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Pure Mathematics 4

Exam Questions by Topic: Mark Scheme

Pearson Edexcel International Advanced Level (WMA14/01) • Calculator allowed

Every question in the booklet, marked line by line the way an examiner marks it

Chapters (tap a chapter to jump to it)	Qs	Marks	Page	My score
1 Proof Proof by contradiction: irrational numbers, primes, integers, inequalities	10	73	3	
2 Partial Fractions Linear and repeated factors, improper fractions	10	78	10	
3 Coordinate Geometry in the (x, y) Plane Parametric equations and conversion to Cartesian form	10	76	18	
4 Binomial Expansion $(a + bx)^n$ for rational n , validity, using partial fractions	10	81	25	
5 Differentiation Parametric and implicit differentiation, rates of change	10	81	33	
6 Integration Volumes of revolution, substitution, parts, partial fractions, differential equations	10	86	40	
7 Vectors 3D vectors, vector equations of lines, intersections, scalar product	10	82	48	

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How to use this mark scheme

The same marking rules Pearson examiners use for WMA14.

Mark every line, not just the answer. Method marks are given for a correct method even when the final answer is wrong. If you lost marks, find the *first* line where your working differs from the scheme. That is the thing to fix.

Code	Meaning
M1	Method mark: for knowing a correct method and attempting to apply it. Small slips in the working do not lose this mark.
dM1	Dependent method mark: can only be given if the method mark before it has been given.
A1	Accuracy mark: for a correct answer or line of working. It can only be given if the method mark it depends on has been given.
B1	Independent mark: for a correct result or statement, whatever method was used.
*	The answer is given in the question. Every step must be shown, and the answer must follow from correct working (cso).
cao	Correct answer only.
cso	Correct solution only: no errors anywhere in the working.
ft	Follow through: the mark can be given for correct work using your own earlier (wrong) answer.
oe	Or equivalent: any correct equivalent form is accepted.

“Solutions relying on calculator technology are not acceptable.” When a question says this, a correct final answer with no algebraic working scores **no marks**. The notes under each scheme line say when this applies.

Marking a proof. A proof by contradiction earns its final mark only when it has all three parts: a clear assumption at the start (“Assume there is ...”), correct deductions in the right order, and a statement of the contradiction with a conclusion. A sketch or calculator values are not a proof.



1

Chapter 1: Proof

Mark scheme. Marks are shown on the right of each line.

Question 1

5 marks

Part	Scheme	Marks
(a)	Line 1 must say that a and b have <i>no common factors</i> (the fraction is in its lowest terms) Without this, finding that a and b are both even contradicts nothing: e.g. $\frac{2}{4} = \frac{1}{2}$ <i>This is the omission that cost most candidates the final mark in the Oct 2021 and June 2023 series.</i>	B1 B1 (2)
(b)	If a were odd, $a = 2k + 1$ and $a^2 = 4k^2 + 4k + 1 = 2(2k^2 + 2k) + 1$ would be odd; so if a^2 is even, a is even *	B1* (1)
(c)	$3b^2 = 2m^2$, so $3b^2$ is even 3 is odd, so b^2 must be even (odd $\times$ odd is odd), and therefore b is even (by part (b)) * <i>Writing "b is even" straight from $2m^2 = 3b^2$ skips the step that uses 3 being odd.</i>	M1 A1* (2)
Total for Question 1: 5 marks		

Question 2

5 marks

Part	Scheme	Marks
	Assume C has a stationary point, so $\frac{dy}{dx} = 0$ for some value of x Differentiates to the form $pe^{3x} + 6 \pm q \cos 3x$ $\frac{dy}{dx} = 3e^{3x} + 6 - 6 \cos 3x$ At this x : $3e^{3x} = 6(\cos 3x - 1)$, and $\cos 3x \leq 1$ so the right-hand side is ≤ 0 But $e^{3x} > 0$ for all x , so the left-hand side is positive: a contradiction. So C has no stationary points * <i>Assuming $\frac{dy}{dx} = 0$ "for all x" is B0. Both facts are needed: $e^{3x} > 0$ and $\cos 3x \leq 1$. A sketch of the curve is not a proof. Alternatively, $3e^{3x} > 0$ and $6 - 6 \cos 3x \geq 0$, so $\frac{dy}{dx} > 0$, contradicting $\frac{dy}{dx} = 0$.</i>	B1 M1 A1 dM1 A1* (5)
Total for Question 2: 5 marks		



Question 3

5 marks

Part	Scheme	Marks
(a)	$(x + y) + (x - y) = 2x$, which is even Two integers with an even sum are both even or both odd (an even and an odd number have an odd sum) * <i>Considering the four cases (both even, both odd, one of each) and showing $x + y$ and $x - y$ match in each is equally valid.</i>	M1 A1* (2)
(b)	Assume there are positive integers x and y with $x^2 - y^2 = 18$, so $(x - y)(x + y) = 18$ Uses (a): if both factors are odd, the product is odd; if both are even, the product is $2p \times 2q = 4pq$, a multiple of 4 18 is even but not a multiple of 4, so neither case is possible: a contradiction. So there are no such positive integers * <i>Alternative: the factor pairs of 18 are 1×18, 2×9, 3×6; each has one odd and one even factor, which contradicts (a).</i>	B1 M1 A1* (3)
Total for Question 3: 5 marks		

Question 4

5 marks

Part	Scheme	Marks
(a)	Assume there is a real number $x > 1$ for which $\frac{x^2}{x-1} < 4$ $x - 1 > 0$, so multiplying by $x - 1$ keeps the inequality: $x^2 < 4x - 4$ $x^2 - 4x + 4 < 0 \Rightarrow (x - 2)^2 < 0$ A square cannot be negative: a contradiction. So $\frac{x^2}{x-1} \geq 4$ for all $x > 1$ * <i>The reason $x - 1 > 0$ must be given: multiplying by a negative number would reverse the inequality.</i>	B1 M1 dM1 A1* (4)
(b)	A correct counter-example, e.g. $x = 0: \frac{0^2}{0-1} = 0 < 4$ <i>Any $x < 1$ works, since then $\frac{x^2}{x-1} \leq 0$. The value of the expression must be shown.</i>	B1 (1)
Total for Question 4: 5 marks		



Question 5

6 marks

Part	Scheme	Marks
(a)	Assume $\log_2 3$ is rational: $\log_2 3 = \frac{a}{b}$, where a and b are positive integers (positive since $3 > 1$) $2^{a/b} = 3$, and raising both sides to the power b: $2^a = 3^b$ 2^a is even (as $a \geq 1$) and 3^b is odd (a product of odd numbers) An even number cannot equal an odd number: a contradiction. So $\log_2 3$ is irrational *	B1 M1 M1 A1* (4)
(b)	$\log_8 9 = \frac{\log_2 9}{\log_2 8} = \frac{2 \log_2 3}{3}$ If $\log_8 9 = r$ were rational, then $\log_2 3 = \frac{3}{2}r$ would be rational, contradicting (a). So $\log_8 9$ is irrational *	M1 A1* (2)
Total for Question 5: 6 marks		

Question 6

10 marks

Part	Scheme	Marks
(a)	Every integer that is not a multiple of 3 is $3k+1$ or $3k+2$; cubes one of them $(3k+1)^3 = 27k^3 + 27k^2 + 9k + 1 = 3(9k^3 + 9k^2 + 3k) + 1$ $(3k+2)^3 = 27k^3 + 54k^2 + 36k + 8 = 3(9k^3 + 18k^2 + 12k + 2) + 2$: neither is a multiple of 3 *	M1 A1 A1* (3)
(b)	Assume $\sqrt[3]{3} = \frac{a}{b}$, where a and b are integers with no common factors $a^3 = 3b^3$, so a^3 is a multiple of 3 So a is a multiple of 3 (by (a): if a were not, a^3 would not be) $a = 3m$: $27m^3 = 3b^3 \Rightarrow b^3 = 9m^3 = 3(3m^3)$ b^3 is a multiple of 3, so b is too. Then 3 is a common factor of a and b: a contradiction, so $\sqrt[3]{3}$ is irrational * <i>"a^3 is a multiple of 3 so a is a multiple of 3" must be justified by part (a): this is the step most often missed (Oct 2021 report).</i>	B1 M1 A1 M1 A1* (5)
(c)	$a^3 = 8b^3$ and $a = 2m$ give $8m^3 = 8b^3$, so $b^3 = m^3$, i.e. $b = m$ Nothing forces b to be even, so there is no common factor and no contradiction (indeed $\sqrt[3]{8} = 2 = \frac{2}{1}$ is rational)	B1 B1 (2)
Total for Question 6: 10 marks		



Question 7

10 marks

Part	Scheme	Marks
(a)	$\frac{1}{\cos^2 x} + \frac{1}{\sin^2 x} = \frac{\sin^2 x + \cos^2 x}{\sin^2 x \cos^2 x}$ $= \frac{1}{\sin^2 x \cos^2 x}$ $\sin x \cos x = \frac{1}{2} \sin 2x, \text{ so } \frac{1}{\frac{1}{4} \sin^2 2x} = 4 \operatorname{cosec}^2 2x \quad *$	M1 A1 A1* (3)
(b)	Assume there is a value of x (not a multiple of $\frac{\pi}{2}$) for which $\sec^2 x + \operatorname{cosec}^2 x < 4$ Then $4 \operatorname{cosec}^2 2x < 4$, so $\operatorname{cosec}^2 2x < 1$, i.e. $\sin^2 2x > 1$ But $\sin^2 2x \leq 1$ for all x: a contradiction. So $\sec^2 x + \operatorname{cosec}^2 x \geq 4 \quad *$	B1 M1 A1* (3)
(c)	$4 \operatorname{cosec}^2 2x = \frac{16}{3} \Rightarrow \sin^2 2x = \frac{3}{4}$ $\sin 2x = \pm \frac{\sqrt{3}}{2}$ $0 < 2x < 2\pi: 2x = \frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}$ $x = \frac{\pi}{6}, \frac{\pi}{3}, \frac{2\pi}{3}, \frac{5\pi}{6}$ <i>Taking only $\sin 2x = +\frac{\sqrt{3}}{2}$ loses the last two solutions. Decimal answers score A0.</i>	M1 A1 M1 A1 (4)
Total for Question 7: 10 marks		



Question 8

8 marks

Part	Scheme	Marks
(a)	Each integer k with $2 \leq k \leq n$ is a factor of $n! = 1 \times 2 \times \dots \times n$ So $n! + 1$ leaves remainder 1 when divided by k : k is not a factor of $n! + 1$ *	M1 A1* (2)
(b)	Assume there are finitely many primes, and let p be the largest Consider $N = p! + 1$, which is greater than p By (a), none of $2, 3, \dots, p$ is a factor of N ; $N > 1$, so N is prime or has a prime factor q , and then $q > p$ Either way there is a prime greater than p : a contradiction. So there are infinitely many primes * <i>N itself need not be prime (see part (c)): the argument only needs a prime factor of N that is bigger than p.</i>	B1 M1 M1 A1* (4)
(c)	Tries values of n , e.g. $2! + 1 = 3$, $3! + 1 = 7$ (both prime), then $n = 4$ $4! + 1 = 25 = 5 \times 5$, which is not prime, so the student is wrong	M1 A1 (2)
Total for Question 8: 8 marks		



Question 9

9 marks

Part	Scheme	Marks
(a)	Assume the curves meet: $2x^2 = 6 - \frac{8}{x^2}$ for some real $x \neq 0$ Multiplies by x^2: $2x^4 - 6x^2 + 8 = 0 \Rightarrow x^4 - 3x^2 + 4 = 0$, a quadratic in x^2 Discriminant $(-3)^2 - 4 \times 1 \times 4 = -7 < 0$ (or $(x^2 - \frac{3}{2})^2 + \frac{7}{4} > 0$) No real x^2, so no real x: a contradiction. So C_1 and C_2 do not meet * <i>The assumption must be written (Oct 2024 report: many lost the first mark by going straight to the algebra). Values of x^2 from a calculator, or reading the figure, score no marks.</i>	B1 M1 dM1 A1* (4)
(b)	$2x^4 - kx^2 + 8 = 0$, i.e. $2u^2 - ku + 8 = 0$ with $u = x^2$; real roots need $k^2 - 64 \geq 0$ Product of roots $4 > 0$, so the roots have the same sign; $u = x^2 > 0$ needs their sum $\frac{k}{2} > 0$ $k \geq 8$ <i>$k \leq -8$ gives two negative values of x^2, so it must be rejected. At $k = 8$ the curves touch at $x = \pm\sqrt{2}$.</i>	M1 M1 A1 (3)
(c)	$2u^2 - 10u + 8 = 0 \Rightarrow u = 1$ or $u = 4$, so $x = \pm 1, \pm 2$ $(-2, 8), (-1, 2), (1, 2), (2, 8)$	M1 A1 (2)
Total for Question 9: 9 marks		



Question 10

10 marks

Part	Scheme	Marks
(a)	Let $a = \frac{m}{n}$ be rational and b irrational. Assume $a + b = \frac{p}{q}$ is rational (m, n, p, q integers, $n, q \neq 0$) $b = \frac{p}{q} - \frac{m}{n} = \frac{pn - mq}{qn}$ $pn - mq$ and qn are integers with $qn \neq 0$, so b is rational: a contradiction. So $a + b$ is irrational *	B1 M1 A1* (3)
(b)	Assume $\sqrt{2} + \sqrt{3} = r$, where r is rational Squares: $2 + 2\sqrt{6} + 3 = r^2$ $\sqrt{6} = \frac{r^2 - 5}{2}$ $\frac{r^2 - 5}{2}$ is rational, but $\sqrt{6}$ is irrational: a contradiction. So $\sqrt{2} + \sqrt{3}$ is irrational *	B1 M1 dM1 A1* (4)
(c)	e.g. $\sqrt{2} + (-\sqrt{2}) = 0$, or $(1 + \sqrt{2}) + (1 - \sqrt{2}) = 2$	B1 (1)
(d)	$(\sqrt{3} - \sqrt{2})(\sqrt{3} + \sqrt{2}) = 3 - 2 = 1$, so $\sqrt{3} - \sqrt{2} = \frac{1}{\sqrt{3} + \sqrt{2}}$ If $\sqrt{3} - \sqrt{2} = s$ were rational ($s \neq 0$), then $\sqrt{3} + \sqrt{2} = \frac{1}{s}$ would be rational, contradicting (b). So $\sqrt{3} - \sqrt{2}$ is irrational *	M1 A1* (2)
Total for Question 10: 10 marks		



2

Chapter 2: Partial Fractions

Mark scheme. Marks are shown on the right of each line.

Question 1

5 marks

Part	Scheme	Marks
(a)	$1^3 - 2(1)^2 - 5(1) + 6 = 0$, so $(x - 1)$ is a factor; divides: $x^2 - x - 6$ $(x - 1)(x + 2)(x - 3)$	M1 A1 (2)
(b)	$4x^2 + 3x - 25 \equiv A(x + 2)(x - 3) + B(x - 1)(x - 3) + C(x - 1)(x + 2)$ Substitutes $x = 1$, $x = -2$ or $x = 3$ to find one constant, e.g. $x = 1$: $-18 = -6A$ $\frac{3}{x-1} - \frac{1}{x+2} + \frac{2}{x-3}$ <i>The final mark needs the whole expression written out, not only $A = 3$, $B = -1$, $C = 2$.</i>	B1 M1 A1 (3)
Total for Question 1: 5 marks		

Question 2

6 marks

Part	Scheme	Marks
(a)	$15t^2 - 10t + 7 \equiv A(3t - 1)^2 + B(t + 1)(3t - 1) + C(t + 1)$ $t = -1$: $32 = 16A$ so $A = 2$; $t = \frac{1}{3}$: $\frac{16}{3} = \frac{4}{3}C$ so $C = 4$ Compares t^2 coefficients: $15 = 9A + 3B$ $A = 2, B = -1, C = 4$ <i>Multiplying through by $(t + 1)(3t - 1)^3$ is a wrong identity (Jan 2024 report). No value of t gives B on its own: compare coefficients, or use $t = 0$: $7 = A - B + C$.</i>	B1 M1 M1 A1 (4)
(b)	$f'(t) = -\frac{2}{(t+1)^2} + \frac{3}{(3t-1)^2} - \frac{24}{(3t-1)^3}$ $f'(0) = -2 + 3 + 24 = 25$ <i>M1 for differentiating at least two terms to the form $\frac{k}{(at+b)^n}$ with n one higher. Using the quotient rule on the original fraction is valid but much longer.</i>	M1 A1 (2)
Total for Question 2: 6 marks		



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Question 3

5 marks

Part	Scheme	Marks
(a)	$x(x-1)^2 = x^3 - 2x^2 + x$; divides: quotient 2 $2x^3 - 2x^2 + x + 3 - 2(x^3 - 2x^2 + x) = 2x^2 - x + 3$, so $g(x) = 2 + \frac{2x^2 - x + 3}{x(x-1)^2}$ *	M1 A1* (2)
(b)	$2x^2 - x + 3 \equiv B(x-1)^2 + Cx(x-1) + Dx$, with $\frac{B}{x} + \frac{C}{x-1} + \frac{D}{(x-1)^2}$ $x = 0: B = 3; x = 1: D = 4; x^2$ coefficients: $B + C = 2$, so $C = -1$ $g(x) = 2 + \frac{3}{x} - \frac{1}{x-1} + \frac{4}{(x-1)^2}$ <i>Leaving out the 2 from part (a) in the final answer is the common error: the answer is $g(x)$, not just its proper part.</i>	B1 M1 A1 (3)
Total for Question 3: 5 marks		

Question 4

5 marks

Part	Scheme	Marks
	$x^2 + px + q \equiv 2(x-2)(x+4) + B(x+1)(x+4) + 3(x+1)(x-2)$ Compares x^2 coefficients: $1 = 2 + B + 3$ $B = -4$ Expands with $B = -4$: $2x^2 + 4x - 16 - 4x^2 - 20x - 16 + 3x^2 - 3x - 6$ $= x^2 - 19x - 38$, so $p = -19, q = -38$ <i>Substituting $x = 2$ gives $18B = 4 + 2p + q$: with p and q unknown this cannot find B on its own.</i>	B1 M1 A1 M1 A1 (5)
Total for Question 4: 5 marks		



Question 5

5 marks

Part	Scheme	Marks
	$(x^2 - 4)(x + 2) = (x - 2)(x + 2)^2$ Correct form: $\frac{A}{x - 2} + \frac{B}{x + 2} + \frac{C}{(x + 2)^2}$, with $3x^2 + x + 2 \equiv A(x + 2)^2 + B(x - 2)(x + 2) + C(x - 2)$ $x = 2: 16 = 16A$, so $A = 1$; $x = -2: 12 = -4C$, so $C = -3$ Compares x^2 coefficients: $3 = A + B$, so $B = 2$ $\frac{1}{x - 2} + \frac{2}{x + 2} - \frac{3}{(x + 2)^2}$ <i>Treating the denominator as $(x - 2)(x + 2)(x + 2)$ with three different single fractions, or using $\frac{A}{x^2 - 4}$, scores MO.</i>	B1 M1 A1 M1 A1
		(5)
Total for Question 5: 5 marks		



Question 6

10 marks

Part	Scheme	Marks
(a)	Divides by $x^2 + 3x$: quotient $2x + 1$ Remainder 9, so $\frac{9}{x(x+3)} \equiv \frac{C}{x} + \frac{D}{x+3}$ $A = 2, B = 1$, and one of $C = 3, D = -3$ $g(x) = 2x + 1 + \frac{3}{x} - \frac{3}{x+3}$ <i>The Oct 2021 report: "the most common error was to set the numerator equal to 6 rather than -6". Here $D = -3$, not 3.</i>	M1 M1 A1 A1 (4)
(b)	Differentiates, $\frac{k}{x} \rightarrow -\frac{k}{x^2}$ for at least one term $g'(x) = 2 - \frac{3}{x^2} + \frac{3}{(x+3)^2}$ <i>Differentiating $\frac{3}{x}$ to $3 \ln x$ is integrating, not differentiating (seen in Oct 2021).</i>	M1 A1 (2)
(c)	$g'(x) < 2 \Leftrightarrow \frac{3}{(x+3)^2} < \frac{3}{x^2}$ For $x > 0, x+3 > x > 0$, so $(x+3)^2 > x^2$ so $\frac{3}{(x+3)^2} < \frac{3}{x^2}$, hence $g'(x) < 2$ for all $x > 0$ * <i>Saying only "$g'(x) \rightarrow 2$ as $x \rightarrow \infty$" does not show $g'(x) < 2$: the two fractions must be compared.</i>	M1 M1 A1* (3)
(d)	A tangent parallel to $y = 2x + 5$ needs $g'(x) = 2$, which is impossible by (c)	B1 (1)
Total for Question 6: 10 marks		



Question 7

10 marks

Part	Scheme	Marks
(a)	$2 \equiv A(r+1)(r+2) + Br(r+2) + Cr(r+1)$ and substitutes $r = 0, -1, -2$ Two of $A = 1, B = -2, C = 1$ $\frac{1}{r} - \frac{2}{r+1} + \frac{1}{r+2}$	M1 A1 A1 (3)
(b)	Writes $\left(\frac{1}{r} - \frac{1}{r+1}\right) - \left(\frac{1}{r+1} - \frac{1}{r+2}\right)$, or lists the terms for $r = 1, 2, 3, \dots, n$ Shows the cancelling: $\left(1 - \frac{1}{n+1}\right) - \left(\frac{1}{2} - \frac{1}{n+2}\right)$ $= \frac{1}{2} - \frac{1}{n+1} + \frac{1}{n+2}$ $= \frac{1}{2} - \frac{(n+2) - (n+1)}{(n+1)(n+2)} = \frac{1}{2} - \frac{1}{(n+1)(n+2)} *$ <i>Listing terms: at least the first three and the last two rows must be shown so the cancelling can be seen.</i>	M1 A1 M1 A1* (4)
(c)	$\sum_{r=1}^{20} - \sum_{r=1}^4 = \left(\frac{1}{2} - \frac{1}{462}\right) - \left(\frac{1}{2} - \frac{1}{30}\right)$ $= \frac{1}{30} - \frac{1}{462} = \frac{12}{385}$ <i>Using $\sum_1^{20} - \sum_1^5$ is the common error: the lower sum must stop at $r = 4$.</i>	M1 A1 (2)
(d)	$\frac{1}{(n+1)(n+2)} > 0$ for every positive integer n , so the sum is less than $\frac{1}{2}$	B1 (1)
Total for Question 7: 10 marks		



Question 8

10 marks

Part	Scheme	Marks
(a)	$A = 1$ (the numerator and denominator have equal degree); remainder $x^2 + 5x + 8 - (x^2 + 5x + 6) = 2$ $\frac{2}{(x+2)(x+3)} \equiv \frac{B}{x+2} + \frac{C}{x+3}$ and substitutes $x = -2$ or $x = -3$ One of $B = 2, C = -2$ $f(x) = 1 + \frac{2}{x+2} - \frac{2}{x+3}$	B1 M1 A1 A1 (4)
(b)	Assume there is a real x ($x \neq -2, -3$) for which $f(x) = 1$ Then $\frac{2}{x+2} = \frac{2}{x+3}$, so $2(x+3) = 2(x+2)$ giving $6 = 4$: a contradiction. So $f(x) \neq 1$ for every real x * <i>Equally: $f(x) - 1 = \frac{2}{(x+2)(x+3)}$, and a fraction with numerator 2 is never zero. Pointing at the asymptote on the figure is not a proof.</i>	B1 M1 A1* (3)
(c)	$\frac{2}{(x+2)(x+3)} > 2$ requires $(x+2)(x+3) > 0$ and $(x+2)(x+3) < 1$ $x^2 + 5x + 5 < 0$: critical values $\frac{-5 \pm \sqrt{5}}{2}$ $\frac{-5 - \sqrt{5}}{2} < x < -3 \text{ or } -2 < x < \frac{-5 + \sqrt{5}}{2}$ <i>Multiplying by $(x+2)(x+3)$ without considering its sign includes $-3 < x < -2$, where $f(x) \leq -7$ (the figure shows the curve is below the x-axis there).</i>	M1 M1 A1 (3)
Total for Question 8: 10 marks		



Question 9

11 marks

Part	Scheme	Marks
(a)	$6u \equiv A(u+3) + B(u+1)$; substitutes $u = -1$ or $u = -3$ $A = -3$ or $B = 9$ $-\frac{3}{u+1} + \frac{9}{u+3}$	M1 A1 A1 (3)
(b)	$f(x) = -\frac{3}{e^x+1} + \frac{9}{e^x+3}$ Chain rule: $\frac{d}{dx} \left(\frac{k}{e^x+c} \right) = -\frac{ke^x}{(e^x+c)^2}$ $f'(x) = \frac{3e^x}{(e^x+1)^2} - \frac{9e^x}{(e^x+3)^2}$ *	B1 M1 A1* (3)
(c)	$f'(x) = 0$ and $e^x \neq 0$: $(u+3)^2 = 3(u+1)^2$ $u > 0$, so $u+3 = \sqrt{3}(u+1)$, giving $u = \frac{3-\sqrt{3}}{\sqrt{3}-1} = \sqrt{3}$ $e^x = \sqrt{3}$, so $x = \frac{1}{2} \ln 3$ <i>Taking $u+3 = -\sqrt{3}(u+1)$ gives a negative u, which is impossible for $u = e^x$.</i>	M1 M1 A1 (3)
(d)	$f = \frac{6\sqrt{3}}{(\sqrt{3}+1)(\sqrt{3}+3)} = \frac{6}{(1+\sqrt{3})^2} = \frac{6}{4+2\sqrt{3}}$ $6-3\sqrt{3}$	M1 A1 (2)
Total for Question 9: 11 marks		



Question 10

11 marks

Part	Scheme	Marks
(a)	$x^2 + 2 \equiv A(x-1)^2 + B(x-1) + C$ $x = 1: C = 3; x^2$ coefficients: $A = 1$ x coefficients (or $x = 0$): $0 = -2A + B$, so $B = 2$ $f(x) = 1 + \frac{2}{x-1} + \frac{3}{(x-1)^2}$	B1 M1 M1 A1 (4)
(b)	$f = 3u^2 + 2u + 1 = 3\left(u + \frac{1}{3}\right)^2 + \frac{2}{3}$ Minimum $\frac{2}{3}$ when $u = -\frac{1}{3}$, i.e. $x - 1 = -3$ $M\left(-2, \frac{2}{3}\right)$ <i>Differentiating $f(x)$ from (a) and solving $-\frac{2}{(x-1)^2} - \frac{6}{(x-1)^3} = 0$ is equally valid.</i>	M1 M1 A1 (3)
(c)	$3u^2 + 2u = 0$, so $u = 0$ or $u = -\frac{2}{3}$; $u = \frac{1}{x-1}$ is never 0, so $u = 0$ is rejected Exactly one solution: $x = -\frac{1}{2}$	M1 A1 (2)
(d)	$3u^2 + 2u + 1 - k = 0$ has two distinct roots when $k > \frac{2}{3}$; each non-zero u gives one x $k = 1$ gives the root $u = 0$ (no x), so $\frac{2}{3} < k < 1 \text{ or } k > 1$ <i>Check with the figure: for $\frac{2}{3} < k < 1$ both solutions have $x < 1$, one on each side of M; for $k > 1$ there is one solution on each branch.</i>	M1 A1 (2)
Total for Question 10: 11 marks		



3

Chapter 3: Coordinate Geometry in the (x, y) Plane

Mark scheme. Marks are shown on the right of each line.

Question 1

5 marks

Part	Scheme	Marks
(a)	$x(t+1) = 2t \Rightarrow t(x-2) = -x \Rightarrow t = \frac{x}{2-x}$	M1
	Substitutes into y : $t-3 = \frac{4x-6}{2-x}$ and $t+1 = \frac{2}{2-x}$	M1
	$y = \frac{4x-6}{2}$, so all points lie on the line $y = 2x - 3$	A1
	<i>The answer must contain no t (Oct 2022 report: y "in terms of both x and t" after a partial rearrangement).</i>	(3)
(b)	Solves with the line: $2t+t-3 = 13(t+1)$ gives $t = -\frac{8}{5}$; or $x = \frac{16}{3}$ on the line $y = 2x - 3$	M1
	$t = -\frac{8}{5}$ is not in $t > -1$ (equivalently $x = 2 - \frac{2}{t+1} < 2$ on C), so C does not meet the line	A1
	<i>Finding the point $(\frac{16}{3}, \frac{23}{3})$ on the full line and stopping there scores M1 A0.</i>	(2)
Total for Question 1: 5 marks		

Question 2

5 marks

Part	Scheme	Marks
(a)	Uses $\sec^2 t = 1 + \tan^2 t$ with $\tan t = x - 2$: $y = 4(1 + (x-2)^2) - 3$	M1
	$f(x) = 4(x-2)^2 + 1$	A1
		(2)
(b)	Domain: $\tan(-\frac{\pi}{4}) = -1$, $\tan \frac{\pi}{3} = \sqrt{3}$, so $1 \leq x \leq 2 + \sqrt{3}$	B1
	$t = 0$ (so $x = 2$) is inside the interval: minimum $y = 1$; ends give $y = 5$ and $y = 13$	M1
	$1 \leq f(x) \leq 13$	A1
	$5 \leq f(x) \leq 13$ (the two end values only) is the common error: the minimum is at $t = 0$, inside the interval.	(3)
Total for Question 2: 5 marks		



Question 3

5 marks

Part	Scheme	Marks
(a)	$y = \sin 2t = 2 \sin t \cos t = x \cos t$	M1
	$y^2 = x^2 \cos^2 t = x^2 \left(1 - \frac{x^2}{4}\right)$	M1
	$4y^2 = x^2 (4 - x^2) \quad *$	A1*
		(3)
(b)	$\sin 2t = 0: t = 0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}$ (or $y = 0$ in the Cartesian equation)	M1
	$(-2, 0), (0, 0), (2, 0)$	A1
		(2)
Total for Question 3: 5 marks		

Question 4

5 marks

Part	Scheme	Marks
	$y = 2 \left(\cos t \cos \frac{\pi}{6} - \sin t \sin \frac{\pi}{6} \right) = \sqrt{3} \cos t - \sin t$	B1
	$\sin t = \frac{x}{2}$, so $y + \frac{x}{2} = \sqrt{3} \cos t$	M1
	Squares and uses $\cos^2 t = 1 - \sin^2 t = 1 - \frac{x^2}{4}$	M1
	$y^2 + xy + \frac{x^2}{4} = 3 - \frac{3x^2}{4}$	A1
	$x^2 + xy + y^2 = 3$, so $k = 3$	A1
	<i>The value of k must be stated (Oct 2024 report: "very common for the value of k to be omitted").</i>	
		(5)
Total for Question 4: 5 marks		



Question 5

5 marks

Part	Scheme	Marks
(a)	$t = e^x - 1: y = (e^x - 1)^2 - 2(e^x - 1)$	M1
	$y = e^{2x} - 4e^x + 3$, so $p = -4, q = 3$	A1
		(2)
(b)	$y = 0: t(t - 2) = 0$ or $(e^x - 1)(e^x - 3) = 0$	M1
	$(0, 0)$ and $(\ln 3, 0)$	A1
		(2)
(c)	$y = (t - 1)^2 - 1$ and $t = 1$ is allowed: $y \geq -1$	B1
		(1)
Total for Question 5: 5 marks		

Question 6

11 marks

Part	Scheme	Marks
(a)	$y = 0: t(t^2 - 3) = 0$	M1
	$t = \pm\sqrt{3}$ (and $t = 0$ at O)	A1
	Both give $A(3, 0)$, with $t = \sqrt{3}$ and $t = -\sqrt{3}$	A1
		(3)
(b)	$y^2 = t^2(t^2 - 3)^2$	M1
	Substitutes $t^2 = x$	M1
	$y^2 = x(x - 3)^2$ *	A1*
		(3)
(c)	$t^3 - 3t = 2t^2 \Rightarrow t(t^2 - 2t - 3) = 0$	M1
	$t = 3$ or $t = -1$	A1
	$(9, 18)$ and $(1, 2)$	A1
		(3)
(d)	$t = 0$ or $t^2 - mt - 3 = 0$, which always has two different non-zero roots ($m^2 + 12 > 0$); two values of t give the same point only at A , when the roots are $\pm\sqrt{3}$	M1
	$m = 0$: the line $y = 0$ meets C only at O and A	A1
		(2)
Total for Question 6: 11 marks		



Question 7

9 marks

Part	Scheme	Marks
(a)	$\cos t = \frac{x+1}{4}, \sin t = \frac{y-3}{4}$ and uses $\cos^2 t + \sin^2 t = 1$ $(x+1)^2 + (y-3)^2 = 16$ centre $(-1, 3)$, radius 4	M1 A1 A1 (3)
(b)	Upper semicircle ($\sin t \geq 0$ so $y \geq 3$) Ends $(3, 3)$ and $(-5, 3)$; highest point $(-1, 7)$	B1 B1 (2)
(c)	$4 \sin t + 3 = 4 \cos t - 1 + k \Rightarrow 4 \sin t - 4 \cos t = k - 4$ $4\sqrt{2} \sin\left(t - \frac{\pi}{4}\right) = k - 4$ $-\frac{\pi}{4} \leq t - \frac{\pi}{4} \leq \frac{3\pi}{4}$: two solutions when $\frac{\sqrt{2}}{2} \leq \sin\left(t - \frac{\pi}{4}\right) < 1$ $8 \leq k < 4 + 4\sqrt{2}$ <i>Alternative: $k = 8$ is the line through the end $(-5, 3)$ (it also meets $(-1, 7)$), and $k = 4 + 4\sqrt{2}$ is the tangent, where the distance from the centre equals 4. Using the whole circle gives $4 - 4\sqrt{2} < k < 4 + 4\sqrt{2}$: M1 M1 M0 A0.</i>	M1 M1 M1 A1 (4)
Total for Question 7: 9 marks		



Question 8

10 marks

Part	Scheme	Marks
(a)	$\cos t = \frac{x}{50}, \sin t = \frac{y}{30}$ and $\cos^2 t + \sin^2 t = 1$ $\frac{x^2}{2500} + \frac{y^2}{900} = 1$	M1 A1 (2)
(b)	$d^2 = 2500 \cos^2 t + 900 \sin^2 t = 2500 \cos^2 t + 900 (1 - \cos^2 t)$ $= 900 + 1600 \cos^2 t$ *	M1 A1* (2)
(c)	$1600 = 900 + 1600 \cos^2 t \Rightarrow \cos t = \pm \frac{\sqrt{7}}{4}$ $t = 0.848$ and the other three by symmetry $t = 0.848, 2.29, 3.99, 5.44$	M1 M1 A1 (3)
(d)	$d > 40$ when $\cos^2 t > \frac{7}{16}$: for $0 \leq t < 0.848, 2.29 < t < 3.99, 5.44 < t < 2\pi$, total $4 \times 0.848 = 3.392$ minutes 204 seconds (3 minutes 24 seconds)	M1 A1 (2)
(e)	e.g. the runner will not keep exactly to one line of the track, or the runner's speed is not likely to follow the model; a real track has straight sections	B1 (1)
Total for Question 8: 10 marks		



Question 9

10 marks

Part	Scheme	Marks
(a)	$\cos 2t = \cos^2 t - \sin^2 t = \cos^2 t (1 - \tan^2 t)$ $\cos^2 t = \frac{1}{\sec^2 t} = \frac{1}{1 + \tan^2 t}$ $y = \frac{1 - x^2}{1 + x^2}^*$	M1 M1 A1* (3)
(b)	$1 - x^2 = 2 - (1 + x^2)$ $y = -1 + \frac{2}{1 + x^2}$	M1 A1 (2)
(c)	Assume there is a point on C with $y \leq -1$ Then $-1 + \frac{2}{1 + x^2} \leq -1$, so $\frac{2}{1 + x^2} \leq 0$ But $1 + x^2 > 0$, so $\frac{2}{1 + x^2} > 0$: a contradiction. So there is no point with $y \leq -1$ *	B1 M1 A1* (3)
(d)	$\frac{1 - x^2}{1 + x^2} = \frac{1}{2} \Rightarrow x^2 = \frac{1}{3}$ (or $\cos 2t = \frac{1}{2}$, $t = \pm \frac{\pi}{6}$) $\left(\frac{\sqrt{3}}{3}, \frac{1}{2}\right)$ and $\left(-\frac{\sqrt{3}}{3}, \frac{1}{2}\right)$	M1 A1 (2)
Total for Question 9: 10 marks		



Question 10

11 marks

Part	Scheme	Marks
(a)	$\left(\frac{x}{2}\right)^{\frac{2}{3}} = \cos^2 t$ and $\left(\frac{y}{2}\right)^{\frac{2}{3}} = \sin^2 t$	M1
	Adds: $\frac{x^{\frac{2}{3}} + y^{\frac{2}{3}}}{2^{\frac{2}{3}}} = 1$	A1
	$x^{\frac{2}{3}} + y^{\frac{2}{3}} = 2^{\frac{2}{3}}$, so $k = \sqrt[3]{4}$	A1
		(3)
(b)	$x^2 + y^2 = 4(\cos^6 t + \sin^6 t)$	M1
	$\cos^6 t + \sin^6 t = (\cos^2 t + \sin^2 t)^3 - 3\cos^2 t \sin^2 t (\cos^2 t + \sin^2 t)$	M1
	$= 1 - 3\sin^2 t \cos^2 t = 1 - \frac{3}{4} \sin^2 2t$, using $\sin t \cos t = \frac{1}{2} \sin 2t$	M1
	$x^2 + y^2 = 4 - 3\sin^2 2t$ *	A1*
(c)		(4)
	Least when $\sin^2 2t = 1$: $x^2 + y^2 = 1$	M1
	Least distance 1	A1
	$t = \frac{\pi}{4} \cdot \left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$	A1
		(3)
(d)	2, at (2, 0), (0, 2), (-2, 0) and (0, -2)	B1
		(1)
Total for Question 10: 11 marks		



4

Chapter 4: Binomial Expansion

Mark scheme. Marks are shown on the right of each line.

Question 1

5 marks

Part	Scheme	Marks
(a)	Takes out $9^{-\frac{1}{2}} = \frac{1}{3}: \frac{1}{3} \left(1 + \frac{4}{9}x\right)^{-\frac{1}{2}}$ Expands with $\left(\frac{4}{9}x\right)$ and correct coefficients, e.g. $\frac{\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)}{2!} \left(\frac{4x}{9}\right)^2$ Two correct simplified terms $\frac{1}{3} - \frac{2}{27}x + \frac{2}{81}x^2 - \frac{20}{2187}x^3$ <i>Taking out 9 or 3 instead of $\frac{1}{3}$ is B0 but can score M1. The factor $\frac{1}{3}$ must multiply every term.</i>	B1 M1 A1 A1 (4)
(b)	$x < \frac{9}{4}$	B1 (1)
Total for Question 1: 5 marks		

Question 2

6 marks

Part	Scheme	Marks
(a)	$1 + \frac{1}{3}(-6x) + \frac{\frac{1}{3}\left(-\frac{2}{3}\right)}{2}(-6x)^2$ $1 - 2x$ $1 - 2x - 4x^2$	M1 A1 A1 (3)
(b)	Constant term: $a = 3$ x terms: $-2a + b = -7$, so $b = -1$ x^2 terms: $-4a - 2b = c$, so $c = -10$ <i>The x^2 term has two parts: $3 \times (-4x^2)$ and $(-x) \times (-2x)$. Missing the second gives $c = -12$.</i>	B1 M1 A1 (3)
Total for Question 2: 6 marks		



Question 3

6 marks

Part	Scheme	Marks
(a)	$nk = -2$ $\frac{n(n-1)}{2}k^2 = 6$ Substitutes $k = -\frac{2}{n}$: $\frac{n(n-1)}{2} \times \frac{4}{n^2} = 6 \Rightarrow 2(n-1) = 6n$ $n = -\frac{1}{2}, k = 4$	B1 B1 M1 A1 (4)
(b)	$\frac{(-\frac{1}{2})(-\frac{3}{2})(-\frac{5}{2})}{6} \times 4^3 = -20$	B1ft (1)
(c)	$ x < \frac{1}{4}$	B1ft (1)
Total for Question 3: 6 marks		

Question 4

6 marks

Part	Scheme	Marks
(a)	$x+7 \equiv A(x-2)+B(1+x); x=-1: A=-2; x=2: B=3$ $-\frac{2}{1+x} + \frac{3}{x-2}$	M1 A1 (2)
(b)	Writes $\frac{3}{x-2} = -\frac{3}{2}\left(1-\frac{x}{2}\right)^{-1}$ Expands both: $-2(1-x+x^2)$ and $-\frac{3}{2}\left(1+\frac{x}{2}+\frac{x^2}{4}\right)$ $-\frac{7}{2} + \frac{5}{4}x - \frac{19}{8}x^2$ Expanding $3(x-2)^{-1}$ as $3(1-2x)^{-1}$, or as $3(-2)(1-\dots)$, is the scheme-of-work error: $(x-2)^{-1} = -\frac{1}{2}\left(1-\frac{x}{2}\right)^{-1}$	M1 M1 A1 (3)
(c)	Valid for $ x < 1$ and $ x < 2$: $ x < 1$	B1 (1)
Total for Question 4: 6 marks		



Question 5

6 marks

Part	Scheme	Marks
(a)	$(4-x)^{-\frac{1}{2}} = \frac{1}{2} \left(1 - \frac{x}{4}\right)^{-\frac{1}{2}}$ Expands with $\left(-\frac{x}{4}\right)$ and correct coefficients Two correct simplified terms $\frac{1}{2} + \frac{x}{16} + \frac{3x^2}{256} + \frac{5x^3}{2048}$	B1 M1 A1 A1 (4)
(b)	$x = 1: (4-1)^{-\frac{1}{2}} = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3} \approx \frac{1181}{2048}, \text{ so } \sqrt{3} \approx 3 \times \frac{1181}{2048}$ $\sqrt{3} \approx \frac{3543}{2048}$ <i>Oct 2024 report: the series approximated "the reciprocal of what they were asked to find". Giving $\frac{1181}{2048}$ as the approximation to $\sqrt{3}$ scores M0.</i>	M1 A1 (2)
Total for Question 5: 6 marks		



Question 6

11 marks

Part	Scheme	Marks
(a)	$(1+x)^{\frac{1}{2}} = 1 + \frac{1}{2}x - \frac{1}{8}x^2 + \frac{1}{16}x^3$ Expands $(1-x)^{-\frac{1}{2}}$ with $(-x)$ $(1-x)^{-\frac{1}{2}} = 1 + \frac{1}{2}x + \frac{3}{8}x^2 + \frac{5}{16}x^3$	B1 M1 A1 (3)
(b)	$\sqrt{\frac{1+x}{1-x}} = (1+x)^{\frac{1}{2}}(1-x)^{-\frac{1}{2}}$ and multiplies, collecting terms to x^3 $x^2: \frac{3}{8} + \frac{1}{4} - \frac{1}{8} = \frac{1}{2}$ $x^3: \frac{5}{16} + \frac{3}{16} - \frac{1}{16} + \frac{1}{16} = \frac{1}{2}$, so $1+x + \frac{1}{2}x^2 + \frac{1}{2}x^3$ *	M1 A1 A1* (3)
(c)	$ x < 1$	B1 (1)
(d)	$x = \frac{1}{5}: \sqrt{\frac{6/5}{4/5}} = \sqrt{\frac{3}{2}} = \frac{\sqrt{6}}{2}$, so $\sqrt{6} \approx 2\left(1 + \frac{1}{5} + \frac{1}{50} + \frac{1}{250}\right)$ $\frac{306}{125}$	M1 A1 (2)
(e)	$\frac{\left \frac{306}{125} - \sqrt{6}\right }{\sqrt{6}} \times 100$ 0.061%	M1 A1 (2)
Total for Question 6: 11 marks		



Question 7

11 marks

Part	Scheme	Marks
(a)	$6x^2 + 3 \equiv A(1 - 2x)^2 + B(1 + x)(1 - 2x) + C(1 + x)$ $x = -1: 9 = 9A, A = 1; x = \frac{1}{2}: \frac{9}{2} = \frac{3}{2}C, C = 3$ x^2 coefficients: $6 = 4A - 2B$ $A = 1, B = -1, C = 3$	B1 M1 M1 A1 (4)
(b)	$(1 + x)^{-1} = 1 - x + x^2$ $(1 - 2x)^{-1} = 1 + 2x + 4x^2$ and $(1 - 2x)^{-2} = 1 + 4x + 12x^2$ Combines: $(1 - x + x^2) - (1 + 2x + 4x^2) + 3(1 + 4x + 12x^2)$ $3 + 9x + 33x^2$	M1 M1 M1 A1 (4)
(c)	$(1 + x)^{-1}$ needs $ x < 1$ and $(1 - 2x)^{-1}, (1 - 2x)^{-2}$ need $ x < \frac{1}{2}$; both must hold, so $ x < \frac{1}{2}$	B1 (1)
(d)	The x^n terms are $(-1)^n, -2^n$ and $3(n+1)2^n$ (from $\binom{-2}{n}(-2)^n = (n+1)2^n$) $(-1)^n + 2^n(3n+3-1) = (-1)^n + (3n+2)2^n$ * Check: $n = 2$ gives $1 + 8 \times 4 = 33$, matching part (b).	M1 A1* (2)
Total for Question 7: 11 marks		



Question 8

10 marks

Part	Scheme	Marks
(a)	$A = \frac{1}{8}$	B1 (1)
(b)	$(2 - kx)^{-3} = \frac{1}{8} \left(1 - \frac{kx}{2}\right)^{-3}$ and expands $B = \frac{3k}{16}$ and $C = \frac{3k^2}{16}$ $\frac{3k^2}{16} = \frac{6k}{16} \Rightarrow k^2 - 2k = 0$ *	M1 A1 A1* (3)
(c)	$k(k - 2) = 0$ and $k > 0$: $k = 2$ $D = \frac{1}{8} \times \frac{(-3)(-4)(-5)}{6} \times (-1)^3$ $D = \frac{5}{4}$	B1 M1 A1 (3)
(d)	$ 2x < 2$: $ x < 1$	B1ft (1)
(e)	$2 - 2x = 1.9$ gives $x = 0.05$: $\frac{1}{8} + \frac{3}{8}(0.05) + \frac{3}{4}(0.05)^2 + \frac{5}{4}(0.05)^3$ 0.1458	M1 A1 (2)
Total for Question 8: 10 marks		



Question 9

10 marks

Part	Scheme	Marks
(a)	Angle $OTP = 90^\circ$ (tangent and radius), so $d^2 = (R+h)^2 - R^2 = 2Rh + h^2$ $d = \sqrt{2Rh \left(1 + \frac{h}{2R}\right)} = \sqrt{2Rh} \left(1 + \frac{h}{2R}\right)^{\frac{1}{2}} *$	M1 A1* (2)
(b)	Expands $\left(1 + \frac{h}{2R}\right)^{\frac{1}{2}}$ with $\left(\frac{h}{2R}\right)$ $1 + \frac{h}{4R}$ $\frac{\frac{1}{2} \left(-\frac{1}{2}\right)}{2} \left(\frac{h}{2R}\right)^2 = -\frac{h^2}{32R^2} *$	M1 A1 A1* (3)
(c)	$\left \frac{h}{2R}\right < 1$ with $h > 0$: $0 < h < 2R$	B1 (1)
(d)	Exact: $d = \sqrt{2 \times 6400 \times 400 + 400^2} = \sqrt{5280000} = 2297.8$ Simple: $\sqrt{5120000} = 2262.7$; percentage error $\frac{2297.8 - 2262.7}{2297.8} \times 100$ 1.5%	M1 M1 A1 (3)
(e)	$d^2 = 2Rh + h^2 > 2Rh$ since $h > 0$, so $d > \sqrt{2Rh}$	B1 (1)
Total for Question 9: 10 marks		



Question 10

10 marks

Part	Scheme	Marks
(a)	Expands with $(2x)$ and $n = \frac{3}{2}$ $1 + 3x + \frac{3}{2}x^2 - \frac{1}{2}x^3, \text{ i.e. } 1 + 3x + \frac{3}{2}x^2 - \frac{1}{2}x^3$	M1 A1 A1 A1 (4)
(b)	$ x < \frac{1}{2}$	B1 (1)
(c)	Assume C_1 and C_2 meet at some x with $0 < x < \frac{1}{2}$ $1 + 3x + \frac{3}{2}x^2 - \frac{1}{2}x^3 = 2 + x + \frac{5}{2}x^2 \Rightarrow \frac{1}{2}x^3 + x^2 - 2x + 1 = 0$ $\frac{1}{2}x^3 + (x-1)^2 = 0, \text{ where } (x-1)^2 > 0 \text{ and } \frac{1}{2}x^3 > 0 \text{ for } 0 < x < \frac{1}{2}$ So the left-hand side is positive and cannot be 0: a contradiction. C_1 and C_2 do not meet * <i>As in the June 2024 scheme, the contradiction needs a reason for both terms being positive, and a conclusion. The figure is not a proof.</i>	B1 M1 M1 A1* (4)
(d)	C_1 is only an approximation (the terms in x^4 and higher are left out), so the true curve could meet C_2 even though C_1 does not	B1 (1)
Total for Question 10: 10 marks		



5

Chapter 5: Differentiation

Mark scheme. Marks are shown on the right of each line.

Question 1

6 marks

Part	Scheme	Marks
	Differentiates implicitly: $xy \rightarrow y + x \frac{dy}{dx}$ or $y^3 \rightarrow 3y^2 \frac{dy}{dx}$	M1
	$4x + y + x \frac{dy}{dx} - 3y^2 \frac{dy}{dx} = 0$	A1
	Substitutes (2, 1): $8 + 1 + 2 \frac{dy}{dx} - 3 \frac{dy}{dx} = 0$	M1
	$\frac{dy}{dx} = 9$	A1
	Normal gradient $-\frac{1}{9}$: $y - 1 = -\frac{1}{9}(x - 2)$	M1
	$x + 9y - 11 = 0$	A1
	The “= 0” must be seen, and any integer multiple is accepted. Finding the tangent ($y = 9x - 17$) scores the first four marks only.	
		(6)
Total for Question 1: 6 marks		

Question 2

6 marks

Part	Scheme	Marks
(a)	$\frac{dx}{dt} = 8 \sin t \cos t$ and $\frac{dy}{dt} = 3 \sec^2 t$	B1
	Divides $\frac{dy}{dt}$ by $\frac{dx}{dt}$	M1
	$\frac{dy}{dx} = \frac{3}{8 \sin t \cos^3 t}$	A1
		(3)
(b)	$t = \frac{\pi}{6}$: $x = 1, y = \sqrt{3}$	B1
	Gradient $\frac{3}{8 \times \frac{1}{2} \times \frac{3\sqrt{3}}{8}} = \frac{2}{\sqrt{3}}$, and $y - \sqrt{3} = \frac{2}{\sqrt{3}}(x - 1)$	M1
	$y = \frac{2\sqrt{3}}{3}x + \frac{\sqrt{3}}{3}$	A1
		(3)
Total for Question 2: 6 marks		



Question 3

5 marks

Part	Scheme	Marks
	$\frac{dV}{dr} = 4\pi r^2$	B1
	$\frac{dS}{dr} = 8\pi r$	B1
	$\frac{dr}{dt} = \frac{50}{4\pi r^2} = \frac{1}{2\pi}$ when $r = 5$	M1
	$\frac{dS}{dt} = \frac{dS}{dr} \times \frac{dr}{dt} = 40\pi \times \frac{1}{2\pi}$	M1
	$20 \text{ cm}^2 \text{ s}^{-1}$	A1
	Alternatively $S = (36\pi V^2)^{\frac{1}{3}}$ and $\frac{dS}{dt} = \frac{dS}{dV} \times 50$ with $V = \frac{500\pi}{3}$.	
		(5)
Total for Question 3: 5 marks		

Question 4

7 marks

Part	Scheme	Marks
(a)	$-3xy \rightarrow -3y - 3x \frac{dy}{dx}$ (product rule)	M1
	$y^2 \rightarrow 2y \frac{dy}{dx}$	M1
	$3x^2 - 3y - 3x \frac{dy}{dx} + 2y \frac{dy}{dx} = 0$	A1
	$\frac{dy}{dx} = \frac{3x^2 - 3y}{3x - 2y}$	A1
		(4)
(b)	Numerator = 0: $y = x^2$; with $y = 4$, $x = \pm 2$	M1
	$p = 2$ (as $p > 0$)	A1
	$k = 8 - 24 + 16$: $k = 0$	A1
		(3)
Total for Question 4: 7 marks		



Question 5

6 marks

Part	Scheme	Marks
(a)	$\frac{d}{dx}(2^x) = 2^x \ln 2$ $xy \rightarrow y + x \frac{dy}{dx}$ $y^2 \rightarrow 2y \frac{dy}{dx}$ and collects the $\frac{dy}{dx}$ terms $2^x \ln 2 + y = (2y - x) \frac{dy}{dx}$, so $\frac{dy}{dx} = \frac{2^x \ln 2 + y}{2y - x}$ * $2^x \rightarrow x2^{x-1}$ or $2^x \rightarrow 2^x$ is B0 (Oct 2024 report: the major error with 8^x).	B1 M1 M1 A1* (4)
(b)	Gradient $\frac{1+\ln 2}{2}$; tangent $y = 1 + \frac{1+\ln 2}{2}x$, then $y = 0$ $x = -\frac{2}{1+\ln 2}$	M1 A1 (2)
Total for Question 5: 6 marks		

Question 6

10 marks

Part	Scheme	Marks
(a)	$\frac{dy}{dx} = \frac{2}{2t}$ $\frac{1}{t}$	M1 A1 (2)
(b)	$P(4, 4)$ Normal gradient $-t = -2$: $y - 4 = -2(x - 4)$ $y = 12 - 2x$ *	B1 M1 A1* (3)
(c)	Substitutes $x = t^2$, $y = 2t$: $2t = 12 - 2t^2$ $t^2 + t - 6 = 0$, $(t + 3)(t - 2) = 0$; $t = 2$ is P , so $t = -3$ $Q(9, -6)$	M1 M1 A1 (3)
(d)	e.g. $\frac{1}{2} x_{PYQ} - x_{QYP} = \frac{1}{2} 4(-6) - 9(4) $ 30 Or: the normal meets the x -axis at $(6, 0)$; area = $\frac{1}{2} \times 6 \times 4 + \frac{1}{2} \times 6 \times 6 = 30$.	M1 A1 (2)
Total for Question 6: 10 marks		



Question 7

10 marks

Part	Scheme	Marks
(a)	$2y \frac{dy}{dx} - 6 \frac{dy}{dx}$ $= 6x + 12$ $\frac{dy}{dx} = \frac{3(x+2)}{y-3}$	M1 A1 A1 (3)
(b)	The ends of the interval are where the tangent is vertical: $y - 3 = 0$ $y = 3: 9 - 18 = 3x^2 + 12x$ $3x^2 + 12x + 9 = 0, 3(x+1)(x+3) = 0$ $p = -3, q = -1$ Otherwise: as a quadratic in y, $y^2 - 6y - (3x^2 + 12x) = 0$ needs $36 + 12x^2 + 48x \geq 0$, i.e. $(x+1)(x+3) \geq 0$.	M1 M1 M1 A1 (4)
(c)	Assume C has a stationary point: $\frac{dy}{dx} = 0$ there, so $x = -2$ Substitutes $x = -2$: $y^2 - 6y = 12 - 24$, i.e. $y^2 - 6y + 12 = 0$ Discriminant $36 - 48 < 0$: no real y, a contradiction. So C has no stationary points * $x = -2$ lies in $-3 < x < -1$, where the curve is not defined: an equally valid reason.	B1 M1 A1* (3)
Total for Question 7: 10 marks		



Question 8

10 marks

Part	Scheme	Marks
(a)	Similar triangles: $\frac{r}{h} = \frac{2.5}{5}$, so $r = \frac{h}{2}$ $V = \frac{1}{3}\pi\left(\frac{h}{2}\right)^2 h = \frac{\pi h^3}{12}$ * <i>Oct 2024 report: substituting $r = \frac{2}{5}h$ without showing where the link comes from cannot score.</i>	M1 A1* (2)
(b)	$\frac{dV}{dh} = \frac{\pi h^2}{4}$ $\frac{dV}{dt} = 2\pi - \frac{1}{2}\pi h$ $\frac{dh}{dt} = \frac{dV}{dt} \div \frac{dV}{dh}$ $= \frac{2\pi - \frac{1}{2}\pi h}{\frac{1}{4}\pi h^2} = \frac{8 - 2h}{h^2}$ *	B1 B1 M1 A1* (4)
(c)	1 m per minute	B1 (1)
(d)	$\frac{dh}{dt} = 0$ when $8 - 2h = 0$ $h = 4$ m ($\frac{dh}{dt} > 0$ for $h < 4$, so the depth rises towards 4 m)	M1 A1 (2)
(e)	The depth approaches 4 m, which is less than the 5 m height, so it does not overflow	B1 (1)
Total for Question 8: 10 marks		



Question 9

10 marks

Part	Scheme	Marks
(a)	$\frac{dx}{dt} = 1 - \cos t, \frac{dy}{dt} = \sin t$ and divides $\frac{dy}{dx} = \frac{\sin t}{1 - \cos t}$ *	M1 A1* (2)
(b)	$\sin t = 2 \sin \frac{t}{2} \cos \frac{t}{2}$ and $1 - \cos t = 2 \sin^2 \frac{t}{2}$ $\frac{2 \sin \frac{t}{2} \cos \frac{t}{2}}{2 \sin^2 \frac{t}{2}} = \cot \frac{t}{2}$ *	M1 A1* (2)
(c)	$P\left(\frac{2\pi}{3} - \frac{\sqrt{3}}{2}, \frac{3}{2}\right)$ Gradient $\cot \frac{\pi}{3} = \frac{\sqrt{3}}{3}$: $y - \frac{3}{2} = \frac{\sqrt{3}}{3} \left(x - \frac{2\pi}{3} + \frac{\sqrt{3}}{2}\right)$ $y = \frac{\sqrt{3}}{3}x + 2 - \frac{2\sqrt{3}\pi}{9}$	B1 M1 A1 (3)
(d)	Normal gradient $-\frac{1 - \cos t}{\sin t}$ through $(t - \sin t, 1 - \cos t)$ Sets $y = 0$: $-(1 - \cos t) = -\frac{1 - \cos t}{\sin t}(x - t + \sin t)$ $\sin t = x - t + \sin t$, so $x = t$: the normal passes through $(t, 0)$ * <i>Dividing by $1 - \cos t$ is valid because $1 - \cos t \neq 0$ for $0 < t < 2\pi$</i>	M1 M1 A1* (3)
Total for Question 9: 10 marks		



Question 10

11 marks

Part	Scheme	Marks
(a)	$\ln y = \cos x \ln x$	M1
	$\frac{1}{y} \frac{dy}{dx}$ on the left	M1
	Product rule on the right: $-\sin x \ln x + \frac{\cos x}{x}$	M1
	$\frac{dy}{dx} = x^{\cos x} \left(\frac{\cos x}{x} - \sin x \ln x \right)$ *	A1*
		(4)
(b)	$x^{\cos x} > 0$, so $\frac{\cos x}{x} = \sin x \ln x$	M1
	Multiplies by $\frac{x}{\cos x}$ ($\cos x \neq 0$): $x \tan x \ln x = 1$ *	A1*
(c)		(2)
	$g(x) = x \tan x \ln x - 1$: $g(1.2) = -0.437$, $g(1.3) = 0.229$	M1
	Sign change and g is continuous on $[1.2, 1.3]$, so $1.2 < \alpha < 1.3$	A1
(d)		(2)
	$g(1.2725) = -0.0027$ and $g(1.2735) = 0.0049$	M1
	Sign change, g continuous, so $\alpha = 1.273$ (3 d.p.)	A1
(e)		(2)
	$1.2729^{\cos 1.2729} = 1.07$	B1
(f)		(1)
Total for Question 10: 11 marks		



6

Chapter 6: Integration

Mark scheme. Marks are shown on the right of each line.

Question 1

5 marks

Part	Scheme	Marks
	Parts with $u = x$, $\frac{dv}{dx} = \sec^2 x$: $x \tan x - \int \tan x dx$	M1
	Correct first application	A1
	$\int \tan x dx = \ln \sec x $ (or $-\ln \cos x $)	M1
	$[x \tan x - \ln \sec x]_0^{\frac{\pi}{4}} = \frac{\pi}{4} - \ln \sqrt{2}$	dM1
	$\frac{1}{4}\pi - \frac{1}{2}\ln 2$	A1
		(5)
Total for Question 1: 5 marks		

Question 2

6 marks

Part	Scheme	Marks
	$x = u^2$, so $dx = 2u du$ (or $\frac{du}{dx} = \frac{1}{2\sqrt{x}}$)	B1
	Full substitution: $\int \frac{2u}{u-1} du$	M1
	Limits $u = 2$ and $u = 3$	B1
	$\frac{2u}{u-1} = 2 + \frac{2}{u-1}$	M1
	$[2u + 2\ln(u-1)]_2^3 = (6 + 2\ln 2) - (4 + 0)$	M1
	$2 + 2\ln 2$, so $a = 2$, $b = 2$	A1
	Using the x limits 4 and 9 with an integral in u loses the last two marks (Oct 2022 report). Writing dx as du is M0.	
		(6)
Total for Question 2: 6 marks		



Question 3

8 marks

Part	Scheme	Marks
(a)	$5 \equiv A(x+2) + B(2x-1)$; substitutes $x = \frac{1}{2}$ or $x = -2$ $A = 2$ or $B = -1$ $f(x) = \frac{2}{2x-1} - \frac{1}{x+2}$	M1 A1 A1 (3)
(b)	Integrates to the form $p \ln(2x-1) + q \ln(x+2)$ $\ln(2x-1) - \ln(x+2)$ Substitutes the limits and combines: $\ln \frac{(4k-1)(k+2)}{(2k+2)(2k-1)}$ $9(4k-1)(k+2) = 14(2k+2)(2k-1) \Rightarrow 4k^2 - 7k - 2 = 0$ $(4k+1)(k-2) = 0$; $k = -\frac{1}{4}$ is rejected as $k > \frac{1}{2}$: $k = 2$ <i>The integral of $\frac{2}{2x-1}$ is $\ln(2x-1)$, not $2\ln(2x-1)$. The June 2023 report: many failed to reject the second root.</i>	M1 A1 M1 M1 A1 (5)
Total for Question 3: 8 marks		

Question 4

5 marks

Part	Scheme	Marks
	$V = \pi \int_1^2 \left(2 + \frac{1}{x}\right)^2 dx$ Squares to three terms: $4 + \frac{4}{x} + \frac{1}{x^2}$ Integrates: $4x + 4 \ln x - \frac{1}{x}$ $\pi \left[\left(8 + 4 \ln 2 - \frac{1}{2}\right) - (4 + 0 - 1) \right]$ $\pi \left(\frac{9}{2} + 4 \ln 2 \right)$ <i>Squaring as $4 + \frac{1}{x^2}$ (each term only) is the Oct 2024 error: M0. A missing π loses the final mark.</i>	B1 M1 M1 dM1 A1 (5)
Total for Question 4: 5 marks		



Question 5

6 marks

Part	Scheme	Marks
	Separates: $\int \sec^2 y \, dy = \int (2x + 1) \, dx$ Integrates both sides $\tan y = x^2 + x + c$ Uses $x = 0, y = \frac{\pi}{4}$ $c = 1$ $y = \arctan(x^2 + x + 1)$	B1 M1 A1 M1 A1 A1 (6)
Total for Question 5: 6 marks		



Question 6

11 marks

Part	Scheme	Marks
(a)	$\frac{dx}{dt} = 2 \sec^2 t$ $\text{Area} = \int y \frac{dx}{dt} dt = \int 4 \cos^2 t \times 2 \sec^2 t dt$ $x = 0 \Rightarrow t = 0, x = 2 \Rightarrow t = \frac{\pi}{4}: \int_0^{\frac{\pi}{4}} 8 dt, \text{ so } k = 8$ $\text{Area } \boxed{2\pi}$ <i>Oct 2024 report: replacing dx by dt without $\frac{dx}{dt}$ scores M0.</i>	B1 M1 B1 A1 (4)
(b)	$\cos^2 t = \frac{1}{\sec^2 t} = \frac{1}{1 + \tan^2 t} = \frac{1}{1 + \frac{x^2}{4}}$ $y = \frac{4}{1 + \frac{x^2}{4}} = \frac{16}{4 + x^2} \quad *$	M1 A1* (2)
(c)	$V = \pi \int y^2 \frac{dx}{dt} dt = \pi \int 16 \cos^4 t \times 2 \sec^2 t dt$ $= 32\pi \int_0^{\frac{\pi}{4}} \cos^2 t dt$ $\cos^2 t = \frac{1}{2}(1 + \cos 2t): 16\pi \left[t + \frac{1}{2} \sin 2t \right]$ $16\pi \left(\frac{\pi}{4} + \frac{1}{2} \right)$ $\boxed{\pi(4\pi + 8)}$	B1 M1 M1 dM1 A1 (5)
Total for Question 6: 11 marks		



Question 7

10 marks

Part	Scheme	Marks
(a)	Parts: $-e^{-x} \sin x + \int e^{-x} \cos x dx$ Correct first application Parts again in the same direction: $\int e^{-x} \cos x dx = -e^{-x} \cos x - \int e^{-x} \sin x dx$ $I = -e^{-x} \sin x - e^{-x} \cos x - I$, so $2I = \dots$ $-\frac{1}{2}e^{-x}(\sin x + \cos x) + c$ <i>Changing direction in the second application returns $I = I$ and scores M0 for the third mark.</i>	M1 A1 M1 M1 A1 (5)
(b)	$\left[-\frac{1}{2}e^{-x}(\sin x + \cos x)\right]_0^{\pi} = \frac{1}{2}e^{-\pi} + \frac{1}{2}$ $\frac{1}{2}(1 + e^{-\pi})$	M1 A1 (2)
(c)	$\int_{\pi}^{2\pi} = -\frac{1}{2}e^{-2\pi} - \frac{1}{2}e^{-\pi}$ Negative (below the axis), so area $R_2 = \frac{1}{2}e^{-\pi}(1 + e^{-\pi})$ $\frac{\text{area } R_1}{\text{area } R_2} = \frac{\frac{1}{2}(1 + e^{-\pi})}{\frac{1}{2}e^{-\pi}(1 + e^{-\pi})} = e^{\pi}$ *	M1 M1 A1* (3)
Total for Question 7: 10 marks		



Question 8

12 marks

Part	Scheme	Marks
(a)	$1 \equiv A(10 - P) + BP$; substitutes $P = 0$ or $P = 10$ $A = \frac{1}{10}$ or $B = \frac{1}{10}$ $\frac{1}{10P} + \frac{1}{10(10 - P)}$	M1 A1 A1 (3)
(b)	Separates: $\int \frac{20}{P(10 - P)} dP = \int dt$ Integrates using (a) to the form $p \ln P + q \ln(10 - P)$ $2 \ln P - 2 \ln(10 - P) = t + c$ $t = 0, P = 2: c = 2 \ln \frac{2}{8} = -2 \ln 4$ Combines and exponentiates: $\frac{P}{10 - P} = \frac{1}{4} e^{\frac{1}{2}t}$ $P = \frac{10e^{\frac{1}{2}t}}{4 + e^{\frac{1}{2}t}} = \frac{10}{1 + 4e^{-\frac{1}{2}t}}$ * <i>The integral of $\frac{1}{10 - P}$ with respect to P is $-\ln(10 - P)$: the sign is often lost. Omitting $+c$ is a critical error (June 2023 report).</i>	B1 M1 A1 M1 M1 A1* (6)
(c)	$e^{-\frac{1}{2}t} \rightarrow 0$: 10 000 fish	B1 (1)
(d)	$8 = \frac{10}{1 + 4e^{-\frac{1}{2}t}} \Rightarrow e^{-\frac{1}{2}t} = \frac{1}{16}$ $t = 2 \ln 16 = 8 \ln 2$ years	M1 A1 (2)
Total for Question 8: 12 marks		



Question 9

12 marks

Part	Scheme	Marks
(a)	$V = \pi \int_0^k x e^{-x} dx$ $\text{Parts: } -x e^{-x} + \int e^{-x} dx$ $-x e^{-x} - e^{-x}$ $\text{Limits: } (-k e^{-k} - e^{-k}) - (0 - 1)$ $V = \pi (1 - (k+1)e^{-k}) \quad *$	B1 M1 A1 dM1 A1* (5)
(b)	π	B1 (1)
(c)	$1 - (\beta + 1)e^{-\beta} = 0.9 \Rightarrow (\beta + 1)e^{-\beta} = 0.1$ $e^{\beta} = 10(\beta + 1), \text{ so } \beta = \ln(10\beta + 10) \quad *$	M1 A1* (2)
(d)	$\beta_2 = 3.9120$ $\beta_3 = 3.8943$	M1 A1 (2)
(e)	$h(\beta) = \beta - \ln(10\beta + 10): h(3.8895) = -0.00018, h(3.8905) = 0.00062$ Sign change and h continuous, so $\beta = 3.890$ (3 d.p.)	M1 A1 (2)
Total for Question 9: 12 marks		



Question 10

11 marks

Part	Scheme	Marks
(a)	Parts: $\frac{1}{2}x \sin 2x - \int \frac{1}{2} \sin 2x \, dx$ $\frac{1}{2}x \sin 2x$ $\frac{1}{2}x \sin 2x + \frac{1}{4} \cos 2x + c$	M1 A1 A1 (3)
(b)	Separates: $\int y \, dy = \int x \cos^2 x \, dx$ $\cos^2 x = \frac{1}{2}(1 + \cos 2x)$, so $x \cos^2 x = \frac{1}{2}x + \frac{1}{2}x \cos 2x$ Integrates using (a) $\frac{1}{2}y^2 = \frac{1}{4}x^2 + \frac{1}{4}x \sin 2x + \frac{1}{8} \cos 2x + c$ $x = 0, y = 1: \frac{1}{2} = \frac{1}{8} + c, c = \frac{3}{8}$ $y^2 = \frac{1}{2}x^2 + \frac{1}{2}x \sin 2x + \frac{1}{4} \cos 2x + \frac{3}{4} *$	B1 M1 M1 A1 M1 A1* (6)
(c)	$y^2 = \frac{\pi^2}{8} + 0 - \frac{1}{4} + \frac{3}{4} = \frac{\pi^2 + 4}{8}$ $y = \sqrt{\frac{\pi^2 + 4}{8}}$ (positive, as $y > 0$)	M1 A1 (2)
Total for Question 10: 11 marks		



7

Chapter 7: Vectors

Mark scheme. Marks are shown on the right of each line.

Question 1

6 marks

Part	Scheme	Marks
(a)	Subtracts: $\vec{QP} = \mathbf{p} - \mathbf{q}$ and $\vec{QR} = \mathbf{r} - \mathbf{q}$ $\vec{QP} = -2\mathbf{i} - 2\mathbf{j} - \mathbf{k}$, $\vec{QR} = 2\mathbf{i} + \mathbf{j} + 2\mathbf{k}$ Oct 2022 report: subtracting the wrong way round gives $\vec{PQ}$ and scores only the method mark.	M1 A1 (2)
(b)	$\vec{QP} \cdot \vec{QR} = -4 - 2 - 2 = -8$ $\cos PQR = \frac{-8}{3 \times 3} = -\frac{8}{9}$ 152.7° Using $\vec{PQ}$ with $\vec{QR}$ gives 27.3°: M1 dM1 A0. The vectors must both point out of Q.	M1 dM1 A1 (3)
(c)	Angle PQR: it is obtuse, and a triangle can have only one obtuse angle (the other two add up to 27.3°)	B1 (1)
Total for Question 1: 6 marks		

Question 2

5 marks

Part	Scheme	Marks
(a)	Direction $\vec{AB} = 2\mathbf{i} + 4\mathbf{j} - 4\mathbf{k}$ (or any multiple, e.g. $\mathbf{i} + 2\mathbf{j} - 2\mathbf{k}$) $\mathbf{r} = (2\mathbf{i} - \mathbf{j} + 3\mathbf{k}) + \lambda(\mathbf{i} + 2\mathbf{j} - 2\mathbf{k})$ The "r =" is required for the A mark (Oct 2024 report). Any point on the line and any multiple of the direction are accepted.	M1 A1 (2)
(b)	j components: $-1 + 2\lambda = 9$, so $\lambda = 5$ $p = 7, q = -7$	M1 A1 (2)
(c)	$\frac{1}{3}\mathbf{i} + \frac{2}{3}\mathbf{j} - \frac{2}{3}\mathbf{k}$ (or its negative)	B1 (1)
Total for Question 2: 5 marks		



Question 3

6 marks

Part	Scheme	Marks
(a)	Equates components: $1 + 2\lambda = 3 - \mu$, $1 - \lambda = q + 2\mu$, $2 + \lambda = 8 + 2\mu$ Solves the i and k equations $\lambda = 2$, $\mu = -2$ j: $1 - 2 = q - 4$, so $q = 3$ <i>The j equation contains q, so it cannot be one of the pair used first.</i>	M1 M1 A1 A1 (4)
(b)	Substitutes $\lambda = 2$ into l_1 (or $\mu = -2$ into l_2) $(5, -1, 4)$	M1 A1 (2)
Total for Question 3: 6 marks		

Question 4

5 marks

Part	Scheme	Marks
	$\begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix} \neq k \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix}$ for any k, so the lines are not parallel Equates two components and solves: $1 + \lambda = 3 + \mu$, $2 - \lambda = 3 + 2\mu$ $\lambda = 1$, $\mu = -1$ Checks the third: $3 + 2\lambda = 5$ but $1 + \mu = 0$ $5 \neq 0$, so the lines do not intersect; they are not parallel, so they are skew * <i>Oct 2022 scheme: the final mark needs both "do not intersect" and "not parallel" with a reason. Stating "skew" straight after finding conflicting values is not enough.</i>	B1 M1 A1 M1 A1* (5)
Total for Question 4: 5 marks		



Question 5

6 marks

Part	Scheme	Marks
(a)	$\vec{OP} = \begin{pmatrix} -4+2\lambda \\ 2+\lambda \\ 6-2\lambda \end{pmatrix} \text{ and } OP^2 = (-4+2\lambda)^2 + (2+\lambda)^2 + (6-2\lambda)^2$ $= 9\lambda^2 - 36\lambda + 56 \quad *$	M1 A1* (2)
(b)	$9(\lambda - 2)^2 + 20$ Least when $\lambda = 2$ $(0, 4, 2)$ Shortest distance $\sqrt{20} = 2\sqrt{5}$ <i>Check: $(0, 4, 2) \cdot (2, 1, -2) = 0$, so OP is perpendicular to l. Differentiating OP^2 and setting it to 0 is equally valid (Jan 2026 scheme).</i>	M1 A1 A1 A1 (4)
Total for Question 5: 6 marks		



Question 6

10 marks

Part	Scheme	Marks
(a)	$\vec{OD} = \vec{OA} + \vec{BC}$ (or $\mathbf{a} + \mathbf{c} - \mathbf{b}$) $D(2, 4, 3)$	M1 A1 (2)
(b)	$\vec{BA} = \begin{pmatrix} -2 \\ 1 \\ -2 \end{pmatrix}, \vec{BC} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$ and $\vec{BA} \cdot \vec{BC} = -6$ $ \vec{BA} = 3, \vec{BC} = \sqrt{14}: \cos \angle ABC = \frac{-6}{3\sqrt{14}}$ $= -\frac{2}{\sqrt{14}} = -\frac{\sqrt{14}}{7} *$	M1 dM1 A1* (3)
(c)	$\sin \angle ABC = \sqrt{1 - \frac{14}{49}} = \sqrt{\frac{5}{7}}$ Area = $ \vec{BA} \vec{BC} \sin \angle ABC = 3\sqrt{14} \times \sqrt{\frac{5}{7}}$ $3\sqrt{10}$ <i>Using a decimal angle loses the A mark: the answer must be exact (Oct 2025 scheme).</i>	M1 M1 A1 (3)
(d)	Area = base $BC \times$ height: $\frac{3\sqrt{10}}{\sqrt{14}}$ $\frac{3\sqrt{35}}{7}$	M1 A1 (2)
Total for Question 6: 10 marks		



Question 7

10 marks

Part	Scheme	Marks
(a)	Direction $4\mathbf{i} + 2\mathbf{j} - 2\mathbf{k}$ (or $2\mathbf{i} + \mathbf{j} - \mathbf{k}$) $\mathbf{r} = (\mathbf{i} + 2\mathbf{k}) + \lambda(2\mathbf{i} + \mathbf{j} - \mathbf{k})$	M1 A1 (2)
(b)	$\vec{PX} = \begin{pmatrix} -2 + 2\lambda \\ -7 + \lambda \\ -5 - \lambda \end{pmatrix}$ $\vec{PX} \cdot \begin{pmatrix} 2 \\ 1 \\ -1 \end{pmatrix} = 0$ $6\lambda - 6 = 0, \text{ so } \lambda = 1$ $X(3, 1, 1)$ <i>June 2023 report: using $\vec{OX} \cdot \mathbf{d} = 0$ instead of $\vec{PX} \cdot \mathbf{d} = 0$ scores M1 only.</i>	M1 dM1 A1 A1 (4)
(c)	X is the midpoint of PP' : $\mathbf{p}' = 2\mathbf{x} - \mathbf{p}$ $P'(3, -5, -5)$	M1 A1 (2)
(d)	$PP' = 2PX = 12\sqrt{2}$ and $AX = \sqrt{6}$ is the perpendicular height; area = $\frac{1}{2} \times 12\sqrt{2} \times \sqrt{6}$ $12\sqrt{3}$	M1 A1 (2)
Total for Question 7: 10 marks		



Question 8

11 marks

Part	Scheme	Marks
(a)	Equates two components, e.g. $\lambda = -2 - 2\mu$ and $-3 + 2\lambda = 3 + \mu$ $\lambda = 2, \mu = -2$ $P(1, 2, 3)$	M1 A1 A1 (3)
(b)	$\begin{pmatrix} 2 \\ 1 \\ 2 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ -2 \\ 2 \end{pmatrix} = 2 - 2 + 4 = 4$ Both direction vectors have magnitude 3: $\cos \theta = \frac{4}{3 \times 3}$ $\cos \theta = \frac{4}{9}^*$	M1 M1 A1* (3)
(c)	$Q(7, 5, 9)$ and $PQ = 3\mathbf{d}_1 = 9$ $PR = \mu + 2 \times 3 = 9$, so $\mu = 1$ or $\mu = -5$ $R(4, -4, 9)$ or $R(-2, 8, -3)$ <i>Finding only one position (one side of P) is the common error: a distance along a line gives two points.</i>	B1 M1 A1 (3)
(d)	$\sin \theta = \frac{\sqrt{65}}{9}$; area = $\frac{1}{2} \times 9 \times 9 \times \frac{\sqrt{65}}{9} = \frac{9\sqrt{65}}{2}$ The angle at P is θ for one position and $180^\circ - \theta$ for the other, and $\sin(180^\circ - \theta) = \sin \theta$	M1 A1 (2)
Total for Question 8: 11 marks		



Question 9

12 marks

Part	Scheme	Marks
(a)	$\vec{OF} = 6\mathbf{i} + 4\mathbf{j} + 3\mathbf{k}$ $\vec{AG} = -6\mathbf{i} + 4\mathbf{j} + 3\mathbf{k}$	B1 B1 (2)
(b)	$\vec{OF} \cdot \vec{AG} = -36 + 16 + 9 = -11$ $\cos \theta = \frac{-11}{\sqrt{61}\sqrt{61}} = -\frac{11}{61}$ Acute angle 79.6° <i>100.4° is the obtuse angle: A0 unless 79.6° is then given (Oct 2024 report).</i>	M1 dM1 A1 (3)
(c)	$\mathbf{r} = t(6\mathbf{i} + 4\mathbf{j} + 3\mathbf{k})$ and $\mathbf{r} = 6\mathbf{i} + s(-6\mathbf{i} + 4\mathbf{j} + 3\mathbf{k})$; equates two components $4t = 4s$ and $6t = 6 - 6s$: $t = s = \frac{1}{2}$, and the $\mathbf{k}$ components agree ($1.5 = 1.5$), so they intersect $(3, 2, 1.5)$	M1 A1 A1 (3)
(d)	$\vec{OA} \cdot \vec{OG} = (6\mathbf{i}) \cdot (4\mathbf{j} + 3\mathbf{k}) = 0$, so angle $AOG = 90^\circ$ Area = $\frac{1}{2} \times 6 \times 5 = 15$	B1 B1 (2)
(e)	$\frac{1}{2} \times AG \times h = 15$ with $AG = \sqrt{61}$ $h = \frac{30}{\sqrt{61}} = \frac{30}{61}\sqrt{61}$	M1 A1 (2)
Total for Question 9: 12 marks		



Question 10

11 marks

Part	Scheme	Marks
(a)	Every point of l has z coordinate 2, but A and B both have $z = 3$	B1 (1)
(b)	$C(\lambda, 1 + \lambda, 2): \vec{CA} = \begin{pmatrix} 5 - \lambda \\ -\lambda \\ 1 \end{pmatrix}, \vec{CB} = \begin{pmatrix} 1 - \lambda \\ 2 - \lambda \\ 1 \end{pmatrix}$ Sets $\vec{CA} \cdot \vec{CB} = 0$ $(5 - \lambda)(1 - \lambda) - \lambda(2 - \lambda) + 1 = 0$ $2\lambda^2 - 8\lambda + 6 = 0$, so $\lambda^2 - 4\lambda + 3 = 0$ *	M1 dM1 A1 A1* (4)
(c)	$(\lambda - 1)(\lambda - 3) = 0$ $C(1, 2, 2)$ or $C(3, 4, 2)$	M1 A1 (2)
(d)	The angle in a semicircle is 90° , so C lies on the sphere with diameter AB A line meets a sphere in at most two points (the condition is a quadratic in λ), so there are at most two such points	B1 B1 (2)
(e)	$\lambda = 1: CA = \sqrt{18} = 3\sqrt{2}$ and $CB = \sqrt{2}$; area = $\frac{1}{2} \times 3\sqrt{2} \times \sqrt{2}$ 3	M1 A1 (2)
Total for Question 10: 11 marks		

