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INTERNATIONAL A LEVEL MATHEMATICS • PURE 2 (WMA12)

Pure 2 Practice Paper 1

Mark Scheme

AIMING FOR B

Written in the style of the Pearson Edexcel International Advanced Level paper (WMA12/01) • 75 marks

Every question marked line by line the way an examiner marks it, with notes on the common errors

Mark every line, not just the answer. Method marks are given for a correct method even when the final answer is wrong. If you lost marks, find the *first* line where your working differs from the scheme. That is the thing to fix.

Code	Meaning
M1	Method mark: for knowing a correct method and attempting to apply it. Small slips in the working do not lose this mark.
dM1	Dependent method mark: can only be given if the method mark before it has been given.
A1	Accuracy mark: for a correct answer or line of working. It depends on the method mark before it.
B1	Independent mark: for a correct result or statement, whatever method was used.
*	The answer is given in the question. Every step must be shown, and the answer must follow from correct working (cso).
cao / cso	Correct answer only / correct solution only, with no errors anywhere in the working.
ft / oe	Follow through from your own earlier answer / or any correct equivalent form.

"Solutions relying on calculator technology are not acceptable." Questions 2, 4, 5, 7, 9 and 10 say this. There, a correct final answer with no algebraic working scores **no marks**.

Rough grade guide (out of 75): A **54** • B **47** • C **40**

Based on recent WMA12 grade boundaries. Real boundaries change every series, so treat this as a guide, not a grade.

Question paper | Exam questions by topic | More practice papers

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What to revise next

Write in your score for each question. Where you dropped marks, tap the chapter to practise ten more exam questions on it.

Q	What it tested	Marks	Practise it	Score
1	Unknown constants from given binomial terms; a coefficient in a product	7	Ch 4 The Binomial Expansion	
2	Factor and remainder theorems; factorising and sketching a cubic	8	Ch 1 Algebraic Methods	
3	Centre and radius from an equation; radius perpendicular to tangent	7	Ch 2 Coordinate Geometry	
4	Exponential graphs; solving $a^x = b$ with logarithms	7	Ch 3 Exponentials and Logarithms	
5	A recurrence relation, a periodic sequence and sigma notation	7	Ch 5 Sequences and Series; Ch 1 Algebraic Methods	
6	Arithmetic and geometric series in context; logarithms to find n	8	Ch 5 Sequences and Series; Ch 3 Exponentials and Logarithms	
7	Using $\tan\theta = \frac{\sin\theta}{\cos\theta}$ and $\sin^2\theta + \cos^2\theta = 1$; an equation in $2x$	7	Ch 6 Trig Identities and Equations	
8	Optimisation in context; the second derivative test	8	Ch 7 Differentiation	
9	Area between two curves with surd limits	7	Ch 8 Integration	
10	Stationary point and its nature; trapezium rule; definite integral	9	Ch 7 Differentiation; Ch 8 Integration	
	Total	**75**		

Where did the marks go? If you lost the first marks of a question, the method needs work: go back to that chapter's key skills page. If you lost the final marks, the method is fine and the gap is accuracy or linking the parts together. Papers 2 and 3 give less help between the parts, so that is the skill to build next.

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Question 1

7 marks

Part	Scheme	Marks
(a)	$a^6 = 64$ and $a > 0$, so $a = 2$	B1
	Term in x: $6a^5b = -576$, so $192b = -576$	M1
	$\boxed{b = -3}$ (and $\boxed{a = 2}$)	A1
	Term in x^2: $p = \binom{6}{2}a^4b^2 = 15 \times 16 \times 9$	M1
	$\boxed{p = 2160}$	A1
	$a = -2$ also gives $a^6 = 64$; it is rejected because $a > 0$. Forgetting to square b in the x^2 term gives $p = -720$.	(5)
(b)	Coefficient of x^2: $3 \times 2160 + (-2) \times (-576) + 1 \times 64$ (at least two of the three products correct)	M1
	$\boxed{7696}$	A1
	The most common error is leaving out 1×64, the x^2 term multiplied by the constant term, giving 7632.	(2)
		Total for Question 1: 7 marks

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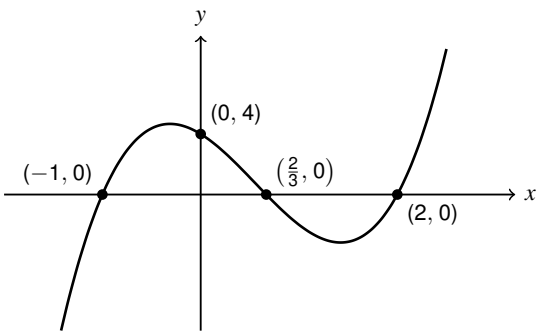
Question 2

8 marks

Part	Scheme	Marks
(a)	$f\left(\frac{2}{3}\right) = 0$: $3\left(\frac{8}{27}\right) + a\left(\frac{4}{9}\right) + b\left(\frac{2}{3}\right) + 4 = 0$ Multiplies by 9: $8 + 4a + 6b + 36 = 0$, so $2a + 3b = -22$ * *Using $f\left(-\frac{2}{3}\right)$ or $f\left(\frac{3}{2}\right)$ scores M0. The "= 0" must be there from the start.*	**M1** **A1*** **(2)**
(b)	$f(1) = -2$: $3 + a + b + 4 = -2$, so $a + b = -9$ $\boxed{a = -5,\ b = -4}$ Divides $3x^3 - 5x^2 - 4x + 4$ by $(3x - 2)$, or compares coefficients: $(3x - 2)\left(x^2 - x - 2\right)$ $\boxed{f(x) = (3x - 2)(x + 1)(x - 2)}$ *Setting $f(1) = 0$ instead of -2 is the common error. Roots found on a calculator, with no division or factorising shown, score M0 A0 here.*	**M1** **A1** **M1** **A1** **(4)**
(c)	 A positive cubic shape with a maximum and a minimum, crossing the x-axis three times Meets the axes at $(-1, 0)$, $\left(\frac{2}{3}, 0\right)$, $(2, 0)$ and $(0, 4)$ only	**B1** **B1** **(2)**
		Total for Question 2: 8 marks

Question 3

7 marks

Part	Scheme	Marks
(a)	$(x-3)^2+(y+5)^2=9+25-9$ Centre $\boxed{Q(3,-5)}$ Radius $\boxed{5}$ *A sign error giving $(-3,5)$ is the most common slip. The radius is $\sqrt{25}$, not 25.*	M1 A1 A1 (3)
(b)	$QP^2=(10-3)^2+(-4+5)^2=50$, and angle $QAP=90^\circ$ because a radius meets a tangent at right angles $PA^2=QP^2-QA^2=50-25=25$, so $PA=5$ * *The right angle must be stated or used. Finding the coordinates of A first is a much longer route.*	M1 A1* (2)
(c)	$PA=PB=QA=QB=5$ and the angles at A and B are 90° (radius and tangent), so $PAQB$ is a rhombus with a right angle: a square Area $\boxed{25}$	B1 B1 (2)
		Total for Question 3: 7 marks

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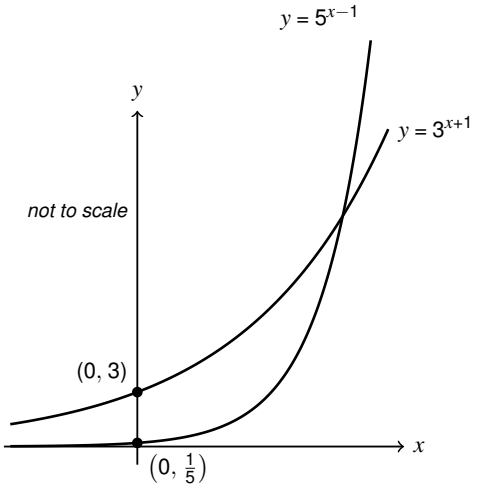
Question 4

7 marks

Part	Scheme	Marks
(a)	 Two increasing exponential curves above the x-axis, each approaching the x-axis on the left	B1
	$y = 3^{x+1}$ through (0, 3) and $y = 5^{x-1}$ through $\left(0, \frac{1}{5}\right)$	B1
	$y = 5^{x-1}$ below $y = 3^{x+1}$ at the y-axis, steeper, and crossing it once for $x > 0$	B1 (3)
(b)	Takes logs of both sides: $(x+1)\log 3 = (x-1)\log 5$	M1
	Collects the x terms: $x(\log 5 - \log 3) = \log 5 + \log 3$, so $x = \dfrac{\log 15}{\log \frac{5}{3}}$	dM1
	$\boxed{x = \log_{\frac{5}{3}} 15}$ $(= 5.30\ldots)$ *Writing* $\log(3^{x+1}) = \log 3^x + 1$*, or giving only 5.30 with no log work, scores M0.*	A1 (3)
(c)	$\boxed{x < \log_{\frac{5}{3}} 15}$ (ft their part (b)) *The sketch shows* 3^{x+1} *is the higher curve to the left of* P*.*	B1 (1)
		Total for Question 4: 7 marks

Question 5

7 marks

Part	Scheme	Marks
(a)	$u_2 = \frac{1}{3},\ u_3 = -\frac{1}{2}$ $u_4 = -3,\ u_5 = 2$	B1 B1 (2)
(b)	$u_{n+2} = \dfrac{u_{n+1} - 1}{u_{n+1} + 1} = \dfrac{\frac{u_n - 1}{u_n + 1} - 1}{\frac{u_n - 1}{u_n + 1} + 1}$ Multiplies through by $u_n + 1$: $\dfrac{u_n - 1 - (u_n + 1)}{u_n - 1 + u_n + 1} = \dfrac{-2}{2u_n} = -\dfrac{1}{u_n}$ *	M1 A1* (2)
(c)	The sequence repeats every 4 terms ($u_5 = u_1$, or using (b) twice), and $u_1 + u_2 + u_3 + u_4 = -\frac{7}{6}$ $S_{68} = 17 \times \left(-\frac{7}{6}\right) = -\frac{119}{6} = -19.8\ldots$, then $S_{69} = -17.8\ldots$, $S_{70} = -17.5$, $S_{71} = -18$, $S_{72} = -21$ $\boxed{n = 72}$ *Solving $-\frac{7}{6}k < -20$ gives $k > 17.1$, and the common error is to stop at $n = 69$. The partial sums inside a block go back up, so the first sum below -20 is at the end of the 18th block.*	M1 M1 A1 (3)
		Total for Question 5: 7 marks

Question 6

8 marks

Part	Scheme	Marks
(a)	$S_{20} = \frac{20}{2}[2(5) + 19(0.4)]$ $= 10 \times 17.6 = 176$ *	M1 A1* (2)
(b)	$3 \times 1.06^{n-1} > 15$ $1.06^{n-1} > 5$, so $n - 1 > \frac{\log 5}{\log 1.06} = 27.6\ldots$ Day 29 *Using 1.06^n gives day 28. Trial values are fine if they show day 28 (14.5 km) and day 29 (15.3 km).*	M1 M1 A1 (3)
(c)	$\frac{n}{2}[10 + 0.4(n-1)] \geqslant 500$, leading to $n^2 + 24n - 2500 = 0$ (oe) Solves the quadratic: $n = \frac{-24 + \sqrt{10576}}{2} = 39.4\ldots$ Day 40 *Rounding 39.4 down to day 39 is the common error. Check: $S_{39} = 491.4$ and $S_{40} = 512$.*	M1 dM1 A1 (3)
		Total for Question 6: 8 marks

Question 7

7 marks

Part	Scheme	Marks
(a)	Uses $\tan\theta = \frac{\sin\theta}{\cos\theta}$ and multiplies by $\cos\theta$: $3\sin^2\theta = 2\cos\theta + 5\cos^2\theta$ $3(1 - \cos^2\theta) = 2\cos\theta + 5\cos^2\theta$, so $8\cos^2\theta + 2\cos\theta - 3 = 0$ *	M1 A1* (2)
(b)	Uses (a) with $\theta = 2x$: $(4\cos 2x + 3)(2\cos 2x - 1) = 0$ $\cos 2x = \frac{1}{2}$ or $\cos 2x = -\frac{3}{4}$ Works in $0 \leqslant 2x < 360°$: $2x = 60°, 300°$ and $2x = 138.6°, 221.4°$ Halves each value to find x $x = 30°, 69.3°, 110.7°, 150°$ *Solving $\cos x = \ldots$ and then doubling, or only working in $0 \leqslant 2x < 180°$, loses the last two marks. Extra answers in the range lose the final A1.*	M1 A1 M1 dM1 A1 (5)
		Total for Question 7: 7 marks

Question 8

8 marks

Part	Scheme	Marks
(a)	Area: $2xh + \frac{1}{2}(2x)^2 \sin 60^\circ = 12$, i.e. $2xh + \sqrt{3}x^2 = 12$	M1
	$h = \dfrac{12 - \sqrt{3}x^2}{2x}$ substituted into $P = 2x + 2h + 2x + 2x = 2h + 6x$	M1
	$P = \dfrac{12 - \sqrt{3}x^2}{x} + 6x = \dfrac{12}{x} + \left(6 - \sqrt{3}\right)x$ *	A1*
	The frame goes round the outside only, so DC is not part of P. Including it gives $8x$ instead of $6x$.	(3)
(b)	$\dfrac{dP}{dx} = -\dfrac{12}{x^2} + 6 - \sqrt{3}$	M1
	Sets $\dfrac{dP}{dx} = 0$: $x^2 = \dfrac{12}{6 - \sqrt{3}}$, so $x = 1.68\ldots$	dM1
	$P = 14.3$ m	A1
	Giving $x = 1.68$ as the answer, without finding P, loses the A1.	(3)
(c)	$\dfrac{d^2P}{dx^2} = \dfrac{24}{x^3}$	M1
	> 0 when $x = 1.68$ (indeed for all $x > 0$), so P is a minimum	A1
		(2)
		Total for Question 8: 8 marks

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Question 9

7 marks

Part	Scheme	Marks
(a)	$x^2 - 4x + 6 = 2 + 4x - x^2$	M1
	$2x^2 - 8x + 4 = 0$, so $x^2 - 4x + 2 = 0$	A1
	$(x-2)^2 = 2$ or $x = \frac{4 \pm \sqrt{8}}{2}$, so $x = 2 \pm \sqrt{2}$ *	A1*
		(3)
(b)	Area = $\int (-2x^2 + 8x - 4)\,dx$ between $2 - \sqrt{2}$ and $2 + \sqrt{2}$ (top curve minus bottom curve)	M1
	$\left[-\frac{2}{3}x^3 + 4x^2 - 4x\right]$	A1
	Substitutes both limits and subtracts: $\left(\frac{8}{3} + \frac{8}{3}\sqrt{2}\right) - \left(\frac{8}{3} - \frac{8}{3}\sqrt{2}\right)$	dM1
	$\frac{16\sqrt{2}}{3}$	A1
	Useful: $(2 \pm \sqrt{2})^2 = 6 \pm 4\sqrt{2}$ *and* $(2 \pm \sqrt{2})^3 = 20 \pm 14\sqrt{2}$. *A decimal answer (7.54) from a calculator scores M1 A1 at most.*	
		(4)
		Total for Question 9: 7 marks

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Question 10

9 marks

Part	Scheme	Marks
(a)	$\frac{dy}{dx} = 2 - 16x^{-3}$	M1
	Sets $\frac{dy}{dx} = 0$: $x^3 = 8$, giving $\boxed{(2, 6)}$	A1
	$\frac{d^2y}{dx^2} = 48x^{-4}$, and finds its value or sign at $x = 2$	M1
	$= 3 > 0$, so $\boxed{\text{minimum}}$	A1
	Differentiating $\frac{8}{x^2}$ as $\frac{8}{2x}$ is the most common error. Write it as $8x^{-2}$ first.	(4)
(b)	$h = 0.5$, with y values $10,\ 6.5556,\ 6,\ 6.28,\ 6.8889$	B1
	$\frac{1}{2} \times 0.5 \times [10 + 6.8889 + 2(6.5556 + 6 + 6.28)]$	M1
	$\boxed{13.6}$ (13.64)	A1
	Using $h = \frac{2}{5}$ (five ordinates, not four strips) and a missing outer bracket are the common errors.	(3)
(c)	$\int (2x + 8x^{-2})\,dx = \left[x^2 - 8x^{-1}\right]_1^3$	M1
	$\left(9 - \frac{8}{3}\right) - (1 - 8) = \boxed{\frac{40}{3}}$	A1
	The trapezium rule overestimates here because the curve bends upwards (it is convex) between $x = 1$ and $x = 3$.	(2)
		Total for Question 10: 9 marks

TOTAL FOR PAPER: 75 MARKS

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