.

(2)

(c) State the least value of the coefficient of friction for which the particle would remain at rest.

(1)

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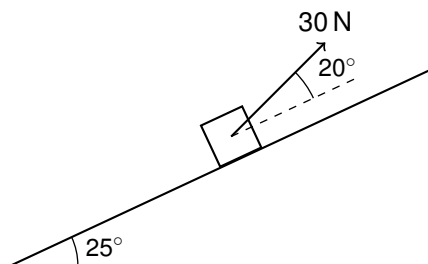


Figure 4

A parcel of mass 3 kg is pulled up a rough plane inclined at  $25^\circ$  to the horizontal by a force of magnitude 30 N. The force acts at  $20^\circ$  to the plane, as shown in Figure 4, and its line of action lies in a vertical plane containing a line of greatest slope of the plane. The coefficient of friction between the parcel and the plane is 0.4. The parcel is modelled as a particle.

(a) Find the acceleration of the parcel.

(5)

The points  $A$  and  $B$  lie on a line of greatest slope of the plane, with  $B$  above  $A$  and  $AB = 5$  m. The parcel passes  $A$  with speed  $2 \text{ ms}^{-1}$ , moving up the plane.

(b) Find the speed of the parcel at  $B$ .

(2)

The 30 N force is removed at the instant the parcel passes  $B$ . The parcel comes to instantaneous rest at the point  $C$ .

(c) Find the distance  $BC$ .

(3)

(d) Determine whether the parcel remains at rest at  $C$ , giving a reason.

(2)



## Question 6 continued

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## Question 6 continued

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The diagram shows a right-angled triangle representing an inclined plane. The angle at the bottom-left vertex is labeled  $\alpha$ . A block labeled 'A' is positioned on the inclined surface. A rope is attached to block 'A', runs parallel to the incline up to the top-right vertex, passes over a pulley (represented by a small circle), and then hangs vertically down to a second block labeled 'B'.

A particle  $A$  of mass  $4\text{ kg}$  lies on a rough plane inclined at an angle  $\alpha$  to the horizontal, where  $\tan \alpha = \frac{3}{4}$ . It is attached to one end of a light inextensible string which passes over a small smooth pulley fixed at the top of the plane. The other end of the string is attached to a particle  $B$  of mass  $5\text{ kg}$ , which hangs freely below the pulley, as shown in Figure 5. The part of the string from  $A$  to the pulley lies along a line of greatest slope of the plane. The coefficient of friction between  $A$  and the plane is  $0.25$ . The system is released from rest with the string taut, and  $A$  moves up the plane.

- (a) Find the frictional force acting on  $A$ . (2)
- (b) Show that the acceleration of  $A$  is  $0.2g$ , and find the tension in the string. (4)
- (c) Find the magnitude of the force exerted on the pulley by the string. (2)
- $B$  was released 1 m above the ground and does not rebound when it reaches the ground.  $A$  does not reach the pulley.
- (d) Find the further distance that  $A$  moves up the plane after  $B$  reaches the ground. (3)

## Question 7 continued

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## Question 7 continued

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(Total for Question 7 is 11 marks)



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- 8.** A particle  $A$  of mass  $2\text{ kg}$  rests on a rough horizontal table. It is attached to one end of a light inextensible string which passes over a small smooth pulley fixed at the edge of the table,  $3\text{ m}$  from  $A$ . The other end of the string is attached to a particle  $B$  of mass  $3\text{ kg}$  which hangs freely,  $1\text{ m}$  above the floor. The coefficient of friction between  $A$  and the table is  $0.3$ . The system is released from rest with the string taut.
- (a)** Find the acceleration of  $A$  and the tension in the string. **(4)**
- (b)** Find the speed of  $B$  at the instant it reaches the floor. **(2)**
- $B$  does not rebound from the floor.
- (c)** Show that  $A$  does not reach the pulley. **(4)**
- (d)** State one physical factor, other than air resistance, that has been ignored in this model. **(1)**

## Question 8 continued

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## Question 8 continued

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## Question 9 continued

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**Question 9 continued**

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## Question 10 continued

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**(Total for Question 10 is 12 marks)****TOTAL FOR CHAPTER 5 IS 86 MARKS****Join students already acing their exams.**Book your FREE lesson at [www.eclassroom.org.uk](http://www.eclassroom.org.uk)

## CHAPTER 6 • KEY SKILLS

## 6

## Momentum and Impulse

Specification M1: Momentum and impulse • 10 questions • 81 marks • about 97 minutes

**1 Momentum**

Momentum =  $mv$ , in N s. It has a direction: choose a positive direction and give each velocity a sign.

**2 Impulse**

Impulse = change in momentum  
 $= mv - mu$ , measured in the direction of the impulse. For a constant force,  
 $I = Ft$

**3 Conservation of momentum**

Total momentum before = total momentum after, when the only impulses are between the two particles.

**4 Directions**

Draw before and after diagrams. A negative answer for a velocity means the particle moves in the opposite direction.

**5 Joining and jerks**

Particles that coalesce, or are joined by a string that becomes taut, move with a common velocity afterwards.

**6 Equal and opposite**

The impulse on  $P$  equals the impulse on  $Q$  in size, in the opposite direction. Use the particle you know most about.

**7 Magnitudes and units**

A speed and the magnitude of an impulse are positive. Impulse in N s; no  $g$  in an impulse.

**8 Before and after**

Use suvat or  $F = ma$  to find the speed just before a collision, then again for the motion after it.

**9 Conditions**

" $P$  reverses direction" means its new velocity has the opposite sign: this gives an inequality, e.g.  $k < 8$

**Skills map** every skill above is tested at least once; interleaved skills come from GCSE or other chapters

| Skill                                                       | Q1 | Q2 | Q3 | Q4 | Q5 | Q6 | Q7 | Q8 | Q9 | Q10 |
|-------------------------------------------------------------|----|----|----|----|----|----|----|----|----|-----|
| Marks                                                       | 6  | 6  | 6  | 6  | 6  | 10 | 10 | 10 | 10 | 11  |
| 1 Momentum                                                  | •  | •  | •  | •  | •  | •  | •  | •  | •  | •   |
| 2 Impulse                                                   | •  | •  | •  | •  | •  | •  | •  | •  | •  | •   |
| 3 Conservation of momentum                                  | •  |    | •  | •  |    | •  | •  | •  | •  | •   |
| 4 Directions                                                | •  | •  | •  |    | •  |    | •  |    | •  | •   |
| 5 Joining and jerks                                         | •  |    |    | •  |    |    |    | •  | •  |     |
| 6 Equal and opposite                                        |    | •  | •  | •  |    | •  | •  | •  |    | •   |
| 7 Magnitudes and units                                      | •  | •  | •  | •  |    | •  |    | •  | •  | •   |
| 8 Before and after                                          |    |    |    |    |    | •  |    | •  | •  | •   |
| 9 Conditions                                                |    |    |    |    | •  |    | •  |    |    | •   |
| Interleaved: constant acceleration, two objects (Chapter 2) |    |    |    |    |    | •  |    | •  | •  | •   |
| Interleaved: $F = ma$ and friction (Chapters 4 and 5)       |    | •  |    |    |    | •  |    | •  |    | •   |
| Interleaved: linear inequalities, algebra (P1)              |    |    |    |    | •  |    | •  |    |    | •   |

**Exam difficulty.** Questions 1 to 5 check each skill on its own. Questions 6 to 10 are multi-step questions built like the last questions on a real WME01 paper, where one part leads into the next and skills from different topics are combined.

**Must learn.** Draw before and after diagrams with arrows and a positive direction. Put the correct mass with each velocity. Give speeds and magnitudes of impulses as positive values, with units N s.



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1. Two railway trucks  $A$  and  $B$ , of masses 25 tonnes and 15 tonnes, move towards each other along a straight horizontal track. Immediately before they collide,  $A$  has speed  $3 \text{ m s}^{-1}$  and  $B$  has speed  $1 \text{ m s}^{-1}$ . In the collision the trucks join together.

(a) Find the speed of the trucks immediately after the collision, and state their direction of motion.

(3)

(b) Find the magnitude of the impulse exerted on  $B$  in the collision, stating the units of your answer.

(3)

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**Question 1 continued**

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2. A ball of mass  $0.2 \text{ kg}$  is moving horizontally with speed  $12 \text{ m s}^{-1}$  when it hits a vertical wall at right angles. It rebounds along the same line with speed  $8 \text{ m s}^{-1}$ . The ball is modelled as a particle.

(a) Find the magnitude of the impulse exerted on the ball by the wall.

(3)

The ball is in contact with the wall for  $0.05 \text{ s}$ .

(b) Find the magnitude of the average force exerted on the ball by the wall.

(2)

(c) State the magnitude and direction of the impulse exerted on the wall by the ball.

(1)

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## Question 2 continued

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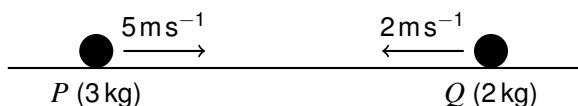


Figure 1

Two particles  $P$  and  $Q$ , of masses 3 kg and 2 kg, move towards each other along the same straight line on a smooth horizontal plane. Immediately before they collide,  $P$  has speed  $5 \text{ m s}^{-1}$  and  $Q$  has speed  $2 \text{ m s}^{-1}$ , as shown in Figure 1. Immediately after the collision,  $P$  moves with speed  $1 \text{ m s}^{-1}$  in the same direction as before.

- (a) Find the speed of  $Q$  immediately after the collision, and state its direction of motion. (3)
- (b) Find the magnitude of the impulse exerted on  $P$  in the collision. (2)
- (c) Write down the magnitude of the impulse exerted on  $Q$ , giving a reason. (1)



**Question 3 continued**

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4. Two particles  $A$  and  $B$ , of masses  $2\text{ kg}$  and  $3\text{ kg}$ , lie at rest on a smooth horizontal plane, connected by a light inextensible string which is slack.  $A$  is projected directly away from  $B$  with speed  $6\text{ ms}^{-1}$ . When the string becomes taut,  $B$  is jerked into motion.
- (a) Explain why  $A$  and  $B$  move with the same speed immediately after the string becomes taut, and find this speed. (3)
- (b) Find the magnitude of the impulse exerted on  $B$  by the string. (2)
- (c) State how you have used the fact that the plane is smooth. (1)

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5. A particle  $P$  of mass  $2m$  is moving in a straight line on a smooth horizontal surface with speed  $3u$ . It receives an impulse of magnitude  $\lambda mu$  in the direction opposite to its motion.
- (a) Given that  $P$  then moves in the opposite direction with speed  $u$ , find the value of  $\lambda$ . (3)
- (b) Given instead that  $\lambda = 4$ , find the speed of  $P$  after the impulse and state its direction of motion. (2)
- (c) Find the range of values of  $\lambda$  for which the direction of motion of  $P$  is reversed. (1)

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## Question 5 continued

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(Total for Question 5 is 6 marks)



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6. A particle  $A$  of mass  $0.4 \text{ kg}$  is projected with speed  $6 \text{ ms}^{-1}$  from a point  $O$  across a rough horizontal table, towards a particle  $B$  of mass  $0.6 \text{ kg}$  which is at rest  $2 \text{ m}$  from  $O$ . The coefficient of friction between each particle and the table is  $0.25$ .  $A$  collides directly with  $B$ . Immediately after the collision,  $A$  moves with speed  $0.5 \text{ ms}^{-1}$  in its original direction.

- (a) Find the speed of  $A$  immediately before the collision. (3)
- (b) Find the speed of  $B$  immediately after the collision. (2)
- (c) Find the magnitude of the impulse exerted on  $B$  in the collision. (2)
- (d) Find the distance between  $A$  and  $B$  when they have both come to rest. (3)



## Question 6 continued

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## Question 6 continued

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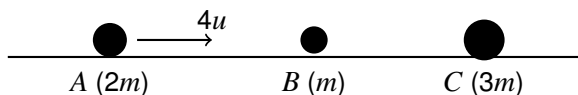


Figure 2

Three particles  $A$ ,  $B$  and  $C$ , of masses  $2m$ ,  $m$  and  $3m$  respectively, lie in a straight line on a smooth horizontal surface, as shown in Figure 2.  $B$  and  $C$  are at rest.  $A$  is projected towards  $B$  with speed  $4u$  and collides directly with  $B$ . Immediately after this collision,  $A$  moves with speed  $u$  in its original direction.

(a) Find the speed of  $B$  immediately after the collision. (2)

(b) Find, in terms of  $m$  and  $u$ , the magnitude of the impulse exerted on  $A$ . (2)

$B$  then collides directly with  $C$ . Immediately after this collision,  $B$  moves with speed  $ku$  in its original direction.

(c) Find, in terms of  $k$  and  $u$ , the speed of  $C$  immediately after this collision. (2)

(d) Find the range of values of  $k$  for which there are no further collisions between the particles. (4)



## Question 7 continued

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## Question 7 continued

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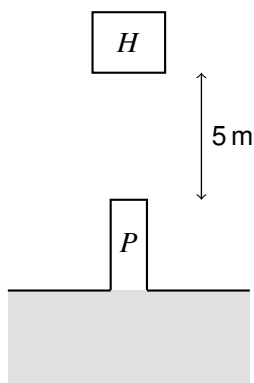


Figure 3

A pile driver consists of a hammer  $H$  of mass 400 kg which is released from rest 5 m above the top of a vertical pile  $P$  of mass 100 kg, as shown in Figure 3. The hammer falls freely and strikes the pile. Immediately after the impact, the hammer and the pile move downwards together. The ground exerts a constant resistance on the pile, and the pile is driven 0.2 m into the ground before coming to rest. The hammer and pile are modelled as particles.

- (a) Find the speed of the hammer immediately before it strikes the pile. (2)
- (b) Find the speed of the hammer and pile immediately after the impact. (2)
- (c) Find the magnitude of the impulse exerted on the pile by the hammer. (2)
- (d) Find the magnitude of the resistance exerted by the ground. (4)



## Question 8 continued

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**Question 8 continued**

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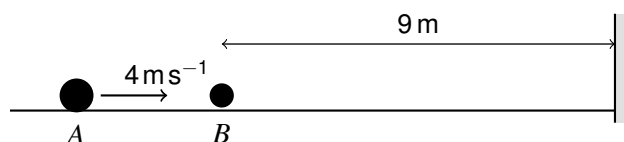


Figure 4

Two particles  $A$  and  $B$ , of masses  $3\text{ kg}$  and  $1\text{ kg}$ , lie on a smooth horizontal floor on a straight line perpendicular to a vertical wall.  $B$  is at rest,  $9\text{ m}$  from the wall.  $A$  moves towards  $B$  with speed  $4\text{ m s}^{-1}$ , as shown in Figure 4, and collides directly with  $B$ . Immediately after the collision,  $A$  moves towards the wall with speed  $2\text{ m s}^{-1}$

(a) Find the speed of  $B$  immediately after the collision.

(2)

$B$  hits the wall and rebounds along the same line with speed  $3\text{ m s}^{-1}$

(b) Find the magnitude of the impulse exerted on  $B$  by the wall.

(2)

(c) Find the time between the two collisions of  $A$  and  $B$ , and the distance from the wall at which the second collision occurs.

(4)

In the second collision,  $A$  and  $B$  join together.

(d) Find their velocity immediately after the second collision.

(2)



## Question 9 continued

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**Question 9 continued**

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10. Two particles  $P$  and  $Q$ , of masses  $2m$  and  $km$  respectively, move towards each other along the same straight line on a smooth horizontal surface. Immediately before they collide,  $P$  has speed  $3u$  and  $Q$  has speed  $u$ . In the collision the direction of motion of  $P$  is reversed and immediately afterwards  $P$  has speed  $u$ .

- (a) Find, in terms of  $m$  and  $u$ , the magnitude of the impulse exerted on  $P$ , and explain why it does not depend on  $k$ . (2)

- (b) Find, in terms of  $k$  and  $u$ , the velocity of  $Q$  immediately after the collision, taking the original direction of motion of  $P$  as positive. (3)

- (c) Find the range of values of  $k$  for which the direction of motion of  $Q$  is reversed. (2)

Given that  $Q$  moves with speed  $3u$  immediately after the collision,

- (d) find the value of  $k$ . (2)

After the collision,  $Q$  moves onto a rough horizontal surface where the coefficient of friction between  $Q$  and the surface is 0.5.

- (e) Find, in terms of  $u$  and  $g$ , the distance that  $Q$  travels on the rough surface before coming to rest. (2)



## Question 10 continued

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## Question 10 continued

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**TOTAL FOR CHAPTER 6 IS 81 MARKS**

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## CHAPTER 7 • KEY SKILLS

## 7

## Statics of a Particle

Specification M1: Statics • 10 questions • 82 marks • about 98 minutes

**1 Equilibrium**

The resultant force is zero: resolve in two perpendicular directions and set each total to 0. No  $ma$  term.

**2 Choosing directions**

Resolve perpendicular to a force you do not need, so it does not appear. On a plane: parallel and perpendicular to it.

**3 Strings**

The tension acts along the string, away from the particle. Over a smooth pulley the tension is the same on both sides.

**4 Triangle of forces**

Three forces in equilibrium form a closed triangle: the sine rule gives

$$\frac{T_1}{\sin A} = \frac{T_2}{\sin B} = \frac{W}{\sin C}$$

**5 Forces in components**

Equilibrium: the  $\mathbf{i}$  (east) parts add to 0 and the  $\mathbf{j}$  (north) parts add to 0.

Removing one force leaves a resultant equal and opposite to it.

**6 Friction at rest**

$F \leq \mu R$ : friction takes whatever value is needed, up to  $\mu R$ .  $F = \mu R$  only when on the point of slipping.

**7 Limiting cases**

About to slip down: friction acts up the plane; about to slip up: friction acts down. Each case gives one end of a range.

**8 Inequalities**

"Remains at rest" gives  $F \leq \mu R$ , so  $\mu \geq \frac{F}{R}$ . Keep the inequality the right way round.

**9 Normal reaction**

$R$  acts perpendicular to the surface and may be reduced (or reversed, for a ring on a rod) by a pull.

**Skills map** every skill above is tested at least once; interleaved skills come from GCSE or other chapters

| Skill                                                           | Q1 | Q2 | Q3 | Q4 | Q5 | Q6 | Q7 | Q8 | Q9 | Q10 |
|-----------------------------------------------------------------|----|----|----|----|----|----|----|----|----|-----|
| Marks                                                           | 6  | 6  | 6  | 6  | 6  | 10 | 10 | 11 | 10 | 11  |
| 1 Equilibrium                                                   | •  | •  | •  | •  | •  | •  | •  | •  | •  | •   |
| 2 Choosing directions                                           | •  |    |    | •  | •  | •  |    | •  |    | •   |
| 3 Strings                                                       | •  |    |    |    |    | •  | •  |    |    | •   |
| 4 Triangle of forces                                            | •  |    |    |    |    |    | •  |    |    |     |
| 5 Forces in components                                          |    | •  |    | •  |    |    |    |    |    |     |
| 6 Friction at rest                                              |    |    | •  |    | •  | •  |    | •  | •  | •   |
| 7 Limiting cases                                                |    |    |    |    | •  | •  |    | •  | •  | •   |
| 8 Inequalities                                                  |    |    | •  |    |    |    |    |    | •  | •   |
| 9 Normal reaction                                               |    |    | •  |    |    | •  |    | •  | •  |     |
| Interleaved: $F = ma$ and friction in motion (Chapters 4 and 5) |    | •  |    |    |    | •  |    |    |    | •   |
| Interleaved: vectors and exact trigonometry (Chapter 3, P1)     |    | •  | •  |    | •  | •  | •  | •  |    | •   |
| Interleaved: linear inequalities (P1)                           |    |    | •  |    |    |    |    | •  | •  | •   |

**Exam difficulty.** Questions 1 to 5 check each skill on its own. Questions 6 to 10 are multi-step questions built like the last questions on a real WME01 paper, where one part leads into the next and skills from different topics are combined.

**Must learn.** Draw every force, including  $R$  and friction, with friction opposing the direction in which the particle would move. Find  $R$  from the perpendicular resolution, including every force with a component in that direction.



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A diagram showing a V-shaped structure. A horizontal line contains points  $A$  and  $B$ . Point  $C$  is located below this line. Line segments  $AC$  and  $BC$  connect the points. A dashed horizontal line extends from point  $C$  to the left, forming a  $30^\circ$  angle with segment  $AC$  and a  $45^\circ$  angle with segment  $BC$ .

A particle  $C$  of mass  $4\text{ kg}$  hangs in equilibrium, supported by two light inextensible strings  $AC$  and  $BC$  attached to fixed points  $A$  and  $B$  on a horizontal ceiling. The strings  $AC$  and  $BC$  are inclined to the horizontal at  $30^\circ$  and  $45^\circ$  respectively, as shown in Figure 1.

**(6)**

## Question 1 continued

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(Total for Question 1 is 6 marks)



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2. [In this question,  $\mathbf{i}$  and  $\mathbf{j}$  are perpendicular horizontal unit vectors.]

A particle  $P$  of mass 2 kg rests on a smooth horizontal surface. Three horizontal forces  $(3\mathbf{i} - 4\mathbf{j})$  N,  $(p\mathbf{i} + 2\mathbf{j})$  N and  $(-5\mathbf{i} + q\mathbf{j})$  N act on  $P$ , and  $P$  is in equilibrium.

- (a) Find the value of  $p$  and the value of  $q$ .

(2)

The force  $(-5\mathbf{i} + q\mathbf{j})$  N is now removed.

- (b) Find the magnitude of the acceleration of  $P$  and the angle its direction makes with  $\mathbf{i}$ .

(4)

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## Question 2 continued

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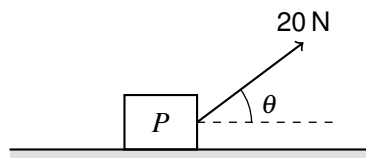


Figure 2

A particle  $P$  of weight 40 N lies at rest on a rough horizontal surface. A force of magnitude 20 N is applied to  $P$  at an angle  $\theta$  above the horizontal, where  $\tan \theta = \frac{3}{4}$ , as shown in Figure 2. The coefficient of friction between  $P$  and the surface is  $\mu$ . The particle remains at rest.

- (a) Find the magnitude of the normal reaction on  $P$ . (2)
- (b) Show that  $\mu \geq \frac{4}{7}$ . (3)
- (c) Describe the equilibrium of  $P$  when  $\mu = \frac{4}{7}$ . (1)



## Question 3 continued

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- (2)

## Question 4 continued

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- (a)** Show that the coefficient of friction between the particle and the plane is  $\frac{7}{24}$

(2)

**(b)** Find the least magnitude of this force that will make the particle move up the plane.

**(3)**

- (c)** State the direction of the friction force in part (b), giving a reason.

(1)

## Question 5 continued

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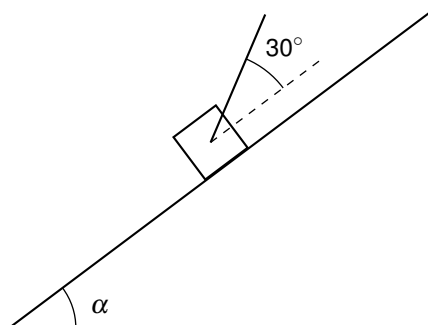


Figure 3

A box of mass  $m$  rests on a rough plane inclined at an angle  $\alpha$  to the horizontal, where  $\tan \alpha = \frac{3}{4}$ . A light string is attached to the box and is held so that it makes an angle of  $30^\circ$  with the plane, above a line of greatest slope, as shown in Figure 3. The tension in the string is  $\frac{1}{2}mg$  and the box is on the point of slipping down the plane. The box is modelled as a particle.

(a) Find, in terms of  $m$  and  $g$ , the normal reaction on the box. (3)

(b) Show that the coefficient of friction between the box and the plane is  $\frac{12 - 5\sqrt{3}}{11}$ . (4)

The string is now cut.

(c) Show that the box slides down the plane, and find its acceleration. (3)



## Question 6 continued

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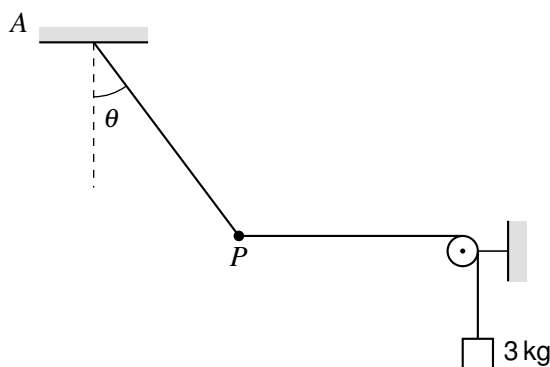


Figure 4

A particle  $P$  of mass 4 kg is attached to one end of a light inextensible string whose other end is fixed at the point  $A$ . A second light inextensible string is attached to  $P$ , passes over a small smooth fixed pulley and carries a particle of mass 3 kg hanging freely.  $P$  hangs in equilibrium with the second string horizontal between  $P$  and the pulley and with  $AP$  inclined at an angle  $\theta$  to the vertical, as shown in Figure 4.

- (a) Find the value of  $\theta$  and the tension in  $AP$ .

(5)

The 3 kg particle is replaced by a particle of mass  $M$  kg, and  $P$  hangs in equilibrium with  $AP$  inclined at  $45^\circ$  to the vertical and the second string still horizontal.

- (b) Find the value of  $M$ .

(2)

- (c) State how you have used the fact that the pulley is smooth.

(1)

- (d) For the original 3 kg particle, find the magnitude of the force exerted on the pulley by the string.

(2)



## Question 7 continued

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## Question 7 continued

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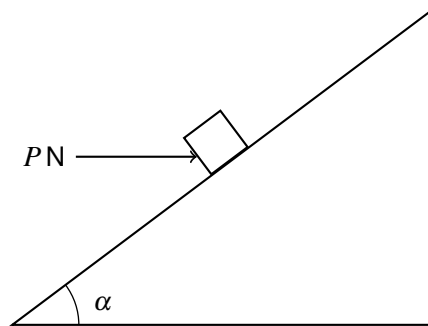


Figure 5

A particle of mass 2 kg is held at rest on a rough plane inclined at an angle  $\alpha$  to the horizontal, where  $\tan \alpha = \frac{3}{4}$ , by a horizontal force of magnitude  $P$  newtons, as shown in Figure 5. The line of action of the force lies in a vertical plane containing a line of greatest slope. The coefficient of friction between the particle and the plane is 0.25.

- (a) Find the least possible value of  $P$ . (5)
- (b) Find the greatest possible value of  $P$ . (4)
- (c) Given that  $P = 15$ , find the magnitude and direction of the friction force acting on the particle. (2)



## Question 8 continued

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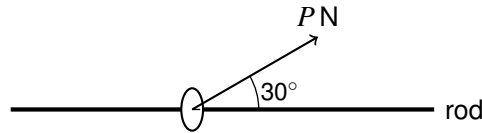


Figure 6

A small ring of mass  $0.4$  kg is threaded on a fixed rough horizontal rod. A force of magnitude  $P$  newtons is applied to the ring at an angle of  $30^\circ$  above the horizontal, as shown in Figure 6, in the vertical plane containing the rod. The coefficient of friction between the ring and the rod is  $0.5$ . The ring is modelled as a particle.

- (a) Given that  $P = 1.5$ , show that the ring remains at rest, and find the friction force acting on it. (4)
- (b) Find the greatest value of  $P$  for which the ring remains at rest. (4)
- (c) Explain why, for large values of  $P$ , the normal reaction on the ring acts downwards, and why this does not allow the ring to stay at rest. (2)



## Question 9 continued

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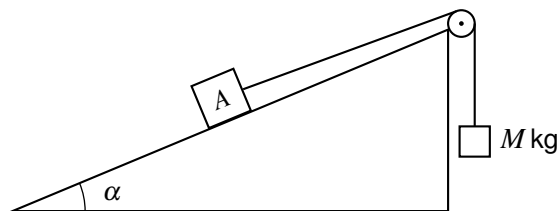


Figure 7

A particle  $A$  of mass  $5\text{ kg}$  lies on a rough plane inclined at an angle  $\alpha$  to the horizontal, where  $\tan \alpha = \frac{5}{12}$ . It is attached to a light inextensible string which passes over a small smooth pulley at the top of the plane and carries a particle of mass  $M\text{ kg}$  hanging freely, as shown in Figure 7. The part of the string from  $A$  to the pulley lies along a line of greatest slope. The coefficient of friction between  $A$  and the plane is  $0.2$ . The system is in equilibrium.

(a) Show that the least possible value of  $M$  is  $1$ . (4)

(b) Find the greatest possible value of  $M$ . (3)

(c) Find the value of  $M$  for which there is no friction acting on  $A$ . (1)

The hanging particle is replaced by one of mass  $4\text{ kg}$  and the system is released from rest.

(d) Find the acceleration of  $A$ . (3)



## Question 10 continued

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**Question 10 continued**

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**(Total for Question 10 is 11 marks)****TOTAL FOR CHAPTER 7 IS 82 MARKS****Join students already acing their exams.**Book your FREE lesson at [www.eclassroom.org.uk](http://www.eclassroom.org.uk)

## CHAPTER 8 • KEY SKILLS

## 8

## Moments

Specification M1: Moments • 10 questions • 82 marks • about 98 minutes

**1 Moment of a force**

Moment = force  $\times$  perpendicular distance from the point, in N m. State clockwise or anticlockwise.

**2 Resultant moment**

Add the moments about one point, counting clockwise as positive (or anticlockwise). Zero resultant moment means no turning.

**3 Equilibrium**

Resolve vertically and take moments about one point, or take moments about two points. Choose a point where an unknown force acts.

**4 Uniform or not**

A uniform rod's weight acts at its midpoint. For a non-uniform rod, let the centre of mass be  $x$  m from one end.

**5 Tilting**

On the point of tilting about  $D$ : the reaction at the other support (or the tension in the other rope) is zero. Take moments about  $D$ .

**6 Ranges**

For no tilting, both reactions must be  $\geq 0$ : each tilting case gives one end of the range of positions.

**7 Relationships**

" $R_C = 2R_D$ " or "the tensions are equal": write the forces in one unknown before taking moments.

**8 Weights and masses**

A mass  $m$  kg has weight  $mg$  N. If a weight is given in newtons, do not multiply by  $g$  again.

**9 Modelling**

Rod: rigid, stays straight. Uniform: weight at the middle. Particle: weight acts at a single point.

**Skills map** every skill above is tested at least once; interleaved skills come from GCSE or other chapters

| Skill                                                  | Q1 | Q2 | Q3 | Q4 | Q5 | Q6 | Q7 | Q8 | Q9 | Q10 |
|--------------------------------------------------------|----|----|----|----|----|----|----|----|----|-----|
| Marks                                                  | 6  | 6  | 6  | 6  | 6  | 10 | 10 | 11 | 10 | 11  |
| 1 Moment of a force                                    | •  | •  | •  | •  | •  | •  | •  | •  | •  | •   |
| 2 Resultant moment                                     |    |    |    |    | •  |    |    |    | •  | •   |
| 3 Equilibrium                                          | •  | •  | •  |    |    | •  | •  | •  | •  | •   |
| 4 Uniform or not                                       | •  |    |    | •  |    |    |    | •  |    | •   |
| 5 Tilting                                              |    |    | •  | •  |    | •  | •  | •  |    |     |
| 6 Ranges                                               |    |    |    |    |    | •  | •  |    |    | •   |
| 7 Relationships                                        |    | •  |    |    |    |    |    | •  |    |     |
| 8 Weights and masses                                   |    |    |    | •  |    |    |    |    |    | •   |
| 9 Modelling                                            | •  | •  |    |    |    |    | •  |    | •  |     |
| Interleaved: forming and solving linear equations (P1) |    |    |    |    | •  |    |    | •  | •  |     |
| Interleaved: linear inequalities (P1)                  |    |    |    |    |    | •  | •  |    |    | •   |
| Interleaved: simultaneous equations (P1)               |    | •  |    | •  |    |    |    | •  |    | •   |

**Exam difficulty.** Questions 1 to 5 check each skill on its own. Questions 6 to 10 are multi-step questions built like the last questions on a real WME01 paper, where one part leads into the next and skills from different topics are combined.

**Must learn.** Draw the rod with every force and every distance marked. Take moments about the point where the force you do not want acts. Each moments equation must be dimensionally correct: force  $\times$  distance in every term.



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A horizontal beam of length 1 m is shown. It is supported by a pin support at point A and a roller support at point C. The distance between the supports is 1 m. The beam is labeled AB, with A at the left end and B at the right end. Point C is located between A and B. A dimension line below the beam indicates the distance between the supports is 1 m.

A uniform plank  $AB$  has length 4 m and mass 20 kg. It rests in equilibrium in a horizontal position on two supports, one at  $A$  and one at the point  $C$ , where  $CB = 1$  m, as shown in Figure 1.

- (a) Find the magnitude of the reaction on the plank at  $C$ . (3)
- (b) Find the magnitude of the reaction on the plank at  $A$ . (2)
- (c) State how you have used the fact that the plank is uniform. (1)

## Question 1 continued

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(Total for Question 1 is 6 marks)



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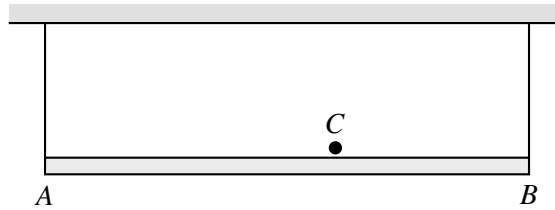


Figure 2

A uniform beam  $AB$ , of length 4 m and mass 30 kg, hangs in equilibrium in a horizontal position from two vertical ropes attached to the beam at  $A$  and at  $B$ . A load of mass 20 kg is attached to the beam at the point  $C$ , as shown in Figure 2. The tension in the rope at  $B$  is 1.5 times the tension in the rope at  $A$ . The load is modelled as a particle and the ropes as light inextensible strings.

- (a) Find the tension in each rope. (3)
- (b) Find the distance  $AC$ . (2)
- (c) State how you have used the modelling assumption that the beam is a rod. (1)



## Question 2 continued

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The diagram shows a horizontal beam of length 2.5 m, labeled A at the left end and B at the right end. A pin support is located at point C, which is 1 m from A. A roller support is located at point D, which is 1.5 m from B. The distance between C and D is 0.5 m.

A uniform rod  $AB$  has length 5 m and mass 20 kg. It rests horizontally on two supports at  $C$  and  $D$ , where  $AC = 1$  m and  $DB = 1.5$  m, as shown in Figure 3. A particle of mass  $m$  kg is placed on the rod at  $B$  and the rod remains in equilibrium.

- (a) Find the greatest possible value of  $m$ . (3)
- (b) Given instead that  $m = 10$ , find the reactions at  $C$  and at  $D$ . (3)

## Question 3 continued

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Diagram of a beam with supports at C and D. The distance from A to C is 1 m, and the distance from D to E is 1 m.

A non-uniform rod  $AB$  has length 4 m and weight  $W$  newtons. It rests horizontally on two supports at  $C$  and  $D$ , where  $AC = DB = 1$  m, as shown in Figure 4. The centre of mass of the rod is  $x$  metres from  $A$ .

Find the value of  $W$  and the value of  $x$ .

**(6)**

## Question 4 continued

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5. A light rod  $AB$  has length 3 m. Particles of mass 2 kg, 5 kg and 3 kg are attached to the rod at  $A$ , at the point  $C$  where  $AC = 1$  m, and at  $B$  respectively.

(a) Find the distance from  $A$  of the point at which the rod can be supported so that it rests horizontally in equilibrium.

(3)

(b) The rod is suspended from this point by a single vertical string. Find the tension in the string.

(1)

A further particle of mass 1 kg is attached to the rod at  $A$ .

(c) Find how far, and in which direction, the string must be moved for the rod to rest horizontally.

(2)

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## Question 5 continued

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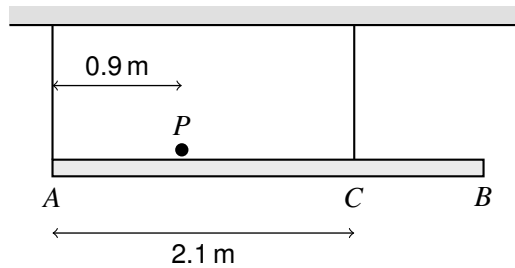


Figure 5

A uniform beam  $AB$ , of length 3 m and mass 60 kg, is held in a horizontal position by two vertical cables, one attached at  $A$  and the other at the point  $C$  on the beam, where  $AC = 2.1$  m. A particle  $P$  of mass 40 kg is attached to the beam 0.9 m from  $A$ , as shown in Figure 5.

(a) Show that the tension in the cable at  $C$  is 588 N.

(3)

(b) Find the tension in the cable at  $A$ .

(1)

The cable at  $C$  will break if the tension in it exceeds 800 N.

(c) Find the greatest distance from  $A$  at which  $P$  can be attached without the cable at  $C$  breaking.

(3)

$P$  is removed, and a particle of mass  $M$  kg is attached to the beam at  $B$ . The beam is now on the point of tilting about  $C$ .

(d) Find the value of  $M$ .

(3)



## Question 6 continued

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## Question 6 continued

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A horizontal beam is shown with points A, C, D, and E marked along its length. Point C is a pin support, and point D is a roller support. The distance from A to C is indicated as 1 m, and the distance from D to E is indicated as 2 m.

A uniform plank  $AB$  has length 6 m and mass 30 kg. It rests horizontally on two supports at  $C$  and  $D$ , where  $AC = 1$  m and  $DB = 2$  m, as shown in Figure 6. A man of mass 70 kg walks along the plank.

- (a) Find the reactions at  $C$  and  $D$  when the man stands 2.5 m from  $A$ . (4)
- (b) Find the range of distances from  $A$  at which the man can stand without the plank tilting. (5)
- (c) State one way in which modelling the man as a particle affects your answer to part (b). (1)

## Question 7 continued

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## Question 7 continued

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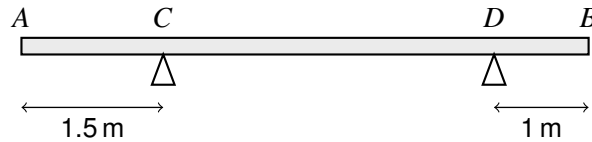


Figure 7

A non-uniform rod  $AB$  has length 6 m and mass 18 kg. It rests horizontally on two supports at  $C$  and  $D$ , where  $AC = 1.5$  m and  $DB = 1$  m, as shown in Figure 7. The reaction on the rod at  $C$  is twice the reaction on the rod at  $D$ .

(a) Find the distance of the centre of mass of the rod from  $A$ . (5)

(b) Find the greatest mass of a particle that can be placed on the rod at  $B$  without the rod tilting. (3)

The support at  $D$  is removed and replaced by a support at the point  $E$ . A particle of mass 5 kg is placed on the rod at  $A$ , and the reactions at  $C$  and  $E$  are now equal.

(c) Find the distance  $AE$ . (3)



## Question 8 continued

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## Question 8 continued

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A horizontal beam is shown with points A, C, and B marked. A fulcrum is located at point C. A dimension line below the beam indicates the distance from A to C is 2.5 m.

### Figure 8

**(d)** Give two criticisms of the model. **(2)**

## Question 9 continued

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## Question 9 continued

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(Total for Question 9 is 10 marks)



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Diagram of a beam ABE with a hinge support at C. The distance from A to C is 1 m, and from C to E is 3 m. A bolt is shown at point A.

## Question 10 continued

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**TOTAL FOR CHAPTER 8 IS 82 MARKS**

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