Trigonometry

Worked Solutions

Suitable for:

Pearson Edexcel International A-Level (iAL) — Pure Mathematics 2

Question 1 — Worked Solution

(a) Find cosθ

Use $\sin^2\theta + \cos^2\theta = 1$:

$$\cos^2\theta = 1 - \sin^2\theta = 1 - \frac{9}{25} = \frac{16}{25}$$

$$\cos\theta = \pm\frac{4}{5}$$

Since θ is obtuse (90° < θ < 180°), cosθ is negative:

$$\therefore \cos\theta = -\frac{4}{5}$$

(b) Find tanθ

$$\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{\frac{3}{5}}{-\frac{4}{5}} = \frac{3}{5} \times \left(-\frac{5}{4}\right)$$

$$\therefore \tan\theta = -\frac{3}{4}$$

Question 2 — Worked Solution

(a) Show cosθ = 1/√5

Use tanθ = sinθ/cosθ = 2, so sinθ = 2cosθ. Substitute into $\sin^2\theta + \cos^2\theta = 1$:

$$(2\cos\theta)^2 + \cos^2\theta = 1 \;\Rightarrow\; 4\cos^2\theta + \cos^2\theta = 1$$

$$5\cos^2\theta = 1 \;\Rightarrow\; \cos^2\theta = \frac{1}{5}$$

$$\cos\theta = \frac{1}{\sqrt{5}} \quad \text{(positive since } \theta \text{ is acute)} \quad ✓$$

(b) Find sinθ

$$\sin\theta = 2\cos\theta = \frac{2}{\sqrt{5}} = \frac{2\sqrt{5}}{5}$$

$$\therefore \sin\theta = \frac{2}{\sqrt{5}} = \frac{2\sqrt{5}}{5}$$

Question 3 — Worked Solution

Solve $2\sin^2 x - \cos x - 2 = 0$, $0° \le x \le 360°$

Replace $\sin^2 x$ with $1 - \cos^2 x$:

$$2(1 - \cos^2 x) - \cos x - 2 = 0$$

$$2 - 2\cos^2 x - \cos x - 2 = 0$$

$$-2\cos^2 x - \cos x = 0$$

$$\cos x(-2\cos x - 1) = 0$$

Case 1: $\cos x = 0 \rightarrow x = 90°, 270°$

Case 2: $\cos x = -1/2 \rightarrow x = 120°, 240°$

$$\therefore x = 90°,\ 120°,\ 240°,\ 270°$$

Question 4 — Worked Solution

Solve $2\cos^2 x + \sin x - 1 = 0$, $0° \le x \le 360°$

Replace $\cos^2 x$ with $1 - \sin^2 x$:

$$2(1-\sin^2 x)+\sin x-1=0$$

$$2-2\sin^2 x+\sin x-1=0$$

$$-2\sin^2 x+\sin x+1=0$$

$$2\sin^2 x-\sin x-1=0$$

$$(2\sin x+1)(\sin x-1)=0$$

Case 1: $\sin x = 1 \rightarrow x = 90°$

Case 2: $\sin x = -1/2 \rightarrow x = 210°, 330°$

$$\therefore x = 90°,\ 210°,\ 330°$$

Question 5 — Worked Solution

Solve $2\cos^2(2x) - 3\sin(2x) = 0$, $0° \le x \le 180°$

Let $u = 2x$. Range of u: $0° \le u \le 360°$.

Replace $\cos^2 u$ with $1 - \sin^2 u$:

$$2(1-\sin^2 u) - 3\sin u = 0$$

$$2 - 2\sin^2 u - 3\sin u = 0$$

$$2\sin^2 u + 3\sin u - 2 = 0$$

$$(2\sin u - 1)(\sin u + 2) = 0$$

$\sin u = -2$ is rejected (outside range -1 to 1).

$\sin u = 1/2 \rightarrow u = 30°, 150°$

Convert back: $x = u/2$

$$\therefore x = 15°,\ 75°$$

Question 6 — Worked Solution

Solve 2sin*x* – 1 = 0,   0 ≤ x ≤ 2π

$$\sin x = \frac{1}{2}$$

Principal value: x = π/6. sin is also positive in the 2nd quadrant:

$$x = \frac{\pi}{6} \quad \text{and} \quad x = \pi - \frac{\pi}{6} = \frac{5\pi}{6}$$

$$\therefore x = \frac{\pi}{6}, \quad x = \frac{5\pi}{6}$$

Question 7 — Worked Solution

Solve $3\cos^2 x - \cos x = 0$, $0° \le x \le 360°$

Factorise:

$$\cos x\,(3\cos x - 1) = 0$$

Case 1: $\cos x = 0$

$$x = 90°, \quad x = 270°$$

Case 2: $\cos x = 1/3$

$$x = \cos^{-1}\left(\frac{1}{3}\right) = 70.5°$$

$$x = 360° - 70.5° = 289.5°$$

$$\therefore x = 70.5°,\ 90°,\ 270°,\ 289.5°$$

Question 8 — Worked Solution

Solve $2\sin^2 x + \sin x - 1 = 0$, $0° \le x \le 360°$

Factorise as a quadratic in sinx:

$$(2\sin x - 1)(\sin x + 1) = 0$$

Case 1: $\sin x = 1/2$

$$x = 30°, \quad x = 150°$$

Case 2: $\sin x = -1$

$$x = 270°$$

$$\therefore x = 30°,\ 150°,\ 270°$$

Question 9 — Worked Solution

Solve $3\sin^2 x - 2\sin x = 0, \quad 0 \le x \le 2\pi$

Factorise:

$$\sin x\,(3\sin x - 2) = 0$$

Case 1: $\sin x = 0$

$$x = 0, \quad x = \pi, \quad x = 2\pi$$

Case 2: $\sin x = 2/3$

$$x = \sin^{-1}\left(\frac{2}{3}\right) = 0.730 \text{ rad (3 s.f.)}$$

$$x = \pi - 0.730 = 2.41 \text{ rad (3 s.f.)}$$

$$\therefore x = 0,\ 0.730,\ 2.41,\ \pi,\ 2\pi$$

Question 10 — Worked Solution

Solve sin(x + 30°) = 1/2, 0° ≤ x ≤ 360°

Let u = x + 30°. Range of u: 30° ≤ u ≤ 390°.

Solve sinu = 1/2 in [30°, 390°]:

$$u = 30°, \quad u = 150°, \quad u = 390°$$

Convert back using x = u – 30°:

$$x = 0°, \quad x = 120°, \quad x = 360°$$

$$\therefore x = 0°,\ 120°,\ 360°$$

Question 11 — Worked Solution

Solve cos(2*x* – π/6) = √3/2, 0 ≤ x ≤ 2π

Let u = 2x – π/6. Range of u when x ∈ [0, 2π]:

$$-\frac{\pi}{6} \leq u \leq \frac{11\pi}{3}$$

Solve cosu = √3/2. Principal value: u = π/6. Also cos is positive in 4th quadrant: u = –π/6. Add 2π: u = π/6 + 2π = 13π/6 and u = –π/6 + 2π = 11π/6.

$$u = -\frac{\pi}{6}, \frac{\pi}{6}, \frac{11\pi}{6}, \frac{13\pi}{6}$$

Convert back using x = (u + π/6)/2:

$$u = -\frac{\pi}{6} : \ x = 0 \qquad u = \frac{\pi}{6} : \ x = \frac{\pi}{6}$$

$$u = \frac{11\pi}{6} : \ x = \pi \qquad u = \frac{13\pi}{6} : \ x = \frac{7\pi}{6}$$

$$\therefore x = 0, \quad \frac{\pi}{6}, \quad \pi, \quad \frac{7\pi}{6}$$